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Study of Analytical Buckling Models for Axial Load Calculation in Tubing Strings

Gabriele K. M. Lins[1], Otávio B. de A. Rodrigues[1], Catarina N. A. Fernandes[1], Thiago B. da Silva[1], William W. M. Lira[1]

[1]*Laboratory of Scientific Computing and Visualization, Federal University of Alagoas*
Av. Lourival Melo Mota s/n, Cidade Universitária, CEP: 57072-900, Maceió/AL, Brazil
gabriele.lins@ctec.ufal.br, otavio.rodrigues@lccv.ufal.br,catarina@lccv.ufal.br, thiago.barbosa@lccv.ufal.br
william@lccv.ufal.br

Abstract. This work proposes to analyze and implement analytical formulations for the calculation of axial loads with buckling in tubing strings in oil wells. These strings are subjected to different operational conditions throughout the well's lifespan. Accurately predicting the axial loads resulting from these operations is essential to prevent accidents and ensure structural integrity. Such loads arise from pressure and temperature variations, self-weight, and buckling—the latter being the main focus of this work. To achieve the proposed objective, the adopted methodology is divided into six main stages: (i) literature review on oil wells; (ii) study of analytical solutions for axial load prediction in tubular strings; (iii) investigation and inclusion of buckling effects in analytical equations; (iv) definition of a vertical well configuration for implementation of the studied equations; (v) Well modeling using the SCORE tool; and (vi) comparative analysis of the obtained results. The analytical solution showed excellent agreement with the reference numerical tool, allowing for the quantification of the buckled length and its impact on the axial forces. The next step of this work involves the numerical modeling of buckling and axial loads in tubing strings, contributing to advancements safe design, especially in environments with complex geometries and more severe operational conditions.

Keywords: Tubing strings; Axial loads; Buckling; Analytical solution; SCORE.

1 Introduction

Tubing strings play a crucial role in oil well operations, as they are responsible for transporting fluids between the reservoir and the surface, in addition to enabling interventions throughout the well's lifespan [1]. In addition to their operational function, these strings act as primary containment barriers and must maintain their structural integrity even under severe pressure, temperature, and mechanical loading conditions, preventing failures that could compromise safety and production [2].

Among the various loads acting on these systems, axial loads stand out for their importance in the design of production and injection tubing, as they result from factors such as self-weight, thermal and pressure variations, and structural instability phenomena such as buckling. Although buckling in tubular elements is a well-studied and consolidated topic, there is still room for improving analytical models that more accurately represent the specific operating conditions of production and injection tubing. These conditions include characteristic geometries, particular combinations of loads, and interactions between mechanical phenomena, whose proper consideration can provide more robust support for assessing structural integrity and for developing safer and more efficient designs.

Although numerical tools are widely used to simulate structural behavior, analytical models remain essential in the early stages of design. They allow quick analyses, provide physical insight into the phenomena, and serve as independent verification of results obtained by numerical methods. In this context, this work proposes the development and implementation of analytical formulations for calculating axial loads in COP/COI, considering the effect of buckling, contributing to safer designs and aligned with the practical demands of the industry. Thus, it aims to provide results that enhance design and operational criteria for these strings, increasing the structural reliability and operational safety of oil wells under different loading conditions.

2 Methodology

The proposed methodology is divided into six macro-steps, as illustrated in Fig. 1.

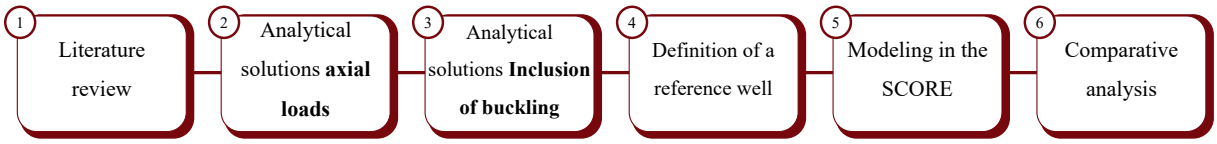


Figure 1. Main stages of the adopted methodology.

The first stage consisted of a comprehensive literature review, grounded in references such as Thomas [1] and Rodrigues [3], to understand the operational and structural functions of tubing strings, as well as the acting loads.

In the second stage, consolidated analytical formulations for estimating axial loads in vertical strings were analyzed, considering self-weight, internal and external pressures, and thermal variations, based on Bellarby [2].

The third stage incorporated the buckling effect, representing structural instability under compressive loads, following the methodology of Souza [4].

In the fourth stage, a reference well was defined, adapted from Bellarby [2], with representative geometric data, mechanical properties, and operational conditions.

Finally, in the last two stages, the well was modeled in the SCORE (Casing Reliability System) Simulator [5], reproducing the analytical model conditions, and a comparison between numerical and analytical results was performed, assessing agreement, discrepancies, and the implications of buckling in the design of tubing.

3 Analytical formulations for axial load calculation in tubing strings

The calculation of axial loads in production and injection tubing can be performed using different approaches, depending on the loading conditions and the structural stability of the string. In the absence of buckling, the analysis considers only the effects of self-weight, thermal variations, and internal and external pressures, allowing the direct application of the differential equations of equilibrium, kinematics, and linear constitutive laws. Conversely, when the string is subjected to compressive forces capable of inducing instability, it becomes essential to incorporate the effects of buckling, which can significantly alter the distribution of loads and the displacements along the structure.

3.1 Axial loads without buckling

In this study, the determination of the axial force followed the formulation described by Bellarby [2], which simultaneously incorporates the effects of temperature, ballooning, piston effect, and self-weight.

The governing differential equilibrium equation for the present problem is derived analogously to that used in the analysis of bars subjected to axial loading, as shown in eq. (1).

$$\frac{dF_a}{dz} = -W_a, \tag{1}$$

where z is the axial coordinate of the string, with the positive axis oriented downward (from top to bottom), F_a is the axial force, and W_a is the distributed weight of the string.

Another relevant result that can be obtained analytically is the total change in the string length between the initial and final operating conditions. This variation (ΔL) is composed of three components: the elastic displacement (eq. (2)), the thermal displacement (eq. (3)), and the displacement associated with the ballooning effect (eq. (4)).

$$\Delta L_1 = \int_0^L \frac{\Delta F_a}{EA}\, dz, \tag{2}$$

$$\Delta L_2 = \int_0^L \alpha \, \Delta T(z) \, dz, \tag{3}$$

$$\Delta L_3 = \int_0^L \frac{-2\nu \left(\Delta p_i A_i - \Delta p_e A_e \right)}{EA} \, dz, \tag{4}$$

where ΔF_a is the change in axial force between the final and initial operating conditions, A is the steel cross-sectional area of the string, A_i and A_e are the internal and external cross-sectional areas, ΔT is the temperature change, L is the total length of the string, E is the modulus of elasticity, ν is Poisson's ratio, α is the thermal expansion coefficient, and Δp_i and Δp_e are the changes in internal and external pressures of the string.

This result is particularly important because, considering that the string is free at its lower end, the resulting length is used, for example, to size the hydraulic seals in the packer.

3.2 Axial loads with buckling

The formulation used for the calculation of axial loads, including the effects of buckling, is based on the methodology described by Souza [4], with minor modifications to account for production and injection tubing.

The calculation of the axial load at the base of the string is carried out as an iterative process, as illustrated in the flowchart of Fig. 2. Initially, the base reaction ΔF is determined without considering buckling, based on the compatibility between the displacement due to the applied loads (ΔL_{load}) and the elastic displacement of the string (ΔL_F). Next, the effective force at the base of the string is calculated using the reaction ΔF, from which the final effective force $F_{ef,bc}$ is obtained after applying the pressure and temperature increments. If $F_{ef,bc} > 0$, it is concluded that no buckling occurs and the initially obtained ΔF value is maintained. If $F_{ef,bc} < 0$, buckling occurs and the additional shortening ΔL_{fl} is included, recalculating ΔF and $F_{ef,bc}$. The process is repeated until the values of ΔF and $F_{ef,bc}$ converge, ensuring the proper consideration of buckling in the axial load calculation.

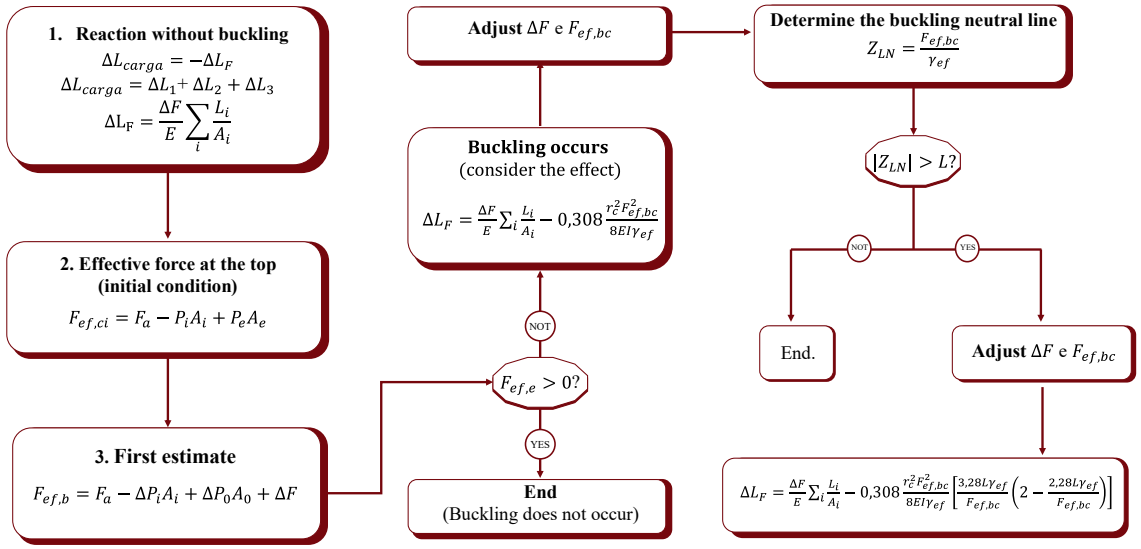


Figure 2. Flowchart of the iterative procedure for calculating axial load with buckling consideration.

Most practical problems involving buckling fall within the second iteration of the process, that is, in the first calculation considering buckling.

4 Case study

The problem analyzed is based on a study proposed by Bellarby (2009), which addresses a production and injection tubing in a vertical well operating under injection conditions. The string, with a total depth of 9,000 ft, is

composed of two distinct sections, and the injected fluid is at a high temperature. Figure 3 shows the geometry of the string under study, as well as the temperature profile along the depth, both in the initial condition and during the injection operation.

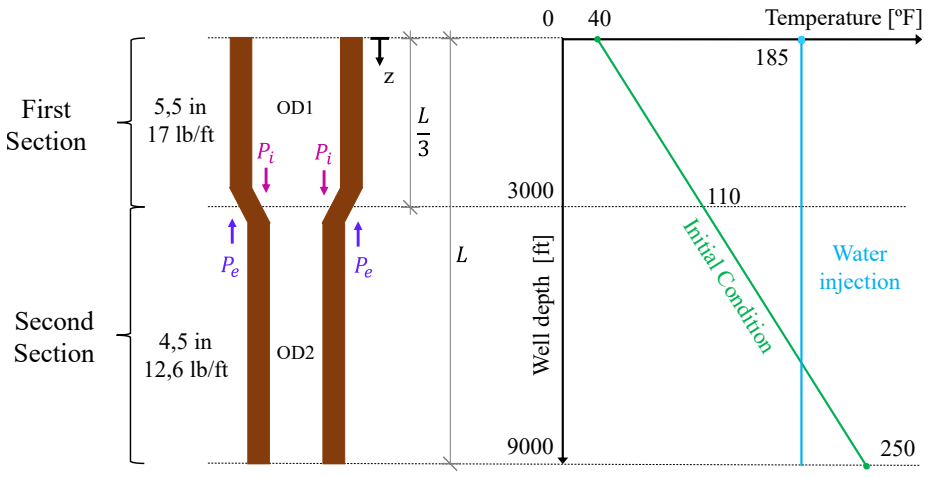


Figure 3. Well column geometry and temperature profile during initial conditions and injection operation.

As the string has two cross-sections, it is necessary to define two differential equilibrium equations, namely,

$$\frac{dF_{a1}}{dz} = -W_{a1} \tag{5}$$

$$\frac{dF_{a2}}{dz} = -W_{a2} \tag{6}$$

where F_{a1} and F_{a2} are the axial forces, and W_{a1} and W_{a2} are the distributed weights corresponding to the first and second sections of the string, respectively.

In this problem, the boundary conditions are governed by the piston forces that arise at the change in diameter and at the bottom of the string, denoted by F_{p1} and F_{p2}, respectively. These conditions are expressed as:

$$F_{a1}\left(\frac{L}{3}\right) - F_{a2}\left(\frac{L}{3}\right) = F_{p1}, \tag{7}$$

$$F_{a2}(L) = F_{p2}, \tag{8}$$

where L is the total string length. The piston effect originates from abrupt variations in the cross-sectional area, which produce discontinuities in the internal (P_i) and external (P_e) pressures at the transition zones (Fig. 3), generating localized axial forces. According to Rodrigues [3], these forces are calculated as:

$$F_{p1} = P_e\left(\frac{L}{3}\right)(A_{e2} - A_{e1}) - P_i\left(\frac{L}{3}\right)(A_{i2} - A_{i1}), \tag{9}$$

$$F_{p2} = -P_e(L)A_{e2} + P_i(L)A_{i2}, \tag{10}$$

where A_{i1} and A_{e1} are the internal and external areas of the first section, and A_{i2} and A_{e2} are the corresponding areas of the second section.

With these parameters defined, the axial force distribution $F_a(z)$ is given by:

$$F_a(z) = \begin{cases} F_{a1}(z), & z < \frac{L}{3}, \\ F_{a2}(z), & z \geq \frac{L}{3}, \end{cases} \tag{11}$$

$$F_{a1}(z) = -W_{a1} z - \frac{1}{3} W_{a1} L + \frac{2}{3} W_{a2} L + F_{p1} + F_{p2}, \tag{12}$$

$$F_{a2}(z) = W_{a1} L - W_{a2} z + F_{p2}. \tag{13}$$

Once F_{p1} and F_{p2} are determined from the pressure values for the initial and final operating conditions, the complete axial force profile for each scenario can be directly obtained.

The graph in Figure 4 presents the axial force along the depth, comparing the results obtained through the analytical approach and the SCORE simulator, for both the final condition with buckling and without buckling. In addition, the graph also shows the buckling force along the depth.

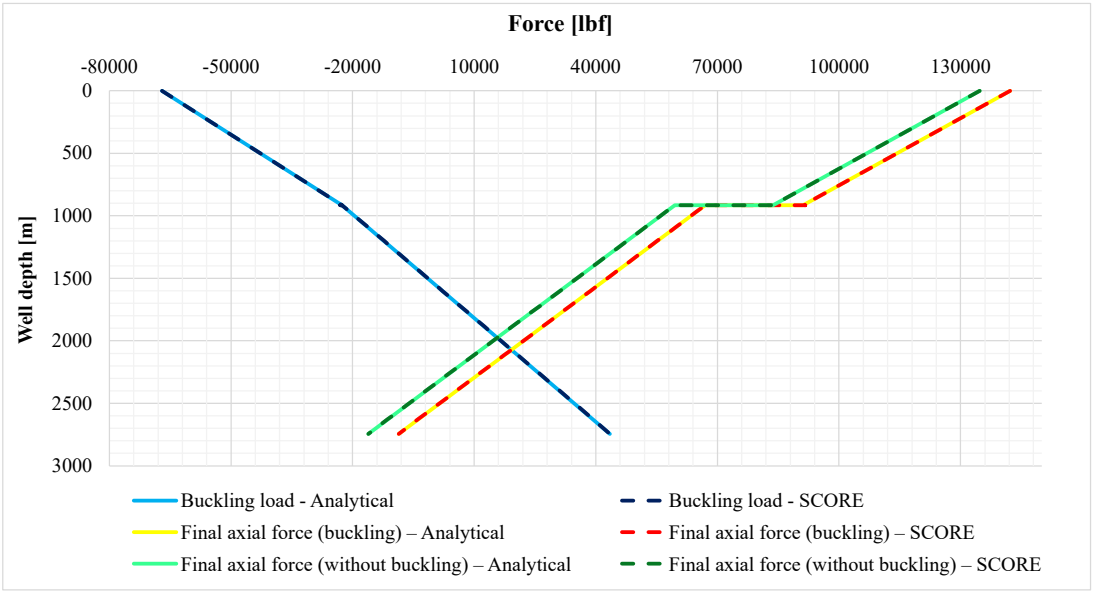


Figure 4. Buckling force and axial force distribution along the well depth with and without buckling.

The comparison between the results obtained from the SCORE simulator and the analytical formulation revealed a high level of agreement under all evaluated conditions. In the case with buckling, at the top of the string (MD = 0 m), the relative difference was only -0.0009%, corresponding to 142,175.81 lbf in the SCORE results and 142,177.07 lbf in the analytical solution. Similar results were observed at other depths, with percentage errors consistently below 0.01%, evidencing the consistency and convergence between the approaches.

For the case without buckling, a similar behavior was observed: at the top of the string (MD = 0 m), the relative difference was -0.0005%, with values of 134,685.56 lbf (SCORE) and 134,686.18 lbf (analytical). At all analyzed depths, the percentage error remained below 0.002%, confirming the accuracy of the analytical formulation compared to the numerical results.

The comparative analysis between the scenarios with and without buckling, using the analytical solution, highlights the direct impact of this phenomenon on the axial force along the string. At the top (MD = 0 m), the difference between the two cases is approximately 7,490.89 lbf, representing the additional reaction generated by buckling, which acts to partially relieve compression. This effect persists along the entire depth, reducing the magnitude of the compressive forces and, in some cases, reversing the sign to tension, as observed at 2,743.2 m, where the axial force without buckling (-16,077.76 lbf) changes to -8,586.87 lbf when buckling is considered.

Overall, the results demonstrate that the analytical formulation can reproduce, with high accuracy, the values obtained from the SCORE simulator, while also enabling a precise quantification of the buckling effect on the force distribution. Therefore, it is established as a reliable tool for assessing the structural integrity of production and injection strings under different operating conditions.

5 Conclusions

Thus, the results confirm that the analytical formulation not only reproduces with high accuracy the values obtained from SCORE, but also consistently quantifies the buckling effect, establishing itself as a reliable tool for assessing the structural integrity of production and injection strings. This finding is particularly relevant, as it demonstrates that the proposed analytical models can serve as a solid basis for preliminary design analyses, reducing the exclusive reliance on more complex numerical simulations.

It was observed that, at shallower depths, buckling tends to increase the axial force in the string, suggesting a favorable redistribution of stresses under certain conditions. In contrast, at greater depths, the effect is reversed, significantly compromising the load-bearing capacity, with expressive reductions in axial force, as observed at 2,743.2 m. This dual behavior highlights the need to incorporate buckling into all stages of the operational cycle, since its impacts are not uniform but rather strongly dependent on loading conditions and depth.

Therefore, beyond attesting to the accuracy of the analytical formulation compared to SCORE, the results underscore the importance of considering buckling as an essential aspect in the structural analysis of production and injection strings. Such an approach contributes not only to enhancing the reliability of design but also to improving the operational safety of wells, mitigating the risk of structural failures that could compromise both productivity and system integrity.

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