XLVI CILAMCE
VITÓRIA - ES - BRAZIL - 2025
XLVI IBERO-LATIN AMERICAN CONGRESS ON COMPUTATIONAL METHODS IN ENGENEERING

Geometric Modeling for Finite Elements: Advances and Contributions in Surface Intersections, Multi-Region Detection, and Topological Structures (2000–2015)

William W. M. Lira[1], Luiz Fernando C. R. Martha[2]

[1]Laboratory of Scientific Computing and Visualization, Federal University of Alagoas (LCCV/UFAL) Campus Ac. Simões. Av. Lourival Melo Mota, S/N, Tabuleiro do Martins, 57072-970, Maceió/AL, Brazil william@lccv.ufal.br
[2]Dept. of Civil and Environmental Engineering, Pontifical Catholic University of Rio de Janeiro (PUC-Rio) Rua Marquês de São Vicente, 225, 22453-900, Rio de Janeiro, RJ, Brazil lfm@tecgraf.puc-rio.br

Abstract. This work presents an overview of the main contributions made between 2000 and 2015 in the field of geometric modeling applied to engineering problems, with emphasis on topological data structures, surface intersections, and automatic recognition of multi-regions. During this period, the studies conducted significantly contributed to pushing the frontier of knowledge in the area, offering conceptual and computational solutions relevant to applications based on the finite element method. The increasing demand for robust numerical simulations in engineering, especially those based on the finite element method, underscores the importance of accurate and efficient geometric representation of complex domains, which is fundamental for generating quality meshes and for the accuracy of simulations. Additionally, some of the works also addressed aspects related to mesh generation and adaptivity, expanding the scope of investigated applications and challenges. The contributions deal with topics such as the robust computation of intersections between parametric surfaces, the efficient handling of Boolean operations on non-manifold solids, and the implementation of topological data structures for the automatic detection and management of multi-regions in three-dimensional models. To present this overview, the main works published by the authors between 2000 and 2015 were selected, aiming to provide a coherent panorama of the proposed approaches, techniques, and solutions. The paper seeks to describe, in an accessible and structured manner, the main concepts, strategies, and results achieved, offering readers a comprehensive perspective of the developed contributions. The main contribution of this work is to provide a consolidated record of the solutions developed over the years, highlighting how they helped to advance the frontier of knowledge in geometric modeling for engineering. This paper establishes itself as a reference for researchers and professionals interested in the evolution of these techniques and their applications in advanced numerical simulations in engineering and related areas.

Keywords: Geometric modeling; Topological structures; Surface intersection; Multi-region detection; Finite element method.

1 Introduction

Geometric modeling plays a fundamental role in the numerical simulation of engineering problems, especially those addressed through the finite element method (FEM). The accuracy of these simulations strongly depends on the faithful and efficient representation of complex geometries, particularly in three-dimensional domains composed of heterogeneous materials, interfaces, and regions of interest with distinct physical behaviors.

Between 2000 and 2015, a series of research efforts by these authors focused on advancing the integration of geometric modeling into FEM-based simulations. These efforts resulted in a significant body of work that explored robust algorithms and data structures for handling critical modeling tasks, such as computing intersections between parametric surfaces, detecting and managing multi-region configurations, and structuring topological data to support mesh generation and model preprocessing.

The state of the art during that period was marked by the emergence of techniques capable of dealing

with non-manifold geometries, surface-surface intersections in parametric space, and Boolean operations involving solids with complex internal boundaries. Publications such as [1–4] illustrate the technical maturity achieved by the group of authors in addressing these challenges.

This article is motivated by the need to consolidate and document this set of contributions. Rather than conducting a comparative or critical analysis of the global literature on the subject, the aim here is to present an integrated overview of the conceptual and computational advancements made by the authors and their collaborators during this 15-year period. This includes highlighting the unique methodological strategies and practical implementations that supported simulations in porous and fractured media, structural mechanics, and other complex engineering contexts.

The main objective is to provide a comprehensive synthesis of the approaches and results developed, emphasizing the impact of these contributions on advancing the frontier of knowledge in geometric modeling for FEM. In doing so, the paper also seeks to serve as a reference for future research, offering insights into the challenges tackled and the solutions proposed.

This overview is structured around four main themes: (i) surface intersection techniques, (ii) detection and representation of multi-region models, (iii) topological data structures for geometric modeling, and (iv) complementary advances in mesh generation, attribute management, and adaptivity. Each of these themes is detailed in the subsequent sections.

2 Methodology

The methodology is based on a retrospective review of the main studies on geometric modeling developed between 2000 and 2015 by the authors. It began with the selection of key publications on FEM applications, later grouped into four research strands.

The first strand develops robust techniques for computing intersections between parametric surfaces, essential for consistent geometric representations in multi-material environments and proper discretization.

The second strand concerns the automatic detection and classification of multi-region domains, distinguishing subdomains that may represent different materials, behaviors, or boundary conditions, thus streamlining simulation setup.

The third strand focuses on topological data structures for representing non-manifold geometries and enabling efficient traversal, modification, and annotation, crucial for preprocessing and meshing.

The fourth strand covers auxiliary developments such as attribute configuration, adaptive mesh refinement, and integration of meshing into simulation pipelines, complementing the core modeling techniques.

The following diagram summarizes the methodology:

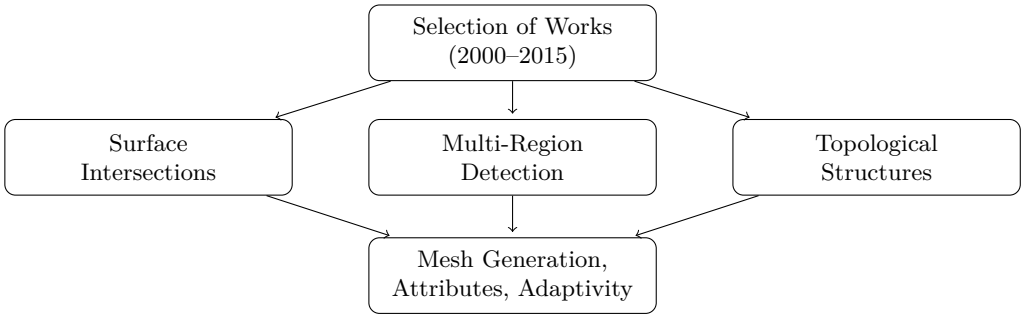


Figure 1. Organization of the main research contributions.

Each of these thematic strands is explored in the following sections with references to the original publications.

3 Surface Intersections

In geometric modeling for finite element applications, robust computation of surface intersections is critical, especially in multi-material domains or contact interfaces where precise region delimitation ensures proper mesh generation and simulation fidelity.

Between 2000 and 2015, the authors introduced strategies for computing intersections between parametric surfaces, particularly NURBS. A central challenge was handling multiple intersections consistently and topologically coherently.

In [2], a technique was proposed that projects the intersection problem into a common parametric domain and solves it via iterative refinement and curve tracing. By exploiting parametric definitions, this approach avoided inconsistencies of purely geometric algorithms and effectively detected intersection curves, including trimmed and irregular surfaces.

The results in [5] extended this methodology to multiple intersections in complex assemblies, including non-manifold edges often present in preprocessing. Identifying intersection segments, loops, and endpoints enabled consistent geometric entity generation and robust model partitioning.

A broader framework was later introduced in [1], treating surface-surface intersection as a core operation in multi-region modeling. It integrated detection with Boolean operations and meshing, allowing intersection curves to be directly applied in solid construction and topological labeling.

Figure 2 illustrates the workflow for parametric surface intersection, adapted from [2], showing key steps: curve computation in 3D, handling of trimming boundaries, and the resulting surface mesh.

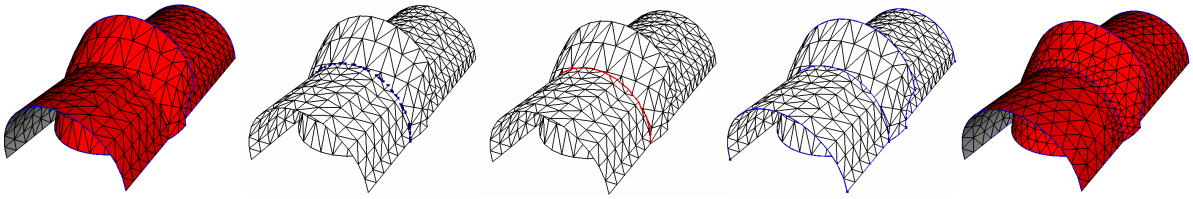


Figure 2. llustrative example of the parametric surface intersection workflow: algorithm steps, intersection curves, and final surface mesh. [2].

These contributions laid the foundation for surface-based domain partitioning in multi-region models, and were later integrated into modeling environments where the geometric kernel had to support parametric entities, topological consistency, and FEM preprocessing.

Beyond algorithmic robustness, special attention was given to the numerical stability and tolerance handling of these intersection algorithms. This was critical for ensuring that small gaps or near-tangent surfaces did not generate invalid results, which would compromise mesh generation and simulation accuracy.

The collective advances in this area not only improved the fidelity of geometric representations used in FEM, but also served as a key enabler for downstream processes such as automatic multi-region detection and Boolean-based model assembly, as discussed in the next sections.

4 Multi-Region Detection

The automatic detection of multi-region is essential for the correct representation of complex models in finite element simulations. Multi-region models arise in contexts where different physical subdomains, such as materials, phases, or structural components, are combined within a single geometric framework. The challenge lies in ensuring that each region is correctly identified, delimited, and connected, especially in three-dimensional domains with non-manifold interfaces.

The methodology proposed by Lira et al. [1, 3, 6] addresses this challenge by combining geometric modeling techniques with topological classification strategies. Their approach starts from a set of parametric surfaces that define the boundaries between regions and uses intersection computations to identify candidate interfaces. A key aspect of the technique is its ability to robustly handle shared surfaces and overlapping boundaries, which are common in CAD-generated models and in simulations involving material interfaces.

One of the main innovations of the approach lies in the use of a volumetric flood-fill algorithm guided by adjacency graphs derived from surface connectivity. This method enables the identification of regions enclosed by the surfaces without requiring prior manual labeling or partitioning. Furthermore, the use of trimmed surfaces and Boolean operations ensures consistency even in the presence of numerical imperfections or surface artifacts.

To support mesh generation and attribute assignment, the technique employs a region-labeling algorithm that traverses the topological structure of the model, assigning unique identifiers to each

connected subdomain. This labeling allows for the automatic differentiation of regions, even when the model presents intricate internal boundaries or when multiple regions meet at a single point or edge.

Figure 3 illustrates an example of automatic multi-region detection, highlighting geometric construction and finite element mesh generation.

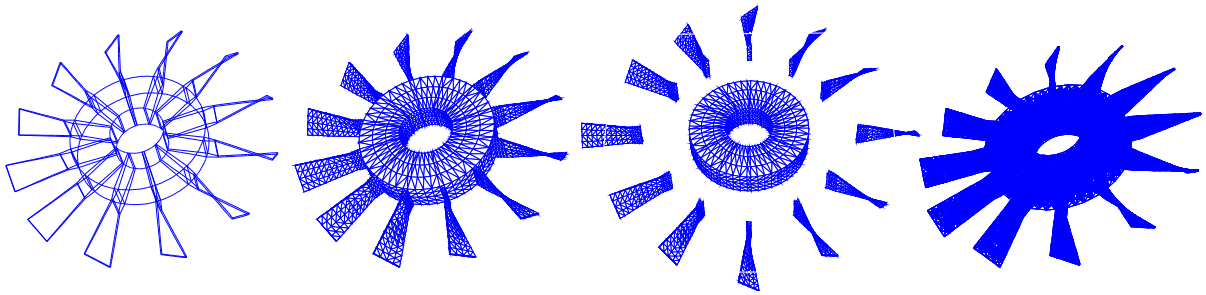


Figure 3. Illustrative example of a multi-region turbine disk model, showing geometric construction and finite element mesh generation. Adapted from [3].

Another important consideration is the integration of region detection into simulation-driven workflows. Further developments, such as those presented in [7–10], integrate the detection phase with adaptive analysis, enabling domain decomposition for parallel execution, local mesh refinement, and targeted error control. The ability to accurately segment the domain into physical subregions is critical for assigning appropriate material properties, boundary conditions, and solver strategies.

In this broader context, multi-region detection becomes not only a preprocessing task but also a central component of the simulation pipeline. The techniques developed contribute to reducing user effort, increasing model robustness, and enabling the handling of increasingly complex scenarios in areas such as structural mechanics, subsurface modeling, and multi-physics analysis.

5 Topological Data Structures

Robust geometric modeling for finite element simulations relies on data structures capable of managing complex topologies, including non-manifold geometries and regions with multiple materials or boundary conditions. In this context, topological data structures play a central role in ensuring consistency and flexibility during model construction, preprocessing, and mesh generation.

5.1 Structures for Parametric Surfaces and Multi-Regions

The representation of parametric surfaces and multi-region models requires a data structure that captures both the geometry and the topological relationships between model components. The approach adopted in the doctoral thesis of Lira [1] builds upon the concept of topological maps to maintain the connectivity between geometric entities such as vertices, edges, and faces, while associating these entities with their corresponding parametric definitions.

This structure enables the consistent assembly of models composed of multiple surface patches, supports the identification of adjacent regions, and facilitates the handling of non-manifold configurations. Each surface patch is defined in a local parametric space and connected to neighboring patches through shared boundaries. The structure also supports attributes such as orientation, patch identifiers, and region tags, which are essential for mesh generation and subsequent finite element analysis.

This representation is especially important when modeling domains with interfaces between different materials, allowing the extraction of topologically coherent regions and enabling downstream operations such as intersection computation and mesh conformity enforcement.

5.2 Boolean-Based Structures for Multi-Region Solids

To model solids with multiple interacting regions, a Boolean-based representation was developed in [4, 11]. It extends the traditional boundary representation (B-rep) by supporting Boolean operations on non-manifold solids and employs a hierarchical scheme combining volumetric primitives through union, intersection, and subtraction.

A key feature is preserving the association between volumetric regions and their topological boundaries, enabling region labeling and geometric trimming in multi-region contexts. The method also identifies overlapping volumes, voids, and interface surfaces, essential for finite element preprocessing.

Figure 4 illustrates Boolean operations on two regions bounded by curved surfaces. Such procedures automatically detect interfaces and generate topologically consistent multi-region models, directly usable in mesh generation and numerical simulation.

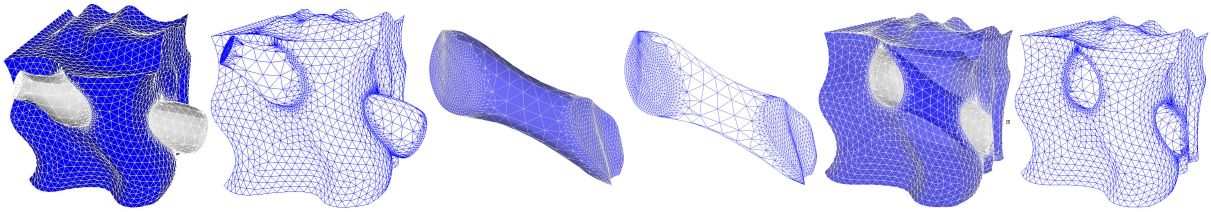


Figure 4. Illustrative example of Boolean operations applied to two regions bounded by generic curved surfaces. Adapted from [4].

This Boolean-based structure enhances modularity and reuse of geometric components and facilitates the implementation of constructive solid geometry (CSG) pipelines tailored to finite element simulations.

6 Mesh Generation, Adaptivity and Attribute Management

Numerical simulations often require not only accurate geometric modeling but also robust mesh generation strategies and flexible mechanisms to control the level of detail in the discretization. In this context, the integration of mesh generation, adaptivity, and extensible attribute configuration into the geometric modeling framework significantly enhances the efficiency, accuracy, and generality of simulation workflows.

6.1 Finite Element Mesh Generation

Mesh generation is fundamental for translating geometric representations into discretized models for numerical simulation. In subsurface engineering, where geological complexity and multi-region configurations are common, meshes must handle heterogeneous materials, curved interfaces, and high aspect-ratio features.

The work in [12] presents a geometric modeling and meshing pipeline for subsurface simulation. It supports parametric surfaces, multi-region topologies, and hierarchical refinement. A key feature is generating conforming meshes directly from Boolean-composed solids, preserving boundaries and ensuring interface compatibility.

Surface intersection and trimming extract valid region boundaries, which feed a volume meshing stage producing tetrahedral or hexahedral meshes as required. The algorithm emphasizes quality through element sizing, grading control, and alignment with physical layers.

Figure 5 shows an example mesh for a geological model with multiple stratigraphic regions, refined near critical interfaces.

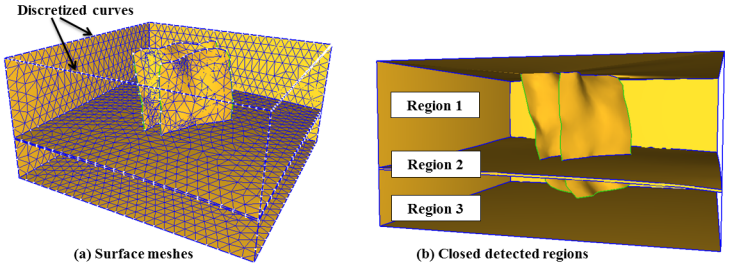


Figure 5. Illustrative example of finite element mesh generation in a subsurface multi-region model. Adapted from [12].

This demonstrates the framework's applicability to realistic subsurface scenarios, bridging conceptual

geological models and FEM-ready meshes.

6.2 Adaptive Mesh Generation

Adaptive mesh refinement (AMR) enables local adjustment of mesh resolution based on error indicators, material interfaces, or geometric features. In multi-region models defined by parametric surfaces, challenges include preserving boundary conformity and topological consistency across interfaces.

The works in [1, 7–9] proposed a framework combining region-aware modeling with hierarchical refinement. Starting from a coarse mesh, refinement operators are applied according to user-defined or simulation-driven criteria, while ensuring inter-region compatibility.

A key feature is using topological information to guide refinement across shared interfaces, preventing non-conforming elements and preserving domain integrity. Figure 6 illustrates a 3D multi-region model with adaptive meshing, where refinement is concentrated in regions of interest.

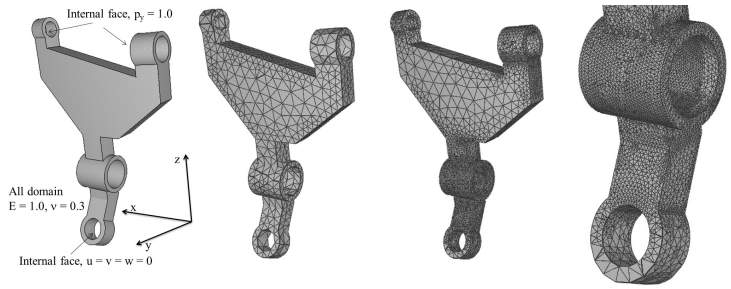


Figure 6. Illustrative example of an adaptive mesh applied to a multi-region model. Adapted from [7].

The ability to selectively refine the mesh without remeshing the entire domain results in significant computational savings and improved simulation accuracy in critical zones.

6.3 Extensible Attribute Management (ESAM)

To manage physical, numerical, or boundary attributes in geometric models, the ESAM (Extensible Strategy for Attribute Management) architecture was developed [13, 14]. ESAM defines a plugin-based mechanism that extends the data model with attribute types, allowing configuration, query, and update without altering the core structure.

It decouples geometry from simulation metadata, enabling flexible association of attributes such as material IDs, boundary conditions, solver parameters, or region tags. Each attribute is an independent module, dynamically registered in the modeling environment.

This promotes modularity and reusability, supports inheritance across hierarchical entities (surfaces, volumes, interfaces), and enables enriched models to be visualized or exported for preprocessing.

In summary, integrating AMR and ESAM into the modeling pipeline enhances the system's capacity to support realistic, complex simulations while preserving flexibility and scalability.

7 Final Considerations

This article presented an overview of contributions developed between 2000 and 2015 in geometric modeling for finite element simulations. The works addressed challenges in surface intersection, multi-region detection, topological structuring, adaptive mesh generation, and attribute management.

All approaches shared the objective of enabling robust and flexible geometric representations for complex engineering simulations. From parametric surface intersection to hierarchical region classification, the methods consistently ensured geometric consistency, topological integrity, and simulation readiness.

A distinctive aspect was the integration of geometry, topology, and simulation data within a coherent framework. Plugin-based attribute management and explicit topological structures highlighted extensibility and computational efficiency. Although published separately, these methods are complementary and evolved into a framework applicable to porous media, structural mechanics, and multiphysics coupling, providing a solid foundation for future geometric preprocessing.

This manuscript was produced with support of artificial intelligence tools, particularly ChatGPT, which assisted in organizing, translating, and editing the content. It is expected that this record will serve both as documentation of past advances and as inspiration for further research in geometric modeling and numerical simulation.

Acknowledgements. The first author acknowledges PETROBRAS for financial and technical support of the project "Desenvolvimento de Produtos Digitais para Gestão de Integridade de Estrutura de Poço" [ANP code 23599-4]. The second author acknowledges support from CAPES [Finance Code 001], CNPq [Grant 308884/2021-3], and FAPERJ [Process No. E-26/201.224/2022].

Authorship statement. The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

References

[1] W. W. M. Lira. Modelagem Geométrica para Elementos Finitos Usando Multi-Regiões e Superfícies Paramétricas. Tese de doutorado, Pontifícia Universidade Católica do Rio de Janeiro - PUC/Rio, 2002.
[2] W. W. M. Lira, A. C. d. O. Miranda, and L. F. C. R. Martha. A technique for computing intersection of parametric-surface meshes. In Proceedings of the XXVII Iberian Latin-American Congress on Computational Methods in Engineering (CILAMCE), 2006.
[3] W. W. M. Lira, P. R. Cavalcanti, L. C. G. Coelho, and L. F. C. R. Martha. A modeling methodology for finite element mesh generation of multi-region models with parametric surfaces. Computers & Graphics, vol. 26, n. 6, pp. 907–918, 2002a.
[4] A. M. B. Pereira, M. C. Arruda, A. C. d. O. Miranda, W. W. M. Lira, and L. F. C. R. Martha. Boolean operations on multi-region solids for mesh generation. Engineering with Computers, vol. 28, pp. 225–239, 2012.
[5] W. W. M. Lira, L. C. G. Coelho, and L. F. C. R. Martha. Multiple intersections of finite-element surface meshes. In 11th International Meshing Roundtable, volume 1, pp. 355–362, 2002b.
[6] W. W. M. Lira, P. R. Cavalcanti, L. C. G. Coelho, and L. F. C. R. Martha. An oop approach for mesh generation of multi-region models with nurbs. In XIV Brasilian Simposium on Computer Graphics and Image Processing, pp. 18–25, 2001.
[7] A. C. d. O. Miranda, W. W. M. Lira, J. B. Cavalcante-Neto, R. A. Sousa, and L. F. C. R. Martha. A three-dimensional adaptive mesh generation approach using geometric modeling with multi-regions and parametric surfaces. Journal of Computing and Information Science in Engineering, vol. 13, 2013.
[8] L. F. C. R. Martha, J. B. Cavalcante-Neto, W. W. M. Lira, A. C. d. O. Miranda, and R. A. d. Sousa. Fe adaptive analysis of multi-region models. In International Conference on Adaptive Modeling and Simulation, 2013.
[9] R. A. Sousa, A. C. d. O. Miranda, W. W. M. Lira, J. B. Cavalcante-Neto, and L. F. C. R. Martha. Two and three dimensional geometry based adaptive finite element analysis. In Proceedings of the XXVIII Iberian Latin-American Congress on Computational Methods in Engineering (CILAMCE), 2007.
[10] W. W. M. Lira, A. C. d. O. Miranda, L. F. C. R. Martha, and J. B. Cavalcante-Neto. Uma metodologia para análise adaptativa 3d usando modelagem geométrica com multi-regiões e superfícies paramétricas. In Proceedings of the XXV Iberian Latin-American Congress on Computational Methods in Engineering (CILAMCE), 2004.
[11] M. C. Arruda, L. F. C. R. Martha, A. C. d. O. Miranda, and W. W. M. Lira. Boolean operations on non-manifold and b-rep solids for mesh generation. In Proceedings of the XXVII Iberian Latin-American Congress on Computational Methods in Engineering (CILAMCE), 2006.
[12] A. C. d. O. Miranda, W. W. M. Lira, R. C. Marques, J. B. Cavalcante-Neto, A. M. B. Pereira, and L. F. C. R. Martha. Finite element mesh generation for subsurface simulation models. Engineering with Computers, vol. 31, pp. 305–324, 2015.
[13] W. W. M. Lira. Sistema integrado configurável para simulações em mecânica computacional. Dissertação de mestrado, Pontifícia Universidade Católica do Rio de Janeiro - PUC/Rio, 1998.
[14] R. Morrot, W. W. M. Lira, and L. F. C. R. Martha. A plugin strategy for extensible configuration of attributes in geometric modelling for finite element simulations. Mecánica Computacional, vol. 29, pp. 6029–6043, 2010.