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Comparison of Simplified Methods for Immediate Deflection Calculation in Prestressed Concrete Beams

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Abstract. Verifying structural deformations is essential to ensure the performance and safety of the designed system. For prestressed concrete beams, deflection calculations involve additional particularities compared to reinforced concrete beams. These include, for instance, the need to account for the effect of prestressing force on the concrete and the presence of prestress-induced hyperstatic effects in continuous beams. In cases of partial prestressing, the calculation of the immediate deflection component requires consideration of the stiffness reduction due to concrete cracking, just as with reinforced concrete beams. Although prestressed beams typically exhibit lower levels of cracking compared to reinforced concrete beams, it is still necessary to account for this effect in deflection calculations. Therefore, it is essential to comprehend how various design standards address these elements. This study aims to compare simplified methods for calculating short-term deflections in simply supported and continuous prestressed concrete beams with partial prestressing levels, following the provisions of the Brazilian standard NBR 6118 [1] and the fib Model Code [2]. Deflection values are computed using both normative models for a set of simply supported and continuous prestressed concrete beams designed with partial prestressing. A comparative analysis is presented, discussing the differences observed between the methods.

Keywords: Immediate deflection; prestressed concrete beams; NBR 6118; fib Model Code.

1 Introduction

The control of deflections is an essential requirement in the design of concrete structures, ensuring compliance with serviceability limit states (SLS), user comfort, and the preservation of architectural aesthetics. In prestressed concrete beams, deflection calculation presents particularities compared to reinforced concrete beams, due to the action of the prestressing force, the construction and loading sequence, and the presence of hyperstatic effects in continuous structures, as evidenced by Naaman [3].

In cases of partial prestressing, concrete cracking may occur under service loads, reducing the effective stiffness of the beam and increasing deflections. According to Branson [4] and Sabnis [5], this behavior requires methods that properly account for stiffness reduction, in a similar way to reinforced concrete beams. Although prestressed beams generally exhibit lower levels of cracking, this effect cannot be neglected, as demonstrated by Abambres and Cavaco [6].

Technical standards for the design of concrete structures present different simplified methods for estimating immediate deflections in reinforced and prestressed beams, aiming to balance accuracy and practical feasibility. Among them, the approaches indicated by NBR 6118 [1] and the fib Model Code [2] stand out, as they present different assumptions and formulations for considering concrete cracking. The Brazilian code NBR 6118 [1] has adopted Branson's simplified method for estimating immediate deflections since its 2007 edition. The American Concrete Institute was the first to adopt this formula in ACI Committee 435 [7], and it continues to be

recommended in ACI 318 [8]. On the other hand, the fib Model Code [2] proposes the so-called "global coefficients method," derived from the bilinear model introduced by the CEB (1985) [9].

The literature on deflections and stresses in reinforced concrete beams is extensive, with numerous studies employing finite element methods and advanced constitutive models to predict structural behavior. Nevertheless, codes still recommend simplified methods for deflection checks due to their practicality. Several studies have compared these normative methods with refined models, such as those by Junges [10], Junges and Rovere [11], and Cruz et al. [12–14].

In contrast, research specifically addressing deflections in prestressed beams, particularly under partial prestressing, remains limited. Some works, such as those by Martins [15] and Carneiro [16], focus on the development of design tools. Others, such as Gregori et al. [17] and Lou [18], propose three-dimensional models considering combined actions, rheological effects, and time-dependent behavior. The model by Marí [19] incorporates nonlinearities and different construction phases.

Despite the accuracy of these advanced models, their application remains restricted mainly to academic contexts. Therefore, simplified methods maintain their practical relevance and continue to be adopted by design standards. Although well-established, these methods still reveal particularities in results depending on the specific case studied. Moreover, their comparison provides a clearer understanding of the differences between normative models — in this case, the Brazilian NBR 6118 and the international reference, the Fib Model Code.

This study aims to compare the results obtained from these two simplified approaches, applied to supported and continuous beams with partial prestressing. The comparison is limited to immediate deflections, without considering long-term effects such as creep, shrinkage, and progressive losses.

2 Literature Review

2.1 Representation of Prestressing Force

The evaluation of immediate deflection in prestressed beams involves the combination of permanent loads, service loads, and the effect of the prestressing force, which induces an initial upward camber. In continuous beams, tendon eccentricity generates secondary moments — known as prestress hyperstatic effects — that modify the distribution of moments and, consequently, the deformations.

In this study, the procedure recommended by NBR 6118 [1] was adopted, which considers the prestressing force as an equivalent external load distributed along the span. This approach simplifies the analysis by avoiding the need to decompose prestress into primary and secondary (hyperstatic) effects, thus facilitating the calculation of immediate deflections. The calculation of the equivalent prestressing force follows the process described by Zanette [20] and Skaf and Stucchi [21].

2.2 Simplified Methods

For calculating the immediate deflection component in reinforced and prestressed concrete beams, the concept of equivalent moment of inertia is widely used. This concept aims to represent the actual stiffness of the beam, taking into account the reduction caused by concrete cracking. The equivalent moment of inertia provides an intermediate value between the moment of inertia of the uncracked cross-section and the inertia in state II, in which the cracked concrete no longer contributes to stiffness. This weighting is able to practically reflect the effective stiffness of the beam under service loads. The equations adopted for calculating the inertia in state II are described in Cruz et al. [14].

This simplification presents clear advantages: it allows quick estimates of immediate deflections without the need for complex or computationally expensive analyses, and it incorporates the gradual transition between uncracked and cracked states, making the assessment closer to real structural behavior. For these reasons, technical standards such as NBR 6118 [1] and the fib Model Code [2] recommend the use of this concept, as it reconciles adequate representativeness and practical applicability in typical design situations.

In the case of NBR 6118 [1], the calculation of the equivalent inertia is based on the equation proposed by Branson [4], which weights the contribution of cracked and uncracked sections as a function of the ratio between the applied moment and the cracking moment. The fib Model Code [2], on the other hand, adopts a bilinear

formulation derived from the CEB-1985 model [9], in which the transition between uncracked and cracked stages is represented more abruptly. In both cases, the central idea is to establish a relationship between the cracked state of the section and the effective applied moment, thereby reflecting the gradual variation in stiffness throughout loading, as noted by Branson [4], Zanette [20], and Skaf and Stucchi [21].

3 Methodology

3.1 Computational Procedure

The computational program AVSer was developed to facilitate the calculation of immediate deflections in reinforced and prestressed concrete beams. The program automates the procedures outlined in NBR 6118 [1] and the Fib Model Code [2], in addition to implementing more refined models. The primary objective is to streamline and expedite a process that, when performed manually, can be both time-consuming and prone to errors. Further details about the AVSer framework can be found in Cruz et al. [14] and Junges [10].

The program initially determines the moments of inertia of the gross section I_{sb} and the cracked section I_{II}, which are fundamental to characterize the structural stiffness under different cracking states of the concrete. For this purpose, geometric properties of the section and mechanical parameters of both concrete and steel are considered, as well as the distribution and position of reinforcement and prestressing tendons, accounting for bonded or unbonded conditions.

Next, the program calculates the cracking moment M_r, a critical threshold that defines the load level from which cracking is expected to occur in the beam. As described in Zanette [20], for prestressed beams this value results from two components: the decompression moment M_0 , which neutralizes the compressive stress induced by prestressing at the section edge (including the effect of prestress-induced hyperstatic moments in continuous beams); and the additional moment required, from this neutralized state, to reach the conventional tensile strength of concrete at the edge. The contribution of the latter is evaluated according to NBR 6118 [1] for the two implemented models, while the formulation for the fib Model Code [2] will be incorporated in future developments.

If the maximum external service moment M_a (calculated from quasi-permanent service load combinations, considering only external loads) exceeds the cracking moment M_r, the equivalent inertia I_{eq} is determined using the formulas prescribed by each standard — NBR 6118 [1] or fib Model Code [2]. To properly assess the cracking level, the prestressing contribution is incorporated by subtracting M_0 from both M_a and M_r, so that M_a includes the prestressing effect, and M_r represents the actual cracking threshold of the section. The deflection is then computed considering the equivalent inertia along the beam under the service load combination, combined with the prestressing force represented as an equivalent external load.

A relevant distinction between the two methods is that NBR 6118 [1] prescribes a segmental evaluation of equivalent inertia in continuous beams, weighting stiffness values for each span portion with a constant moment sign. Conversely, the fib Model Code [2] does not provide such a procedure; thus, the value of I_{eq} calculated for the most critical section of the span is extended across the entire beam length.

3.2 Studied Beams

To perform the comparison between the two normative models, two theoretical beams, calculated by Zanette [20], were analyzed: a simply supported beam and a continuous beam, both designed for partial prestressing using unbonded (greased) tendons. The concrete adopted has a characteristic compressive strength f_{ck} = 30 MPa and an elastic modulus E_{cs} = 26.070 MPa. The passive reinforcement consists of CA-50 steel with a characteristic yield strength f_{yk} = 500 MPa and elastic modulus E_s = 210.000 MPa. The unbonded tendons are CP190-RB strands, with a diameter ø = 12,7 mm, cross-sectional area A_p = 1,014 cm², initial prestressing force P_0 = 130 kN, and effective prestressing force after losses P_∞ = 120 kN.

The effects produced by the tendons were considered through the representation of equivalent external loads, which was implemented in the AVSer program based on the tendon geometry defined in the input data file. Validation confirmed that the equivalent load calculated by the program matched the manual results obtained by Zanette [20].

The simply supported beam analyzed has a free span of 12.0m, total section height h=85cm, and web width b_w = 20 cm (Figure 1). It is subjected to a permanent load of 15.0 kN/m and a variable load of 5.0kN/m. Three prestressing tendons were used, generating a total force corresponding to 43% of the permanent load at the most critical section, where the maximum prestressing eccentricity is 31.9 cm.

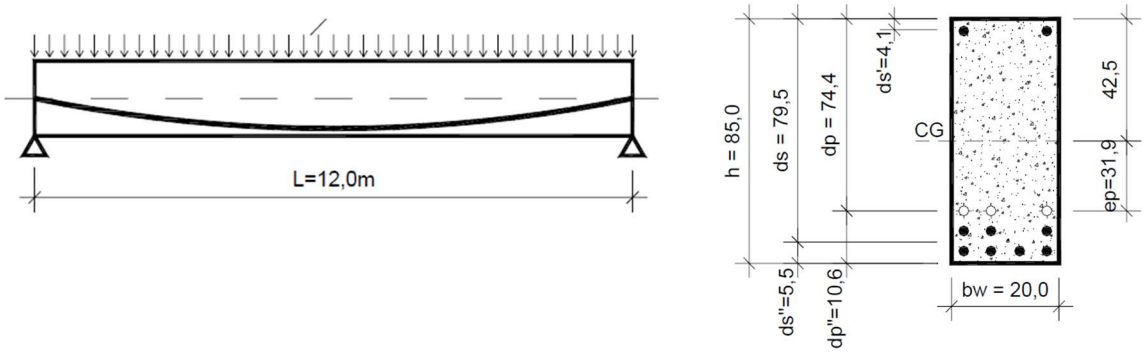


Figure 1 – Geometry and tendon layout of the simply supported beam. Source: Zanette [20].

The continuous beam has a total length of 26 m, with a section height h=60 cm and width b_w = 20 cm. It is subjected to a permanent load of 23.0 kN/m and a variable load of 16.0 kN/m. The tendon layout follows the bending moment diagram of the beam, as illustrated in Figure 2. Three prestressing tendons were used, generating a total force corresponding to 70% of the permanent load.

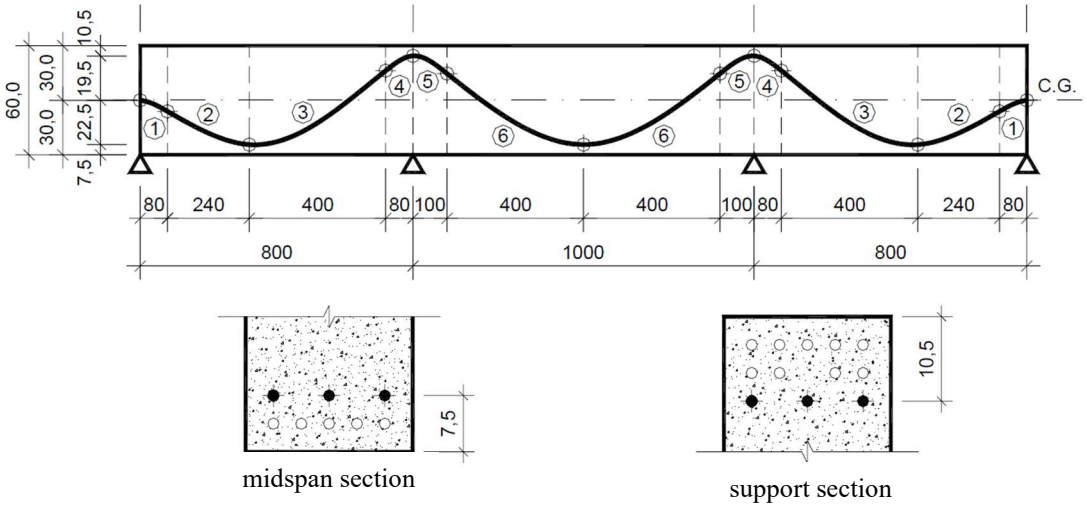


Figure 2 – Tendon layout of the continuous beam. Source: Zanette [20].

4 Results and Discussion

4.1 Simply Supported Beam

For the simply supported beam, the manual calculation by Zanette [20], using the NBR 6118 [1] method, was initially taken as a reference to validate the results obtained with the AVSer program. Figure 3 shows the distribution of the applied bending moments (M_a) and the cracking moment of the section (M_r) along the span. It can be observed that, in the midspan region, the applied moment exceeds M_r, indicating the occurrence of cracking and the consequent need to use the equivalent moment of inertia by simplified methods.

It should be noted that the AVSer program can compute a value of M_r for each beam element discretized in

the analysis. However, since the calculation presented here follows the simplified method, the value of M_r adopted for the evaluation of the equivalent inertia corresponds to the most critical (most cracked) section of the beam (see Junges [10] for further details).

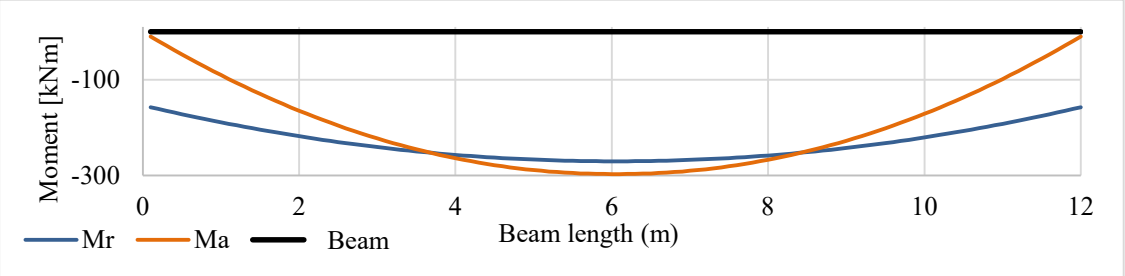


Figure 3 – Bending moment values in the simply supported beam. Source: Author.

The immediate deflections calculated according to NBR 6118 [1] and the fib Model Code [2] are compared with the value obtained by Zanette [20] in Table 1. It is observed that NBR 6118 [1] resulted in a deflection of 1.57 cm, very close to Zanette's value of 1.50 cm, which used the same procedure, with a difference of only 4.7%. This small difference may be attributed to possible rounding in the manual calculation. The fib Model Code [2] provided a slightly lower value of 1.38 cm, approximately 12% smaller than the value obtained with NBR 6118 [1].

Table 1 – Immediate deflection results for the simply supported beam. Source: Author.

	Zanette [20]	NBR 6118 [1]	*fib* Model Code [2]
Immediate deflection value	1.5 cm	1.57 cm	1.38 cm

Figure 4 illustrates the evolution of deflection as a function of the applied load at midspan, highlighting the transition between the uncracked and cracked states. For this calculation, the external load was incrementally varied, while the equivalent prestressing load remained constant. As a result, an initial upward camber is observed in the absence of external loading. It can be seen that both normative methods adequately follow the trend of the manual calculation, with only minor discrepancies appearing after the onset of cracking. For this example of a simply supported beam under partial prestressing, both simplified models showed good agreement.

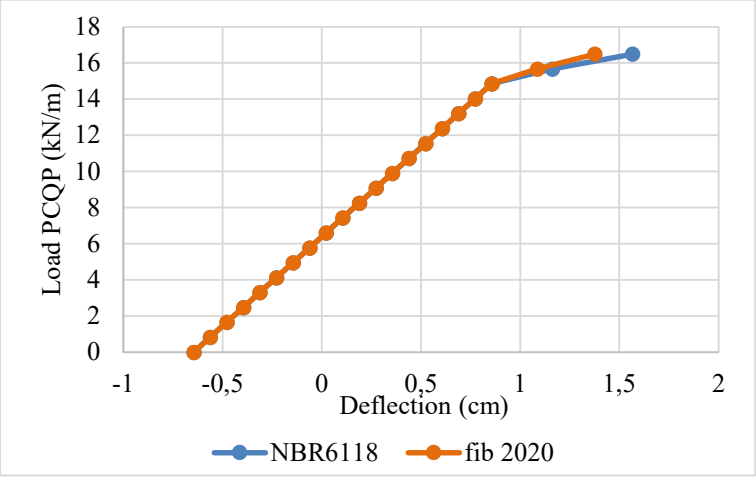


Figure 4 – Applied load versus midspan deflection for the simply supported beam. Source: Author.

4.2 Continuous Beam

Figure 5 shows the distribution of applied bending moments (M_a) compared with the cracking moment (M_r), indicating that the applied moment exceeds the cracking resistance only in the regions near the central supports. In this context, it is essential to highlight the difference between the normative models. While NBR 6118 [1] adopts a weighted approach to inertia along the entire beam, reflecting the spatial variation of cracking, the fib Model Code [2] considers stiffness corresponding to the most critical section and applies it to the whole span.

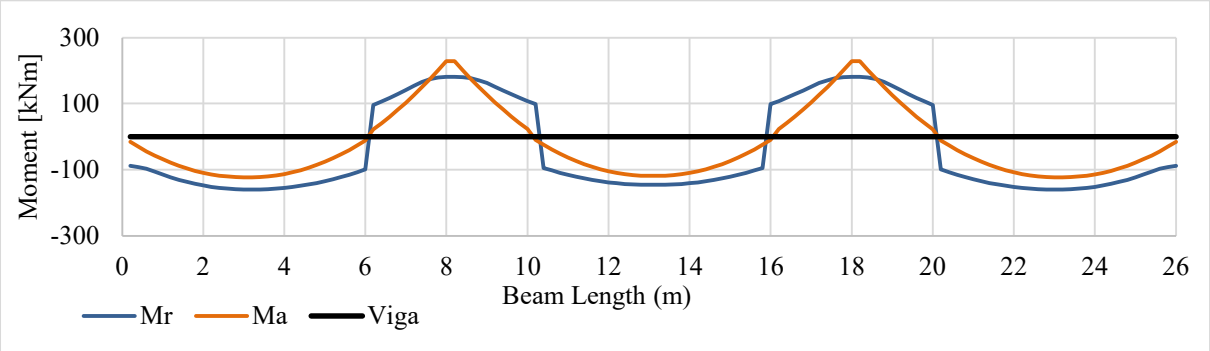


Figure 5 – Bending moment values in the continuous beam. Source: Author.

The immediate deflection values are presented in Table 2. At the central span, NBR 6118 [1] resulted in a deflection of 0.516 cm, which practically coincided with the value obtained by Zanette [20] (0.510 cm), with a difference of only 1.2%. The fib Model Code [2], in turn, provided a higher deflection value of 0.658 cm, 29% greater than the reference, which evidences the more conservative nature of this model for continuous structures. The same trend was also observed in the deflections of the side spans.

Table 2 – Immediate deflection results for the continuous beam. Source: Author.

	Zanette [20]	NBR 6118 [1]	*fib* Model Code [2]
Immediate deflection at the central span	0.51 cm	0.516 cm	0.658 cm
Immediate deflection at the side spans	-	0.297 cm	0.377 cm

Figure 6 complements this analysis by illustrating the relationship between applied load and midspan deflection. It can be observed that, after cracking, due to the differences in the equivalent inertia calculated by each method, the two standards produce curves with distinct slopes, leading to the immediate deflection differences previously noted.

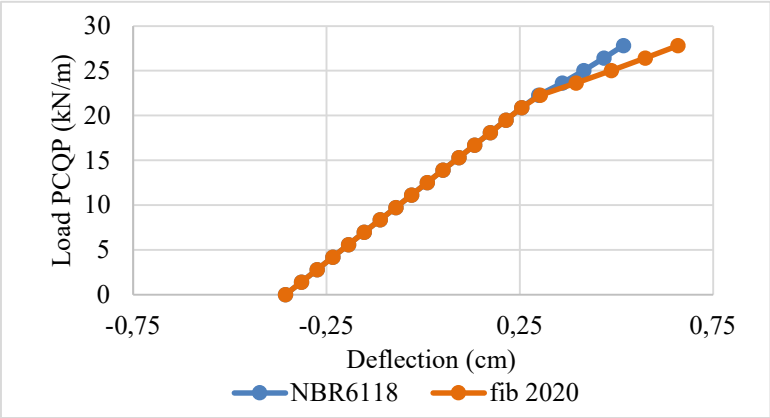


Figure 6 – Applied load versus midspan deflection of the central span in the continuous beam. Source: Author.

5 Conclusions

Based on the results obtained, the following points can be highlighted:

- The implementation of the simplified method of NBR 6118 [1] in the computational tool AVSer was validated against the manual results of Zanette [20].
- For the simply supported beam under partial prestressing, both NBR 6118 [1] and the fib Model Code [2] gave very close results, confirming the adequacy of the equivalent inertia approach in cases of low cracking, typical of beams with predominantly elastic behavior.
- In the continuous beam analysis, NBR 6118 [1], by weighting inertia according to maximum moments of each span segment, produced lower deflections than the fib Model Code [2], which instead adopts the inertia of the most critical section for the whole span, leading to more conservative results.
- This divergence underscores the importance of understanding the assumptions of each standard, especially for continuous structures where model choice significantly affects design and performance evaluation.
- The findings highlight the need for further studies on localized cracking and stiffness weighting procedures, to support future updates in standards and improve computational tools such as AVSer.

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