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Numerical Modeling and Simulation of Wind-Induced Water Elevation By Tropical Cyclones Using Shallow Water Equations

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Abstract. Modeling the interaction between storms such as tropical cyclones over the ocean is highly relevant in ocean, coastal, and offshore engineering, as it supports forecasting of loads and risks, along with the mitigation of coastal damage from these extreme events. This study investigates sea surface elevation caused by the formation of tropical cyclones, using a fourth-order explicit Runge-Kutta method to solve the Shallow Water Equations. Cyclone-induced winds were simulated using an analytical model based on meteorological parameters and their interaction with water bodies, modeled as tangential stresses applied to the free surface. Simulations were first conducted for a stationary cyclone center over constant bathymetry. The effects of model parameters describing the tropical cyclone were investigated and discussed. As a result, it was found that the most significant factors influencing water elevation are the radius at which the cyclone's maximum wind speed occurs and the pressure difference between the storm center and its periphery, leading to considerable water elevation in the most severe cases. In contrast, the wind inflow angle of the cyclone had little impact on water elevation. Finally, an additional investigation is presented, considering a moving storm scenario to illustrate the potential applications of the proposed model.

Keywords: Tropical cyclones, Shallow water equations, Computational fluid dynamics

1 Introduction

In meteorology, understanding the dynamic behavior of the sea surface is essential for forecasting extreme weather events, such as storms and tropical cyclones (TCs). In this context, computational modeling emerges as a valuable tool analysis and prediction of potential risks and impacts of these events. Such models support more effective emergency planning and evacuation strategies, by enabling the identification of vulnerable areas in advance, contributing to the reduction of material damage and human losses [1].

When the fluid depth is much smaller than the horizontal dimensions, the Navier-Stokes Equations can be simplified into a form that describes a free-surface incompressible flow [2]. Under these assumptions, the Shallow Water Equations (SWE) are derived, offering a widely applicable framework, especially in ocean hydrodinamics and large-scale atmospheric flows. This formulation significantly reduces the computational cost to simulate the oceanic movement, as it solves a sistem of Partial Differential Equations (PDEs) in which the vertical dimension (fluid depth) is treated implicitly. In this context, the present work develops a numerical model to simulate the sea surface elevation induced by tropical ciclones, solving the Shallow Water Equations with an explicit fourth order Runge-Kutta scheme. The cyclone is represented through an analytic wind field model, as studied by [1], and incorporated into the SWE as a drag force acting on the free surface.

A parametric study of sea surface elevation was investigated on a simplified domain with uniform bathymetry to assess the influence of key variables in the tropical cyclone modeling. The proposed methodology incorporates cyclone-induced effects on the elevation field, providing an efficient and accessible numerical approach for evaluating ocean hydrodynamics. This framework can support shoreline vulnerability analysis, urban planning and

early warning sistems in risk-prone areas. Unlike studies involving complex bathymetry and storm behavior, this simplified yet representative case enables a clearer investigation of individual model parameters. Variables such as bathymetry depth, wind inflow angle, cyclone's central pressure and radius of maximum wind velocity were analyzed. Coefficients of variation along time were computed to quantify the influence of each parameter, providing a deeper understanding of their role in cyclone-induced ocean dynamics.

2 Methodology

2.1 Shallow Water Equations

The Shallow Water Equations (SWE) is a set of Partial Differential Equations (PDE) obtained from the integration of the Navier-Stokes Equations along the depth of the fluid, which are derived with the Mean Value Theorem and the Leibniz Formula [3]. In this way, the resulting model describes an incompressible flow with a free surface, considering that the horizontal dimensions are much greater than the vertical dimension (depth). By solving this system of PDEs in a three-dimensional cartesian space (x_1, x_2, x_3) it is possible to obtain the temporal evolution of the free surface elevation, together with the components of the horizontal mean velocity of the fluid along directions x_1 and x_2.

First, a horizontal reference datum plane is defined as the mean level of the sea free surface. From this plane, three parameters are defined: H, which represents the time-independent ocean bed depth; η, a positive or negative value representing the sea level elevation, relative to the reference plane; and h, which is the total depth of the fluid. As shown in Fig. 1, these variables are related through the expression $h(x_1, x_2, t) = H(x_1, x_2) + \eta(x_1, x_2, t)$.

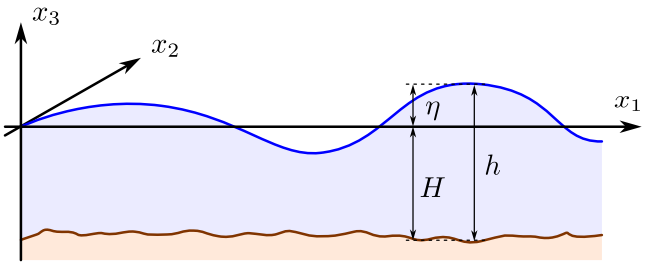


Figure 1. The shallow water domain of study

The velocity components are written as $u_i = hU_i$ $(i = 1, 2)$, i. e. a product between the horizontal mean velocities in the i-th direction U_i and the total depth h. The bathymetry H is considered constant over time. Therefore, the continuity equation is given by the following expression:

$$\frac{\partial \eta}{\partial t} + \frac{\partial \left(hU_i\right)}{\partial x_i} = 0. \tag{1}$$

Similarly, by substituting $u_i = hU_i$ into the Navier-Stokes Equations leads to [3]:

$$\frac{\partial \left(hU_i\right)}{\partial t} + \frac{\partial \left(hU_iU_j\right)}{\partial x_j} + gh\frac{\partial \eta}{\partial x_i} - \frac{1}{\rho}\left(\tau_{3i}^S - \tau_{3i}^B\right) + \frac{h}{\rho}\frac{\partial p_{atm}}{\partial x_i} = 0 \tag{2}$$

where g is the acceleration due to gravity (9.81 m/s²), ρ is the water density (1000 kg/m³), p_{atm} is the atmospheric pressure field, τ_{3i}^S represents the i-th component of the wind stress at the free surface and τ_{3i}^B denotes bottom friction stress exerted by the seabed. In eq. (2), turbulence effects and the Coriolis force were not considered. Additionally, bottom friction was modeled using the Manning formulation, which is widely adopted in the literature [1, 4]. In this approach, the value of τ_{3i}^B is given by:

$$\tau_{3i}^B = \frac{\rho g n^2}{h^{1/3}} \left\|\boldsymbol{U}\right\| U_i \tag{3}$$

where n is the Manning coefficient, equal to 0.025 in this study. The Shallow Water Equations are then written as:

$$\frac{\partial (hU_i)}{\partial t} + \frac{\partial (hU_iU_j)}{\partial x_j} + gh\frac{\partial \eta}{\partial x_i} - \frac{\tau_{3i}^S}{\rho} + \frac{gn^2}{h^{1/3}} \|\boldsymbol{U}\| U_i + \frac{h}{\rho}\frac{\partial p_{atm}}{\partial x_i} = 0. \tag{4}$$

The system of PDE was solved numerically with a fourth order explicit Runge-Kutta scheme. This method has been extensively explored by Ern and Guermond [5] for its accuracy and efficiency.

2.2 Wind stress on SWE

The wind interacts dynamically with the oceanic flow trough tangential stress exerted on the sea surface. Although this stress is typically of low magnitude, its cumulative effect over a large body of water can be critical [2]. Various approaches have been proposed in the literature to estimate tangential stress. Among these, the model developed by Wu [6] remains the most widely adopted. In this model, the horizontal components of the surface stress τ_{3i}^S $(i = 1, 2)$ are calculated on the mean surface of the water and given by:

$$\tau_{3i}^S = \rho_{ar} \cdot C_d \cdot \|\boldsymbol{V}_r\| \cdot V_{r,i} \tag{5}$$

where $V_{r,i}$ is the i-th wind velocity component at 10 m height above the mean surface, ρ_{air} is the air density (1.22 kg/m³) and C_d is the drag coefficient, considered constant in this study and set to 0.0026 [7]. To investigate the influence of tropical cyclones in ocean surface elevations it is essential to accurately characterize the wind-induced velocity and pressure fields. Several models are available in the literture for this purpose. In the present study, the model proposed by Xinian et al. [7] was adopted, which represents an improvement of Fujita's Model [8]. In this formulation, the moving position of the TC center at time step $(k+1)\Delta t$, located by coordinate components $x_{c,i}$, is determined by:

$$x_{c,i}^{k+1} = x_{c,i}^{k} + V_{c,i} \cdot \Delta t \tag{6}$$

with $V_{c,i}$ denoting the i-th component of the translational velocity of the TC center. The pressure and wind-induced velocity fields associated with TCs are typically described by a radial coordinate system. Accordingly, the radial distance r between the storm center and each grid node is defined by the following expression:

$$r = \sqrt{(x_1 - x_{c,1})^2 + (x_2 - x_{c,2})^2}. \tag{7}$$

Based on eq. (7) the radial pressure field can be formulated as follows:

$$P(r) = P_\infty - \frac{\Delta P}{\sqrt{1 + 2\,(r/R)^2}}. \tag{8}$$

In this expression, P_∞ represents the atmospheric pressure in a region far from the TC center, equals to 101325 Pa in the present study. The term $\Delta P = P_\infty - P_c$ denotes the pressure drop between P_∞ and the pressure at the storm center P_c. Tipically, P_c ranges between 87000 Pa in intense events and up to 101000 Pa in weaker storms [9]. The parameter R corresponds to the radius of maximum wind (RMW), which is the distance from the cyclone center where the tangential wind speed reaches its peak. The velocity field is defined with the aid of a gradient wind term V_g, computed as:

$$V_g\,(r) = \begin{cases} -\frac{f}{2} + \sqrt{\frac{f^2}{4} + \frac{2\Delta P}{\rho_{ar} R^2}\left[1 + 2\left(\frac{r^2}{R^2}\right)\right]^{-3/2}} & \text{if } 0 \le r \le 2R \\ -\frac{f}{2} + \sqrt{\frac{f^2}{4} + \frac{\Delta P}{\rho_{ar}(1+r/R)^2 Rr}} & \text{if } r > 2R \end{cases} \tag{9}$$

where $f = 2\omega \sin\phi$ is the Coriolis parameter, which takes the value 8.3304×10^{-5} s^{-1} for a latitude ϕ of 39 degrees and an angular velocity ω of 7.292×10^{-5} s^{-1}. This value was adopted to enable an order-of-magnitude comparison with results presented by Sun et al. [1] and Xinian et al. [7]. All other variables are expressed in SI units. With this, the values of V_g are combined with the translational velocity components of the cyclone center $V_{c.i}$, to yield the velocity field $V_{r,i}$ as follows:

$$V_{r,1} = C_1 V_{c,1} \exp\left(-\frac{\pi}{4}\frac{|r-R|}{R}\right) - C_2 V_g \left[(x_1 - x_{c,1})\sin\theta + (x_2 - x_{c,2})\cos\theta\right] \tag{10}$$

$$V_{r,2} = C_1 V_{c,2} \exp\left(-\frac{\pi}{4}\frac{|r-R|}{R}\right) + C_2 V_g \left[(x_1 - x_{c,1})\cos\theta - (x_2 - x_{c,2})\sin\theta\right]. \tag{11}$$

Here, θ represents the wind inflow angle, which influences the TC's shape. C_1 and C_2 are constants equal to, respectively, 1.0 and 0.8 [7]. By substituting eq. (10) and eq. (11) into eq. (5), the distribution of wind-induced shear stresses on the sea surface is obtained. Finally, incorporating this stress distribution into eq. (4) enables the solution of the PDE sistem given by eq. (1) and eq. (4), yielding the free surface elevation field $\eta\,(x_1, x_2, t)$ and the depth-averaged horizontal velocity components $U_i\,(x_1, x_2, t)$ of the fluid.

3 Results and Discussion

The study domain covers an area of $500 \times 500km$, discretized with an uniform grid containing 201 nodes in x_1 and x_2 directions. This resolution was selected based on a mesh convergence study performed with grids of 101×101 and 151×151 nodes. A parametric assesment was performed to evaluate the impact of each relevant variable on sea surface elevation, using a domain with uniform bathymetry. The study individually varied parameters including bathymetry depth H, pressure in the storm center P_c, radius of maximum wind R and inflow wind angle θ. In each simulation a time step of 5 seconds is adopted, covering the simulation of a period of 100 hours. A stationary typhoon position is considered at the center domain. The mesh and time step parameters are selected to satisfy the Courant-Friedrichs-Lewy (CFL) condition, which is a necessary requirement for ensuring the numerical stability of explicit schemes used in the solution of hyperbolic partial differential equations. Orlanski open boundary conditions are applied to maintain continuity at the open sea surface [10]. A total of twelve simulations were carried out, with three simulations performed for each of the four parameters investigated in this study. The corresponding peak values observed at the center of the domain, along with the times at which these peaks occur, are summarized in Table 1.

Table 1. Peak values

Variable	Simulation	H [m]	P_c [Pa]	R [m]	θ [°]	Peak value at center [m]	Peak time [h]
H	(01)	2000	92500	50000	0	1.4513	76.88
	(02)	2500	92500	50000	0	1.4408	68.76
	(03)	3000	92500	50000	0	1.4368	62.77
θ	(04)	2000	92500	50000	30	1.4675	76.87
	(05)	2000	92500	50000	45	1.4879	76.87
	(06)	2000	92500	50000	60	1.5055	76.87
R	(07)	2000	92500	20000	0	1.2325	76.91
	(08)	2000	92500	30000	0	1.2746	76.90
	(09)	2000	92500	40000	0	1.4023	76.88
P_c	(10)	2000	95000	50000	0	1.0335	76.88
	(11)	2000	97500	50000	0	0.6219	76.88
	(12)	2000	100000	50000	0	0.2145	76.88

The presented results demonstrate that increasing the bathymetric depth H induces a slight reduction in sea surface elevation, with the peak occurring earlier in time. Variations in the wind inflow angle θ, on the other side, exhibit negligible influence on the timing of peak elevation. However, the magnitude of η causes also a small increase when θ approaches 90 degrees, suggesting a modest stronger hydrodynamic response under almost perpendicular wind forcing. As expected, an increase in R leads to a broader spatial impact of the tropical cyclone, resulting in a relevant higher sea surface elevations. The most expressive increase in that case occurs between simulations (08) and (09), highlighting the sensitivity of η to R. Finally, consisted with theoretical expectations, a smaller pressure deficit $\Delta P = P_\infty - P_c$ corresponds to a weaker storm intensity. When the central pressure P_c approaches the ambient pressure $P_\infty = 101325$ Pa, the storm-induced surge reduces, confirming the direct relationship between pressure differential and event severity. Furthermore, the pressure drop was the parameter that exhibited the greatest increase in magnitude, with an amplitude of 0.819 m between simulations (10) and (12), making it the most significant variable in this study.

To illustrate the results more clearly, simulations (06) and (12) were selected for detailed examination, corresponding to the maximum and minimum peak values, respectively. For both cases, a series of elevation profiles along the horizontal centerline was plotted at 20-hour intervals. A velocity field at the final of each simulation was also generated. The resulting plots are presented in Fig. 2. As discussed by [1] and confirmed here,variations in the wind inflow angle have minimal effect on the shape of the elevation profile, influencing only the magnitude of η. In contrast, the velocity field is strongly affected by the θ, becoming increasingly distorted as the angle increases. Finally, the pressure drop ΔP continues to exhibit a significant influence on the elevation profile.

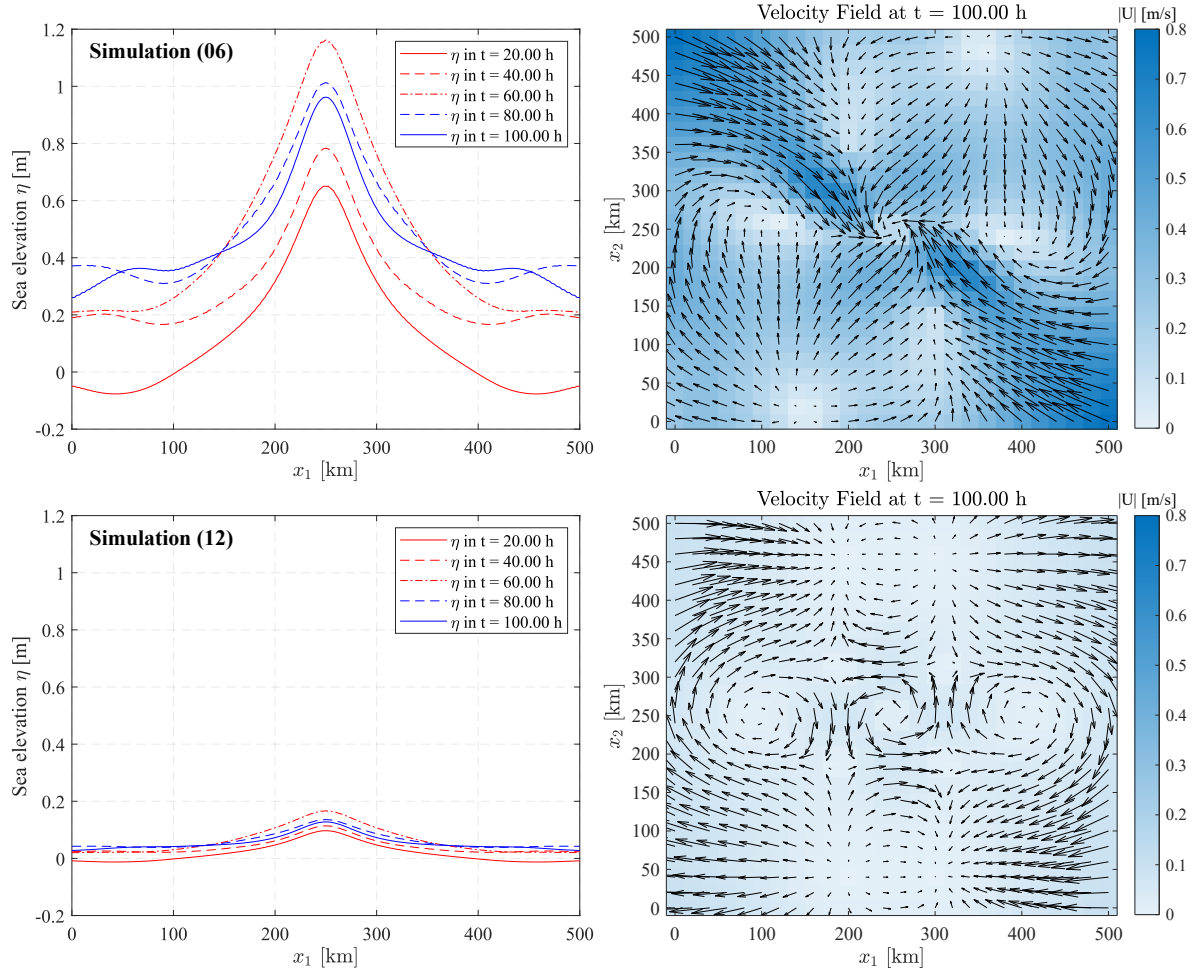


Figure 2. Water elevation and velocity field for each simulation set

To quantify the relative influence of each parameter, Coefficients of Variations (CVs) of the water elevation were computed for H, P_c, R and θ (denoted as CV_H, CV_{P_c}, CV_R and CV_θ, respectively). These coefficients were defined as the ratio of the instantaneous mean of water elevation η across simulations to the instantaneous

standard deviation over time. Figure 3 presents the temporal evolution of these CVs. To emphasize their long-term trends, a moving average filter with a 25-hour window was applied. This smoothing technique effectively reduces high-frequency noise while preserving the underlying signal, allowing a clearer interpretation of the dominant variability patterns.

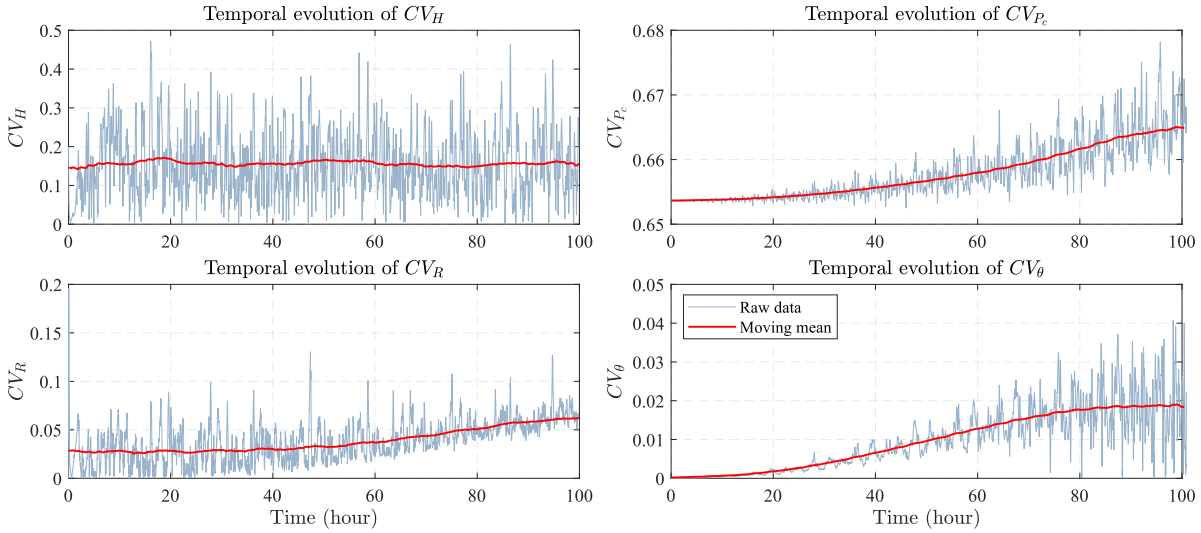


Figure 3. Sensitivity analysis for the uniform bathymetry domain

The time series analysis indicates that variations in bathymetric depth result in a relatively constant 15% change in sea surface elevation throughout the simulation. The variations of the pressure drop had the greatest impact, with values ranging from 65 to 68%, and increasing over time. A similar upward trend, though less pronounced (2 to 3%) was observed in the moving average curves of CV_R and CV_θ over the 100-hour period. These gradual increases likely reflect nonlinear interactions between variables and growing divergence in storm structure, which enhance the variability in sea surface elevation as the simulation progresses. This discussion emphasizes the most influential parameters P_c and R in TC modeling, while also recognizing that H and θ has a non-negligible effect on η.

The effect of the TC's translational velocity was evaluated by simulating its northward movement along the vertical centerline of the domain, with $V_{c,2} = 15$ km/h. The parameters H, θ, R P_c were similar to the simulation case (01) previously presented in Table 1. Elevation maps at 4 and 16 hours are presented in Fig. 4. In this scenario, the maximum sea surface elevation reached 1.2426 m, located at the center of the TC, as expected due to the pronounced inverse barometer effect under this isolated condition. This result, along with other presented in this study, aligns well with findings from the literature, with studies as Sun et al. [1] and Xinian et al. [7] reporting elevations of the same order of magnitude.

4 Conclusions

This work investigated the impact of tropical cyclones over sea surface elevation. To achieve this, a fourth-order explicit Runge-Kutta scheme was employed to solve the Shallow Water Equations, incorporating seabed friction, wind-induced drag stresses from the cyclone and spatial variations in atmospheric pressure. This efficient and accessible modeling approach enabled a parametric study within a simplified domain. The results indicated that the wind inflow angle had minimal influence on the sea surface elevation, but significantly affected the distortion of the velocity field. Moreover, parameter H exhibited a non-negligitible impact, with a coefficient of variation of approximately 15%. Finally, the pressure drop and the radius of maximum wind were identified as the most influential parameters, yielding sea surface elevation amplitudes of 0.819 m in the most extreme scenario, corresponding to a pressure drop of 92500 Pa and radius of maximum wind of 50 km. A moving cyclone was also simulated using the parameters from simulation case (01) and a constant translational velocity of 15 km/h along x_2 direction, illustrating the inverse barometer effect induced by TCs.

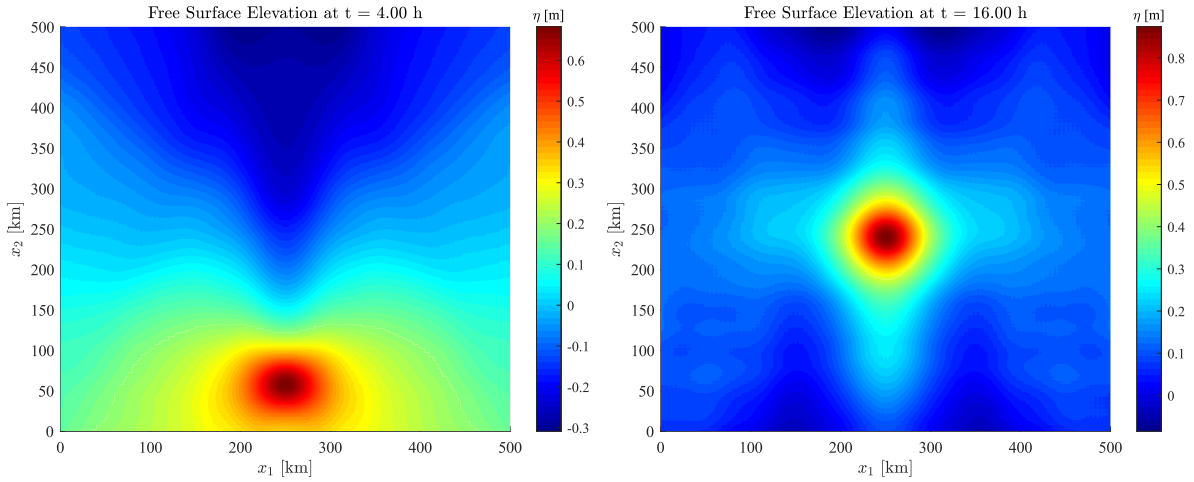


Figure 4. Effect of a moving tropical cyclone center on sea surface elevation

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