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Computational Solution of the Direct Heat Transfer Problem in Laminated Composites Using the Finite Volume Method

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Abstract. Laminated composite materials are widely used in engineering applications due to their excellent thermal and mechanical properties. However, internal defects such as delamination at the interfaces between layers can significantly affect their performance and compromise structural integrity. In this context, accurate thermal analysis becomes essential for evaluating the reliability of these materials and supporting the early detection of failures. This project proposes the development of a robust numerical model to solve the direct heat transfer problem in laminated composites using the finite volume method (FVM). The study considers transient, two-dimensional heat conduction under different boundary conditions and thermal properties. The governing partial differential equations are discretized using the FVM, and the solution is obtained through an implicit time integration scheme to ensure numerical stability and precision. A computational code is being developed in Python, and its validation will be carried out using benchmark problems or analytical solutions available in the literature. The model aims to simulate the temperature distribution within laminated composites and analyze how variations in material properties and contact conditions influence heat flow. The results will provide insights into the thermal behavior of these materials, helping to assess their structural integrity. The proposed approach may also serve as a foundation for future studies involving inverse problems, failure detection, and thermal characterization of advanced composite systems.

Keywords: Finite volume method, Heat transfer, Laminated composites, Numerical simulation

1 Introduction

According to Kvapil et al. [1], the study of heat transfer has proven to be extremely important due to its various applications, such as aerodynamic heating, thermal control of satellites, cooling of nuclear reactors, and electronic equipment. In this context, it is essential to map the composite interfaces, since failures in this region cause undesirable thermal resistance and compromise structural performance. Such modeling makes it possible to understand the thermophysical properties of the materials.

The industrial sector frequently seeks lightweight, easy-to-handle, highly resistant materials with specific physical and chemical properties. In this scenario, composite materials have been widely disseminated across various sectors, with laminated composites standing out. According to Rezende and Botelho [2], these composites are made from layers with different components that, when combined, generate superior properties compared to conventional materials.

As a way of investigating interface failures in these materials, Abreu et al. [3] combined the generalized integral transform technique and finite differences for the direct problem, in addition to Bayesian inference via the Monte Carlo method for the inverse problem. The experimental measurements obtained identified failures with precision and demonstrated the technique's robustness in the face of measurement errors, proving the method's efficiency. Thus, the relevance of studying the contact surface of laminated composites can be understood.

This work develops a numerical model to solve the composite materials' direct heat transfer problem via the finite volume method. With the implemented approach, it will be possible to evaluate and qualify internal failures, especially at the plate interfaces, contributing to optimizing the use of composites in various engineering applications. The problem formulation, solution methodology, and numerical results are presented below.

2 Problem Formulation

The physical problem under study involves an analysis of the thermal behavior in a two-dimensional system composed of two materials with distinct properties, as shown in Fig. 1.

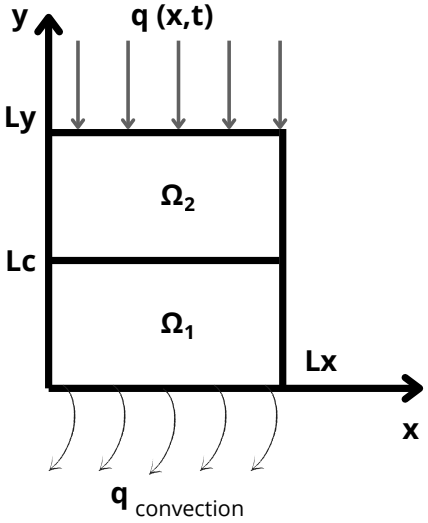


Figure 1. Physical problem

The mathematical formulation for this case is established by the following equations:

$$\frac{1}{\alpha_1}\frac{\partial T_1}{\partial t} = \frac{\partial^2 T_1}{\partial x^2} + \frac{\partial^2 T_1}{\partial y^2}, \quad \text{for } (x,y) \in (0, L_x) \times (0, L_c),\ t > 0 \tag{1}$$

$$\frac{1}{\alpha_2}\frac{\partial T_2}{\partial t} = \frac{\partial^2 T_2}{\partial x^2} + \frac{\partial^2 T_2}{\partial y^2}, \quad \text{for } (x,y) \in (0, L_x) \times (L_c, L_y),\ t > 0 \tag{2}$$

$$-k_1\frac{\partial T_1}{\partial y} + h_0 T_1 = h_0 T_\infty, \quad \text{for } x \in (0, L_x),\ y = 0,\ t > 0 \tag{3}$$

$$k_2\frac{\partial T_2}{\partial y} = q(x,t), \quad \text{for } x \in (0, L_x),\ y = L_y,\ t > 0 \tag{4}$$

$$\begin{cases} k_1\dfrac{\partial T_1}{\partial y} = k_2\dfrac{\partial T_2}{\partial y} \\ k_1\dfrac{\partial T_1}{\partial y} = h_c(x,t)(T_2 - T_1) \end{cases}, \quad \text{for } x \in (0, L_x),\ y = L_c,\ t > 0 \tag{5}$$

$$\frac{\partial T}{\partial \vec{n}} = 0, \quad \text{for } y \in (0, L_y),\ x = 0 \text{ or } x = L_x,\ t > 0 \tag{6}$$

$$T_1 = T_2 = T_\infty, \quad \text{for } t = 0, \text{ in } \Omega_1 \cup \Omega_2 \tag{7}$$

in which the subscripts 1 and 2 refer to the materials; q is the heat flux; α is the thermal diffusivity; h_0 is the heat transfer coefficient in the convection propagation process; h_c is the thermal conductance; T_∞ is the ambient temperature; Ly, Lc, and Ω define the domain, and k refers to the thermal conductivity.

3 Numerical Methodology

3.1 The finite volume method

The finite volume method converts differential equations into algebraic equations through the discretization of the domain into discrete volumes, according to Moukalled et al. [4]. The conservation equations are applied to each control volume, allowing the calculation of dependent variables at points located at the center of these volumes, known as nodes. Interpolations between adjacent nodes are used to estimate values at the boundaries between volumes, facilitating the imposition of different boundary conditions.

Furthermore, as stated by Pinto [5], the method guarantees local energy conservation, since the heat flux entering a given control volume is exactly equal to that leaving the neighboring volume. Based on this, the finite difference technique can be applied to estimate the fluxes at the interfaces between volumes.

According to Eymard et al. [6], such characteristics make the finite volume method a highly appropriate tool for numerical simulations in various fields, being widely used in modeling complex physical phenomena and advanced applications. In industry, it stands out for its robustness and low computational cost, offering reliable results even in highly complex problems, with relatively simple and efficient implementation.

3.2 Mathematical Discretization

For the spatial discretization of the problem, the finite volume method was used, and for the temporal discretization, the fully implicit formulation was selected, ensuring the method's stability.

Initially, it is necessary to divide the case into control volumes, as shown in Fig. 2.

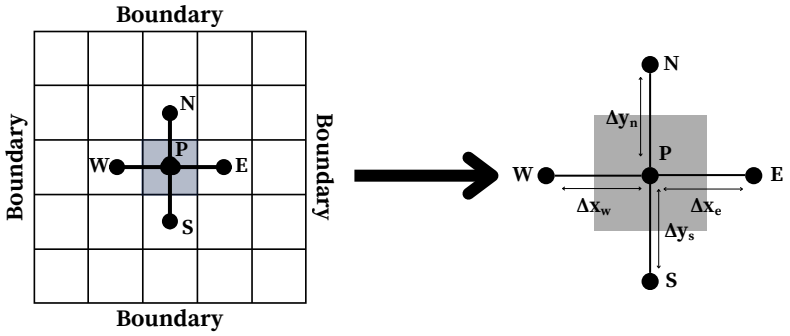


Figure 2. Two-dimensional mesh and control volume centered at point P

The letter P represents the central point of the control volume under analysis, while the letters N, S, W, and E correspond, respectively, to the neighboring volumes in the north, south, west, and east directions. To indicate the interfaces between the control volumes, the same notation in lowercase letters (n, s, w, e) will be adopted, thus distinguishing the centers from the boundaries.

In this way, the integration for eq. (1) and eq. (2) is performed as follows:

$$\int_w^e \int_s^n \int_t^{t+\Delta t} \left(\frac{1}{\alpha} \frac{\partial T}{\partial t} \right) dx\, dy\, dt = \int_w^e \int_s^n \int_t^{t+\Delta t} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) dx\, dy\, dt \tag{8}$$

$$\int_w^e \int_s^n (T - T_P^0)\, dy\, dx = \int_t^{t+\Delta t} \left[\alpha \left(\left.\frac{\partial T}{\partial y}\right|_n - \left.\frac{\partial T}{\partial y}\right|_s \right) + \alpha \left(\left.\frac{\partial T}{\partial x}\right|_e - \left.\frac{\partial T}{\partial x}\right|_w \right) \right] dt \tag{9}$$

Considering that the phenomenon under analysis is purely diffusive and that the temperature distribution between the centers of the control volumes can be assumed linear, the obtained expression is:

$$\frac{T_P - T_P^0}{\Delta t} = \alpha \left[\left(\frac{T_N + T_S - 2T_P}{\Delta y^2} \right) + \left(\frac{T_E + T_W - 2T_P}{\Delta x^2} \right) \right] \tag{10}$$

Furthermore, for the boundaries, the mentioned boundary conditions will be applied, discretized for each case, yielding the following results:

$$\frac{T_P - T_P^0}{\Delta t} = \alpha \left[\frac{1}{\Delta y} \left(\frac{q(t,x)}{k} - \frac{T_P - T_S}{\Delta y} \right) + \left(\frac{T_E + T_W - 2T_P}{\Delta x^2} \right) \right] \text{em} \begin{cases} 0 < x < L_x \\ y = L_y \end{cases} \tag{11}$$

$$\frac{T_P - T_P^0}{\Delta t} = \alpha \left[\frac{1}{\Delta y} \left(\frac{T_N - T_P}{\Delta y} - \frac{h_0(T_P - T_\infty)}{k} \right) + \left(\frac{T_E + T_W - 2T_P}{\Delta x^2} \right) \right] \text{em} \begin{cases} 0 < x < L_x \\ y = 0 \end{cases} \tag{12}$$

Finally, for the interface region between different materials, the following conditions are established:

$$k_1 \frac{T_P - T_S}{\Delta y} = h_c(T_N - T_S) \quad \text{em} \quad y \to L_c^- \tag{13}$$

$$k_2 \frac{T_N - T_P}{\Delta y} = h_c(T_N - T_S) \quad \text{em} \quad y \to L_c^+ \tag{14}$$

As a way to analyze the accuracy of the method, the Root Mean Square (RMS) was used to compare the results obtained by the numerical solution with the analytical solution, with the calculation performed as shown in eq. (15).

$$E_{RMS} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (X_i - \hat{X}_i)^2} \tag{15}$$

4 Results and Discussions

Before applying the model to the proposed physical problem, preliminary tests were conducted on simplified geometries and conditions to validate the numerical implementation and analyze the model's sensitivity to mesh refinement. The tests considered two distinct scenarios: a one-dimensional case with two different materials and a two-dimensional case with a homogeneous material.

The one-dimensional test with two materials was represented by a bar composed of Styrofoam and nylon, with an interfacial thermal conductance nearly zero ($h_c = 1 \times 10^{-4}$). The analytical solution considered assumes the presence of thermal insulation between the two regions, allowing conduction only from the nylon, with heat flux entering solely in this material. Thus, the comparative analysis between numerical and analytical results is restricted to the portion of the bar composed of nylon. Fig. 3 shows the temperature profile obtained for this case.

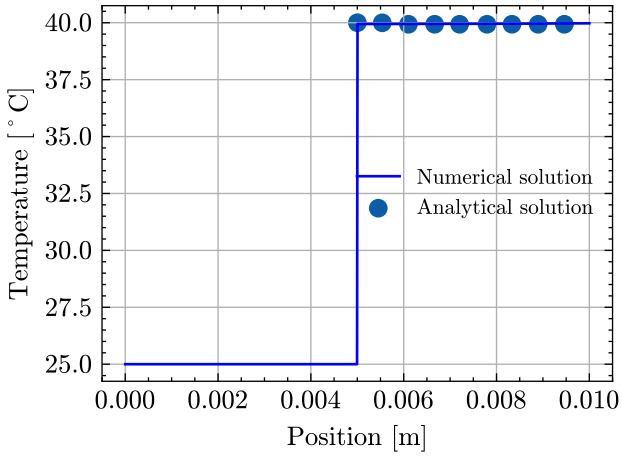


Figure 3. Temperature variation along the bar position at 100 seconds

The verification of the results was based on a comparison with analytical data obtained from the work of Abreu [7], which addresses similar problems using a compatible methodology. The RMS was used as a metric to quantify the closeness between the numerical and analytical results.

Table 1. Temperature values for each mesh, with respect to the y-position, applied to two materials in one dimension.

$T(y) \setminus ny$	20	30	40	50	100	Analytical
T(2,564)	31,605	31,761	31,836	31,88	31,966	32,364
T(17,436)	37,029	37,129	37,097	37,079	37,045	36,727
T(41,538)	39,413	39,472	39,459	39,453	39,447	39,086
T(71,795)	39,924	39,91	39,9	39,895	39,884	39,322
T(93,718)	39,987	39,983	39,981	39,98	39,977	40,068
RMS	0,545	0,475	0,443	0,425	0,389	—

Table 2. Temperature values for each mesh, with respect to the x-position, applied to one material in two dimensions with y = Ly.

$T(x) \setminus nx$	20	30	40	50	100	Analytical
T(0,01)	37,297	37,335	37,352	37,361	37,377	37,267
T(0,019744)	31,797	32,152	32,446	32,623	32,834	36,721
T(0,049744)	26,702	27,395	27,94	28,253	28,619	29,799
T(0,094744)	23,189	24,129	24,872	25,295	25,792	27,109
RMS	1,493	1,387	1,308	1,246	1,063	—

Table 1 shows the one-dimensional case with two materials, while Table 2 presents the two-dimensional case with a homogeneous material. The notation $T(y) \setminus ny$ and $T(x) \setminus nx$ denotes temperatures as a function of position, where the first column indicates the evaluated position and the first row the number of mesh divisions. In both cases, the error decreases with mesh refinement, confirming convergence.

It is worth noting that increasing mesh refinement implies a higher computational cost. In this sense, it was found that the mesh with 30 divisions in each direction offers a good balance between accuracy and efficiency,

presenting low errors and results very close to the analytical solution in both the 1D and 2D tests. Therefore, this configuration was adopted to simulate the proposed physical problem.

Based on these results, it was possible to ensure the reliability of the numerical model for application to the proposed problem. The convergence analysis proved essential to ensure that the results obtained in the simulations were not compromised by numerical errors associated with domain discretization.

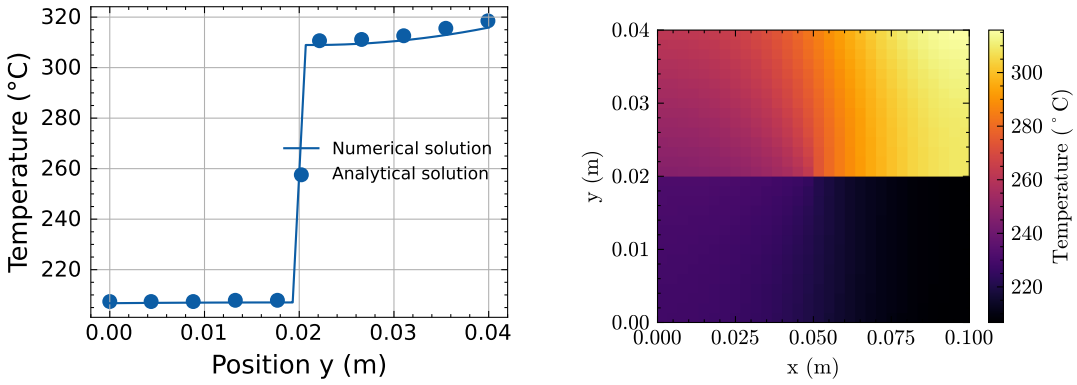
For the proposed problem, the interfacial thermal conductance was defined by eq. (16), which represents that, from the midpoint of the observation time onward, the right half of the interface behaves as a perfect insulator.

$$h_c = \begin{cases} 0, & \text{if } t \geq \frac{t_f}{2} \text{ and } x \geq \frac{L_x}{2} \\ 1000, & \text{otherwise} \end{cases} \tag{16}$$

Table 3. Parameters used in the numerical simulation

Parameter	**Notation**	**Value**
Plate height	L_y (m)	0.04
Plate width	L_x (m)	0.1
Initial temperature	T_0 (°C)	25
Temperature at boundary 1	$T_{\infty 1}$ (°C)	25
Temperature at boundary 2	$T_{\infty 2}$ (°C)	25
Convective heat transfer coefficient	h_0 (W/m²°C)	10
Thermal diffusivity of AISI 1050 steel	α_{steel} (m²/s)	1.474×10^{-5}
Thermal diffusivity of titanium	$\alpha_{titanium}$ (m²/s)	1.26×10^{-6}
Thermal conductivity of AISI 1050 steel	k_{steel} (W/m°C)	54
Thermal conductivity of titanium	$k_{titanium}$ (W/m°C)	21.9
Heat flux	q (W/m²)	20000

The parameters described in Table 3 were entered into the computational code developed in Python and used as input data for the simulation of thermal behavior. Figure4(a) illustrates the temperature distribution along the thickness of the plate at the final time of 1500 seconds and Fig. 4(b) shows the spatial temperature distribution, providing a global view of the thermal behavior within the domain.



(a) Temperature variation along the y-position of the plate at 1500 seconds with x = Lx

(b) Temperature distribution in space at the end of 1500 seconds

Figure 4. Temperature distribution results at the end of the simulation for different spatial perspectives.

It is observed that the results obtained through the numerical simulation show excellent agreement with the reference analytical solution. The difference between the curves is minimal along the entire plate length, with an RMS of 1.6°C, even in a scenario with high temperatures and thermal discontinuity. This performance reinforces the robustness of the model in predicting thermal fields and its ability to handle more realistic physical conditions.

The proposed model proves to be particularly useful for thermal integrity analysis in components operating under severe conditions, such as aeronautical structures, composite panels in the automotive industry, and heat exchange equipment. By identifying and simulating interfacial failures, the numerical tool can assist in defining critical zones for thermal monitoring, contributing to predictive maintenance strategies and extending the service life of systems.

5 Conclusions

The work successfully met its objectives by developing a robust numerical model based on the finite volume method to simulate heat transfer in laminated composites. Implemented in Python, the model was verification against analytical results, demonstrating high accuracy even with coarse meshes and confirming its reliability at a lower computational cost. The methodology effectively captured the thermal behavior of composites, including interfacial failures—an essential factor in structural applications—with results consistent with the literature. The model also showed good numerical stability and adaptability to various physical configurations in one and two dimensions. As a future direction, the model can be enhanced by incorporating variable thermal properties and extending the simulations to three dimensions, broadening its applicability in both academic and industrial settings.

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