Multi-objective Optimization for Bridge and Viaduct Design

Eduardo V. W. Trentini[1], Guilherme A. Parsekian[2], Túlio N. Bittencourt[3]

[1]*Faculty of Civil Engineering, Federal University of Uberlândia*
Av. João Naves de Ávila, 2121 – Bloco 1Y, 38408-100, Uberlândia, MG, Brazil
etrentini@ufu.br
[2]*Department of Civil Engineering, Federal University of São Carlos*
Rodovia Washington Luís (SP-310), km 235, 13565-905, São Carlos, SP, Brazil
parsekian@ufscar.br
[3]*Department of Structural and Geotechnical Engineering, University of São Paulo*
Av. Prof. Almeida Prado, Travessa 2, No. 271, 05508-900, São Paulo, SP, Brazil
tbitten@usp.br

Abstract. This study explores the multi-objective optimization of bridge and viaduct design, considering three objectives: minimizing construction costs, reducing environmental impacts through CO_2 emission reductions, and maximizing structural durability. Viaducts play a critical role in Brazil's transportation infrastructure, demanding efficient and sustainable design. Traditional approaches, based mainly on empirical knowledge, often lead to suboptimal results. To address this, three metaheuristic algorithms were evaluated: Multiple-objective Particle Swarm Optimization (MOPSO), Nondominated Sorting Genetic Algorithm II (NSGA-II), and Strength Pareto Evolutionary Algorithm 2 (SPEA2). MOPSO outperformed the others in terms of hypervolume and solution diversity. Its parameters were then calibrated via the Taguchi method to enhance performance and robustness. The improved MOPSO was applied to two real viaducts on the BR-376 highway in Paraná, Brazil, achieving construction cost reductions of 9.1%–23.2%, CO_2 reductions of 8.9%–23.5%, and lifespan increases up to 540.7%. Analysis also showed that larger cross-sectional dimensions often resulted in lower construction costs, while smaller ones reduced emissions. The proposed method provides engineers with a practical tool for achieving balance between cost, sustainability, and durability. The developed software and source code are freely available at: https://www.optimusviaduto.ufscar.br.

Keywords: viaduct, bridge, MOPSO, NSGA-II, SPEA2.

1 Introduction

The design of bridges and viaducts is traditionally based on a trial-and-error approach, in which engineers define an initial configuration guided by experience. This involves selecting values for independent variables such as component dimensions, column spacing, and concrete strength. The structure is then analyzed and verified against design criteria. However, many other combinations of variables can also satisfy these requirements, potentially offering advantages such as lower cost, reduced environmental impact, shorter construction time, or extended service life. Optimization seeks to identify the combination of variables that best meets one or more objectives. As the number of degrees of freedom increases, the potential for improved solutions also grows.

In the realm of multi-objective optimization, where different objectives may conflict with one another, it is misleading to label any solution as absolutely "optimal". As Hashimoto [1] notes, a solution may favor one objective at the expense of others, leading to inevitable trade-offs.

The field of structural cost optimization is extensive, yet research specifically targeting bridges and viaducts is relatively scarce. The literature reviews by Hassasain and Loov [2] and Trentini [3] present a total of twenty-three studies.

Torres et al. [4] used linear programming to optimize single-span prestressed bridges, identifying optimal girder spacing. Lounis and Cohn [5] applied nonlinear optimization, determining efficient span arrangements based on deck length. Olivieri [6] and Cortês [7] employed genetic algorithms to optimize prestressed concrete bridge decks, achieving cost reductions of up to 26.8%. Ahsan et al. [8] applied evolutionary optimization methods reaching a 35.4% cost saving. Reported reductions depend on the quality of the baseline solution. Trentini [3] implemented an exhaustive search approach for optimizing cross-sections of decks with multiple I-girders, evaluating all possible combinations to identify the optimal configuration. El Mourabit [9] extended optimization to entire bridge structures, reducing costs by 12.8%, though excluding foundation costs. Yepes et al. [10] applied multi-objective optimization to I-girder decks, increasing service life by 233% and cutting CO_2 emissions by 11%, with only a 4% cost increase. García-Segura et al. [11] studied cellular decks using a harmony search algorithm and a neural network trained on CSiBridge data, balancing cost, emissions, and safety.

The evolution of deck optimization research has involved a gradual increase in the number of degrees of freedom (DOFs), leading to more refined optimal solutions. Trentini et al. [12] conducted a survey of published studies highlighting this trend over time, from the early work of Torres et al. [4] to the more complex models used by Ahsan et al. [8]. El Mourabit [9] broadened the optimization scope to the entire bridge, including column positioning. García-Segura et al. [11] explored the relationship between corrosion onset and construction costs, a topic still unexplored in the context of precast post-tensioned I-girder bridges and viaducts.

This study proposes a methodology for the multi-objective design of entire viaduct and bridge systems—including superstructure, substructure, and foundations—considering cost, sustainability, and service life criteria. The investigated typology consists of discontinuous decks with precast, post-tensioned I-girders (without intermediate crossbeams), rectangular pier caps, circular columns, and deep caisson foundations. The methodology involves selecting appropriate structural analysis and design techniques compatible with the optimization process, followed by the evaluation of different metaheuristic algorithms to identify the most effective for this application. The selected algorithm is then calibrated to the specific characteristics of the problem to enhance its performance. The proposed approach yields a set of high-performance trade-off solutions that optimize geometric configurations, concrete types, and post-tensioning levels. Results are assessed not only in terms of construction cost but also from a life-cycle perspective, with demonstrated reductions in CO_2 emissions and increases in service life.

The effectiveness of the proposed methodology is assessed through its application in two real-world case studies involving viaducts located along the BR-376 highway in Paraná, Brazil, one at the entrance to Atalaia and the other in Mandaguaçu. These case studies allow for the validation of the proposed optimization methodology under practical conditions. In addition to evaluating cost, environmental impact, and service life improvements, the study explores the relationships among objective functions along the Pareto front and identify patterns linking the degrees of freedom of the structural design variables to the performance of the obtained solutions.

2 Optimization problem description

This research develops a multi-objective optimization methodology for the design of viaducts and bridges, encompassing their superstructure, substructure, and foundation systems. The proposed optimization problem considers three objectives: minimizing construction cost, reducing environmental impact (measured through CO_2 emissions), and maximizing the structure's design service life. The problem is defined by a vector of decision variables $\boldsymbol{x}$, which includes parameters such as the number and geometric configuration of the girders, cross-sectional dimensions, slab height, concrete strength, pier cap dimensions, column diameters, caisson depth and diameter, reinforcement covers, and prestressing configurations. The total number of design variables (n) depends on the number of spans (N_v) and is given by

$$n = 14 + 24N_v. \tag{1}$$

The multi-objective optimization problem is subject to inequality and equality constraints, which ensure that the structural design complies with Brazilian standards, particularly ABNT NBR 7187:2021 [13] for bridge design and NBR 6118:2014 [14] for reinforced and prestressed concrete structures. These constraints enforce structural safety and serviceability conditions for all relevant combinations of loading during construction and operation.

The three objective functions are evaluated as follows:

- Cost is calculated based on material quantities and unit prices obtained from the SICRO [15] database

(maintained by DNIT), using the compositions specific to the state of São Paulo as of April 2022.

- CO_2 emissions are computed using the QE-CO_2 Method proposed by Costa [16], which accounts for emissions generated during raw material extraction, processing, transport, and application. Emission factors are associated with each material or service and multiplied by the corresponding quantities.
- Design service life is estimated through a semi-probabilistic approach based on the carbonation-induced depassivation time of reinforcement. The methodology proposed by Possan et al. [17] is employed, using a Monte Carlo simulation to evaluate the statistical distribution of the service life based on factors such as concrete strength, cover depth, relative humidity, pozzolanic content, and atmospheric CO_2 concentration.

Constraints for the structural optimization problem addressed in this study are based on the Brazilian design standards ABNT NBR 7187:2021 [13] and ABNT NBR 6118:2014 [14]. These standards establish the criteria for ensuring structural safety, durability, and serviceability, requiring verification of ultimate and service limit states for all relevant combinations of loads during both construction and operation phases.

The structural analysis methodology begins by defining a geometric configuration based on the established degrees of freedom. The analysis comprises two main stages. First, the transverse load distribution across the deck is evaluated considering variations in vehicle positioning according to ABNT NBR 7188:2013, using the Fauchart method. Next, the load distribution obtained from the first stage is applied to a three-dimensional beam model representing the entire viaduct structure, including the foundations. Soil-structure interaction is modeled according to the Winkler approach.

To facilitate automated structural analyses, a custom-developed algorithm named AMEBP3D was implemented. This algorithm utilizes the stiffness method and effectively accounts for hinges, deformation constraints, and flexible supports, efficiently analyzing the structure under various loading scenarios. AMEBP3D's workflow is divided into two phases: a preprocessing stage, executed once per analytical model, to handle loading-independent computations, and a processing stage, executed for each load case, generating displacement vectors, support reactions and, internal forces.

Structural component design begins with the slabs, which are classified according to their support conditions (end slabs and center slabs). Bending moments are derived using Rüsch tables, with corrections applied to continuous spans. Shear forces are evaluated via effective widths, and fatigue limit state verifications are performed using frequent load combinations. Slab reinforcement is determined following ABNT NBR 6118:2014 [14], considering simplified stress diagrams and iterative checks for fatigue compliance.

Girder design incorporates deflection and cracking criteria according to El Debs [18], considering time-dependent effects. Additionally, girder stability during construction phases is analyzed following the geometric nonlinear method proposed by Krahl et al. [19]. The composite action between precast girders and cast-in-place slabs is explicitly considered, accounting for strain discontinuities at the interface as detailed by Trentini et al. [20].

Pier caps are designed to resist bending moments and torsion, following ABNT NBR 6118:2014 [14] guidelines. Columns, subjected to combined bending and axial loads, are analyzed considering second-order effects through standard methods prescribed by the same standard, utilizing interaction diagrams (M, N, 1/r). Caissons, similarly subjected to axial-flexural loads, are designed following column criteria. Due to differences in cross-sectional dimensions, foundation blocks transition stresses via horizontal stirrups as recommended by Bastos [21]. The allowable bearing capacity of caisson bases is computed using empirical correlations with soil penetration resistance, as proposed by Cintra et al. [22].

3 Methodology

This study adopts metaheuristic methods to solve a multi-objective optimization problem involving structural design. NSGA-II, SPEA2, and MOPSO were selected as candidate algorithms due to their prominence in the literature, as identified by Zavala et al. [23]. Following the No Free Lunch theorem proposed by Wolpert and Macready [24], which states that no optimization algorithm is universally superior, these algorithms were tested for performance. The best-performing algorithm was selected and further refined through parameter calibration using the Taguchi method. This statistical technique, originally developed for industrial quality control, enables efficient tuning of algorithm parameters by reducing the number of required simulations through orthogonal experimental design. The calibration process involves maximizing the signal-to-noise ratio associated with

solution quality (measured via hypervolume), helping identify the optimal combination of parameter levels. This approach improves convergence, robustness, and computational efficiency, as demonstrated in prior studies applied to structural optimization problems. The Taguchi method's integration into the metaheuristic calibration ensures that the selected algorithm is tailored to the specific characteristics of the viaduct and bridge optimization problem addressed in this research.

3.1 Multiple Objective Particle Swarm Optimization (MOPSO)

The MOPSO algorithm is a metaheuristic inspired by the social behavior observed in bird flocks during food searches. Originally proposed for single-objective problems, Particle Swarm Optimization was adapted by Coello et al. [25] for multi-objective contexts. In MOPSO, solutions (particles) adjust their positions within a multidimensional decision space based on individual and collective experiences, represented by personal best and global best solutions. Each particle's movement is influenced by velocity vectors incorporating cognitive (individual experience) and social (swarm experience) learning factors, guiding the swarm toward Pareto-optimal solutions

3.2 Nondominated Sorting Genetic Algorithm II (NSGA-II)

The NSGA-II, introduced by Deb et al. [26], is a widely-used evolutionary algorithm for multi-objective optimization. It improves upon its predecessor by reducing computational complexity and incorporating elitism through a Pareto-based sorting mechanism. NSGA-II classifies populations into fronts of non-dominated solutions and employs a crowding distance estimator to maintain diversity. Solutions are evolved via genetic operators—simulated binary crossover and polynomial mutation—and iteratively selected based on dominance ranking and solution density to converge efficiently toward the Pareto front while preserving solution diversity

3.3 Strength Pareto Evolutionary Algorithm 2 (SPEA2)

The SPEA2, proposed by Zitzler et al. [27], addresses previous limitations of the original SPEA by integrating a fine-grained fitness assignment and density-based selection mechanisms. SPEA2 maintains an external archive separate from the main population, storing high-quality non-dominated solutions. Fitness is assessed using Pareto strength and solution density measures, guiding selection and survival operations. To promote diversity and uniform solution distribution along the Pareto front, SPEA2 uses a k-th nearest neighbor density estimator and a truncation procedure, ensuring efficient exploration and exploitation of the solution space.

4 Results and discussion

To evaluate the performance of the selected metaheuristic algorithms, a comparative analysis was conducted using a hypothetical viaduct design scenario. The structure features a 40 m longitudinal span, a 16.2 m width, and a level road grade. The optimization problem was solved using MOPSO, NSGA-II, and SPEA2, each executed 30 times with a termination criterion of 10 million solution evaluations. The simulations were performed using a 20-core computing cluster at the Federal University of São Carlos (UFSCar), significantly reducing computation time compared to a standard desktop setup.

Performance assessment was based on two key metrics: hypervolume and Pure Diversity. Hypervolume quantifies the extent of the solution space covered by the Pareto front, with higher values indicating better performance. Pure Diversity measures the spread of solutions using an L_p-norm (with $p < 1$), where higher values suggest a more diverse and representative solution set.

The results, summarized in Figure 1 through boxplots, reveal that MOPSO achieved the highest average performance in both hypervolume and diversity. To statistically verify these findings, the Wilcoxon Rank-Sum test was applied with a 5% significance level. MOPSO demonstrated a statistically significant advantage over NSGA-II and SPEA2 in both metrics. In contrast, no significant difference was observed between NSGA-II and SPEA2, suggesting similar performance between them.

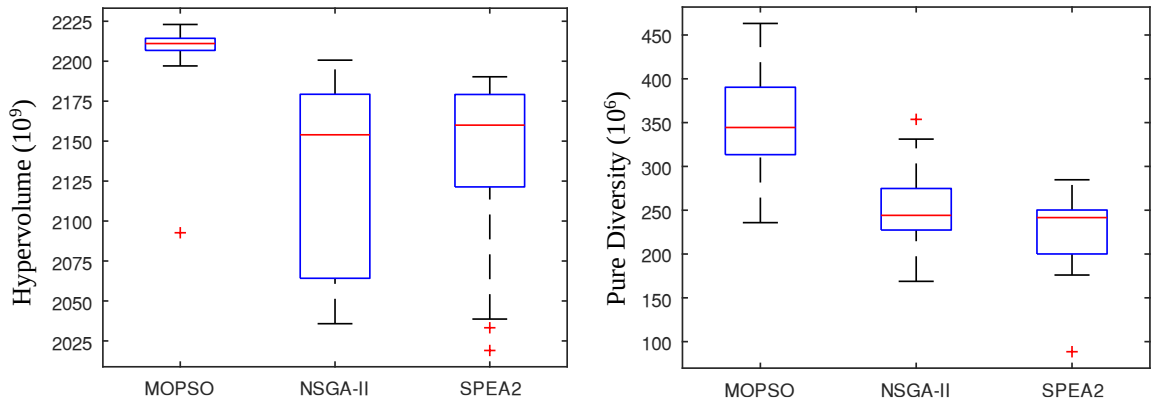


Figure 1. Box plot of performance metrics: hypervolume and Pure Diversity

Overall, the results indicate that MOPSO is the most effective algorithm for this optimization problem, offering both extensive coverage of the Pareto front and high solution diversity. Accordingly, MOPSO was selected as the preferred method for subsequent parameter calibration.

To improve the quality of solutions, reduce execution time, and enhance the robustness of the selected method, a parameter tuning process was conducted for MOPSO. Five parameters were considered: inertia weight (W), cognitive (C_1) and social (C_2) learning factors, mutation index (η_m), and swarm size. Instead of performing a full factorial design, which would be computationally intensive, the Taguchi method was adopted using an orthogonal matrix with 54 experiments repeated five times. This approach enabled efficient exploration of the parameter space while maintaining statistical relevance.

Based on the signal-to-noise ratios calculated from the hypervolume results, several candidate combinations were evaluated to identify the most effective configuration. After further refinement, the combination that yielded the highest average hypervolume across repeated runs was adopted as the calibrated configuration for MOPSO, as summarized in Table 1.

Table 1. Calibrated MOPSO parameters

W	C_1	C_2	η_m	swarm size
0.4	0.75	1.0	20	130

Although improvements in optimal cost and CO_2 emissions were modest (0.65% and 0.62%, respectively), the coverage metric ($\mathcal{C}$) further highlights the advantages of the calibrated configuration. Set $\boldsymbol{A}$ represents the solutions obtained using the original (non-calibrated) parameters, while set $\boldsymbol{B}$ corresponds to the solutions obtained with the calibrated configuration. The results were $\mathcal{C}(\boldsymbol{A},\boldsymbol{B}) = 0.146$ and $\mathcal{C}(\boldsymbol{B},\boldsymbol{A}) = 0.576$, indicating that only 14.6% of the solutions in set $\boldsymbol{B}$ are dominated by set $\boldsymbol{A}$, whereas 57.6% of the solutions in set $\boldsymbol{A}$ are dominated by set $\boldsymbol{B}$. This confirms that the calibrated configuration yields a more favorable distribution of trade-off solutions among the objective functions.

To assess its real-world applicability, the proposed optimization methodology was applied to actual viaduct structures on the BR-376 highway in Paraná, Brazil, located in the cities of Atalaia and Mandaguaçu.

In the Atalaia case study, the optimization process produced superior solutions compared to the original design, achieving construction cost reductions between 10.5% and 22.7%, decreases in CO_2 emissions from 8.9% to 21.2%, and improvements in service life ranging from 24.0% to 540.7%. Analyzing selected optimal solutions indicated a nuanced relationship between cost and CO_2 emissions, demonstrating that, for example, a 4.8% reduction in CO_2 emissions could be achieved with only a 1.0% cost increase, while a 4.3% cost reduction resulted in just a 0.5% increase in emissions. Additionally, each additional decade of service life was associated with a cost increase ranging from R$ 829.88 to R$ 4,681.65, depending on the level of CO_2 emissions. These findings suggest that significant improvements in durability can, in some cases, be achieved at a relatively low cost. Another noteworthy observation is that degrees of freedom that increase the cross-sectional dimensions tend to yield solutions with lower costs, whereas those involving smaller cross-sectional dimensions typically lead to solutions with lower CO_2 emissions.

In the Mandaguaçu case study, the optimization process also outperformed the original design, achieving construction cost reductions between 9.1% and 23.2%, decreases in CO_2 emissions from 12.7% to 23.5%, and increases in service life of up to 540.7%. Examination of selected optimal solutions revealed a clear trade-off between cost and environmental impact; for instance, a 5.2% reduction in CO_2 emissions was achieved with only a 0.74% increase in cost, while a 3.6% cost reduction resulted in just a 1.4% increase in emissions. Regarding service life, the cost associated with each additional decade varied from R$ 1,932.37 to R$ 2,856.33, depending on the level of CO_2 emissions. These results indicate that substantial gains in structural longevity can be obtained with modest additional investment. As in the Atalaia case, it was observed that degrees of freedom that enlarge cross-sectional dimensions tend to produce lower-cost solutions, while reductions in these dimensions generally lead to lower CO_2 emissions.

5 Conclusions

This study proposed and validated an optimization methodology for the design of viaduct and bridge structures, emphasizing cost efficiency, environmental sustainability, and structural durability. The methodology, which incorporated automated structural analysis and calibrated optimization techniques, was applied to two real-world case studies along the BR-376 highway in Paraná, Brazil.

The optimized solutions consistently outperformed the original designs, achieving reductions in construction costs (approximately 9% to 23%) and CO_2 emissions (approximately 9% to 23%), along with enhancements in design service life. Notably, emission reductions were possible with minimal cost increases, demonstrating favorable trade-offs. Additionally, extending service life incurred relatively modest incremental costs, indicating potential for cost-effective durability improvements.

Analysis of design variables revealed clear trends: larger cross-sectional dimensions generally reduced costs, while smaller dimensions were associated with reduced environmental impact. The results confirmed MOPSO's effectiveness and robustness as a metaheuristic method for structural optimization, particularly after parameter calibration, enhancing the diversity and quality of the solution set.

Overall, the calibrated optimization methodology proved to be a practical tool for achieving economically viable, environmentally sustainable, and structurally durable viaduct designs, providing valuable support for decision-making in infrastructure projects.

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