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Application of Topology Optimization to Prestressed Slabs with Unbonded Tendons

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Abstract. The use of prestressing, especially unbonded type, has become increasingly common in building slabs, bringing benefits to concrete by reducing or even eliminating tensile stresses in the structure. Researchers and structural engineers have been seeking design concepts that maximize the efficiency of the concrete-prestressing interaction. One tool for this is the use of topology optimization (TO), although it is understood that TO, generates conceptual designs that may not be practical or fully detailed, often requiring the application of shape and size optimization to refine the solution. The solutions obtained must be analyzed considering various alternative elements of the problem, such as: the optimization models (design variables, objective function, and constraints), the finite element models used, the numerical modeling of prestressing, the constitutive models of materials, material parameterization, the type of loading, boundary conditions, and the optimization algorithm. The aim of this work is to conduct an in-depth study on the application of topology optimization to prestressed concrete slabs, particularly unbonded ones, to analyze the influence of the above cited elements involved in the optimized solutions. An example from the literature, numerical and experimental, and the finite element software Abaqus and its TO tool are used in this work.

Keywords: Topology Optimization, Finite Elements, prestressed Concrete, SIMP.

1 Introduction

The use of prestressed structures has become increasingly common, as this technique significantly increases the durability of the structure. The inclusion of prestressing reduces or even eliminates the tensile stresses acting on the concrete, allowing it to work predominantly in compression. The use of prestressing also reduces the volume of concrete, offering structural and economic advantages over reinforced concrete, enabling the design of longer spans. In this context, the use of unbonded prestressing stands out, which offers execution advantages when compared to bonded prestressing.

Topological optimization has become attractive in the design of these structures, as it allows adjusting the quantity and distribution of material to achieve structural efficiency. This type of optimization is essential in prestressed structures, as it enables efficient material distribution, reducing weight and cost without compromising strength and stiffness. By using computational algorithms, it is possible to identify critical areas that require more reinforcement and regions where material can be removed, resulting in lighter and more efficient designs. This approach is particularly valuable in prestressed structures, where the cable tension must be balanced with the structure's geometry, ensuring better mechanical performance and durability.

This study aims to explore topology optimization applied to concrete slabs with unbonded post-tensioning, highlighting relevant aspects of the numerical modeling involved. For this purpose, an experimental slab previously evaluated numerically by Ellobody and Bailey [1] was used, in addition to the finite element software *Abaqus* and its optimization tool *Tosca*. The research seeks to understand, above all, how the application of post-tensioning influences the material distribution patterns resulting from the optimization process.

First, the unidirecional prestressed slab from Ellobody and Bailey [1] was analyzed considering both bonded and unbonded prestressing, nonlinear behavior, and compared with literature results. Then, TO was applied using the classical compliance minimization formulation, varying loading and boundary conditions, and using both the optimality criteria (OC) algorithm and the classical mathematical programming method (MMA). For the topology optimization, the material was considered to be linear elastic.

In this context, a detailed simulation of a slab with unbonded prestressing was carried, the tendons and anchorages were modeled as structural elements, and to ensure the internal sliding of the tendon, a tangential relationship between frictionless contact surfaces was employed. Additionally, the use of reduced integration elements proved to be an essential alternative for reducing computational cost while maintaining accuracy.

2 Finite Element Analysis

The entire model was developed using eight-node hexahedral solid elements with reduced integration, Therefore, the cables and the concrete matrix, were all modeled as resistant elements. As done by Ellobody and Bailey [1], the contact regions between the cables and the interior of the concrete matrix were defined as frictionless surfaces with low slip, considering the physical properties of unbonded prestressing.

In order to reproduce the mechanical behavior of the reference model by Ellobody and Bailey [1], which used hexahedral and prismatic elements, an equivalent mesh was simulated. However, considering topology optimization, an equivalent mesh using eight-node hexahedral elements with reduced integration was adopted. Kang et al. [2] recommend that the mesh should have at least five layers to adequately represent bending effects, so nine layers were used in the model.

The slab was modeled as simply supported, where the prestressing was applied as a stress field, introduced before applying the loads. As in the proposed by Ellobody and Bailey [1], the model's symmetry was explored, and the maximum eccentricity was adopted for the cable at the mid-span.

The loads applied experimentally were also reproduced. For that, a distributed load on the top face was considered, representing the self-weight of the slab, along with the loading tree mentioned by Ellobody and Bailey [1]. Additionally, the incremental load was simulated using a pressure controlled by an amplitude curve.

For the linear elastic behavior of the material, the secant modulus of elasticity defined by the Brazilian standard NBR 6118:2023 - Projeto de estruturas de concreto — Procedimento [3] was adopted, along with a Poisson's ratio in accordance with the values specified in the same standard.

For the plastic behavior, the CDP (Concrete Damaged Plasticity) model was used, with properties recommended by the Abaqus 2024 Documentation [4]. This model requires the material behavior under compression and tension, which can be provided as an inelastic stress-strain curve or fracture energy (for tension). The curve adopted for concrete in compression follows the reference article by Ellobody and Bailey [1], based on EC2 [5] (Eurocode 2), considering concrete at room temperature. For concrete in tension, the model was based on fracture energy and a tensile strength equal to 10% of the cylindrical compressive strength.

The constitutive model adopted for the steel, used in the tendons and anchorages, considered linear elastic behavior with a typical modulus of elasticity and Poisson's ratio for the material. Plasticity was represented by the curve obtained from the experimental study provided by Ellobody and Bailey [1].

The use of reduced integration elements is justified by computational cost savings, since the full integration solid element (brick) with 8 nodes has 8 integration points, while its reduced integration counterpart has only 1, drastically reducing the computational cost. The use of symmetry is also justified for the same reason, since considering only a quarter of the structural model elements represents a processing time saving. It is important to note that for this to be done properly, one must enforce the symmetry boundary conditions on the faces of the structural model.

Several tests were performed during the finite element analysis of the model to ensure the proper functioning of prestressing. One test consisted of applying only the prestressing load, which proved effective, as it resulted in upward vertical displacements and represented bending behavior properly. Another verification was the analysis of stress concentration at the anchorage regions, a behavior consistent with practice, since in unbonded prestressing, stresses are transferred at the ends through the anchorages and also via the parabolic profile of the tendons, but allowing the cable to slide inside the concrete matrix.

In the context of the experiment conducted by Ellobody and Bailey [1], it is important to emphasize that, due to the incremental loading and the uncertainty regarding the material's strength limit, material nonlinearity (MNL) must be accounted for in the model. This approach aims to more accurately represent the structure's behavior, as the material properties may vary significantly under different stress levels.

3 Topology Optimization

The classical TO model for compliance minimization with volume constraint in the design domain Ω was adopted in the form:

$$\begin{aligned} \min_{\rho} \quad & c = \mathbf{f}^T \mathbf{u}(\rho) \\ \text{s.t.} \quad & \frac{\sum_{i=1}^{N} v_i \rho_i}{V_0} - f \leq 0, \end{aligned} \tag{1}$$

where c is the compliance; $\mathbf{f}$ is the internal force vector; $\mathbf{u}(\rho)$ is the nodal displacement vector of the model as a function of its density; v_i is the volume of the ith element; ρ_i is the density of the ith element; V_0 is the initial volume, and f is the volume fraction used in the constraint.

The studied model was discretized into 20183 eight-node linear brick elements (C3D8R) using the *Abaqus* finite element software. This discretization strategy was adopted to ensure a sufficient level of detail in capturing the structural behavior throughout the optimization process. The choice of element type and mesh density is particularly justified by the functioning of the SIMP (Solid Isotropic Material with Penalization) method, which forms the basis of the topology optimization formulation employed in this study.

$$E_i = \rho_i^p E_0 \quad \rho_{min} \leq \rho_i \leq 1, \tag{2}$$

where E_i and E_0 are the updated and initial elastic moduli, respectively; ρ_i is the density of the ith element; ρ_{min} is a predefined value to avoid singularities when operating with zero, and p is the penalization factor.

Topology optimization was carried out for distributed and point loads, respectively, with only the supports varying between continuous and point types. The cases analyzed are illustrated in Fig. 1. The material was assumed to be in the elastic regime in all topology optimizations.

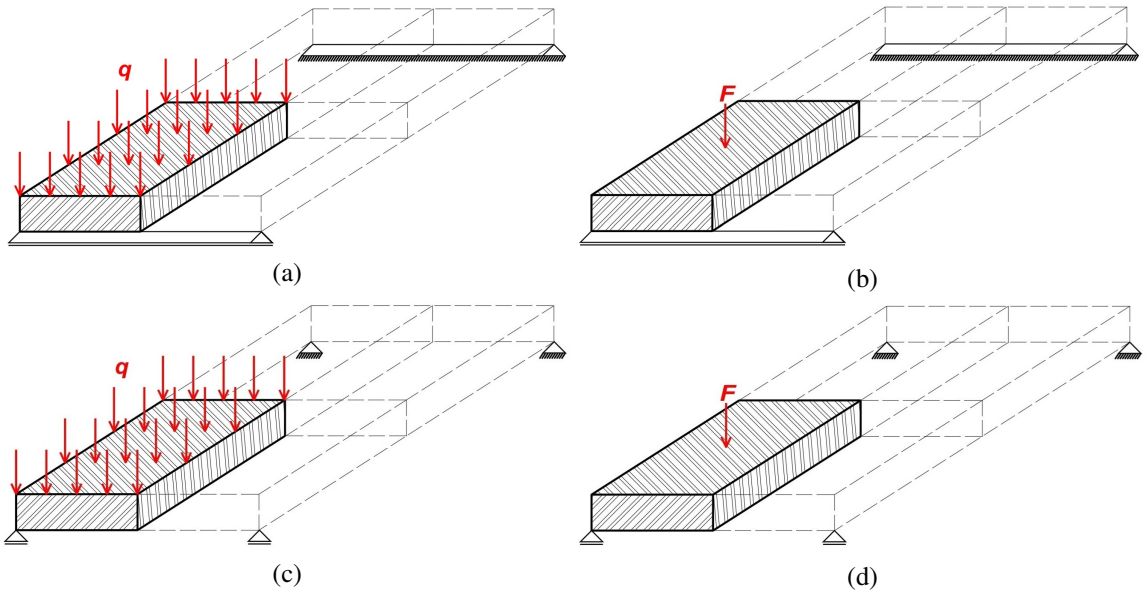
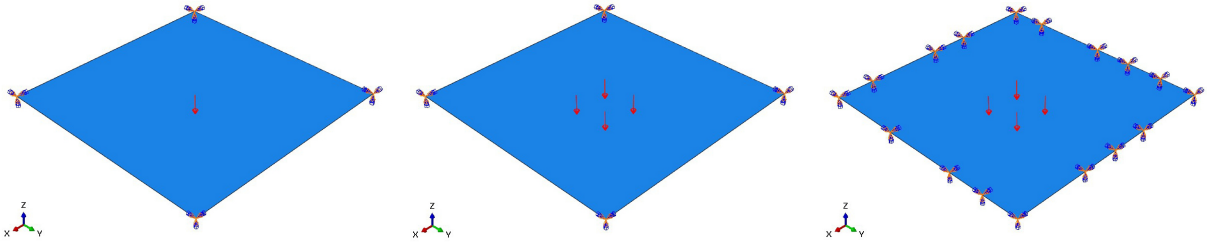


Figure 1. Optimized load and support conditions

Two optimization algorithms are employed in this study: the optimality criteria (OC) method and the Method of Moving Asymptotes (MMA). The optimality criteria method, as outlined by Bendsøe and Sigmund [6], consists of an iterative process in which the design variables—in this case, the material densities— are progressively adjusted to satisfy a predefined optimality condition. This approach seeks to ensure an efficient distribution of material throughout the structure, maximizing performance within the given constraints. Due to its simplicity and computational efficiency, it is particularly well-suited for problems with relatively straightforward formulations. In contrast, the MMA algorithm, introduced by Svanberg [7], is based on a more sophisticated and robust mathematical framework. It constructs convex approximations of the original problem, which are updated at each iteration through the strategic adjustment of moving asymptotes. This feature allows MMA to handle more complex and nonlinear optimization problems with greater stability and convergence reliability. Within the context of this work, both algorithms were employed and their performance was systematically evaluated. For additional details regarding the use of the OC method in *Abaqus*, the reader is referred to the work of Bakhtiary et al. [8].

4 Tool Validation

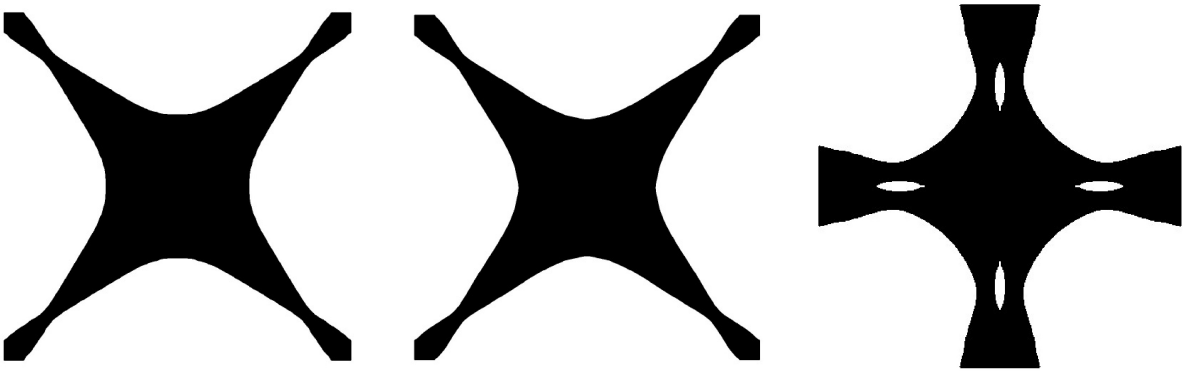
This section addresses the validation of the use of the TOSCA tool through the replication of a study presented in the literature by Ma et al. [9], using the BESO (Bi-directional Evolutionary Structural Optimization) method, considering the following loading and support cases:



(a) (b) (c)

Figure 2. Loading and support cases used for model validation

For this, TOSCA Structure was used in the optimization of the load models, as done by Ma et al. [9], who obtained the solutions using their own algorithm. The optimization aimed at minimizing compliance, subject to a 30% volume constraint (formulation of the eq. (1)), resulting in the topologies shown below:



(a) (b) (c)

Figure 3. Optimized topologies obtained with TOSCA Structure

The topologies obtained by Ma et al. [9], using the SIMP method (Fig. 4a) and BESO (Fig. 4b and Fig. 4c), are presented below for comparison. The topologies are similar and we emphasize that Ma et al. [9] used BESO instead of SIMP in the material parametrization. It is also worth noting that, according to Ma et al. [9], a mesh with 10000 four-node shell elements with reduced integration was used.



(a) (b) (c)

Figure 4. Optimized topologies obtained by Ma et al. [9]

5 Application and Results

To reproduce the analysis results presented by Ellobody and Bailey [1], a mesh with 6238 finite elements was used. From the model, the load versus displacement curve was extracted and compared with the experimental and numerical data provided by the authors, as shown in Fig. 5. It is noted that in all numerical analyses monotonic loading was applied.

The applied loads consisted of a distributed load of 6.12 kN/m², and an incremental point load controlled by an amplitude curve. The model adopted a span of 4 meters for the slab with an eccentricity of 38 mm at mid-span for the prestressing cable, as described by Ellobody and Bailey [1]. For the prestressing steel, an elastic modulus of 200 GPa and a Poisson's ratio of 0.3 were considered.

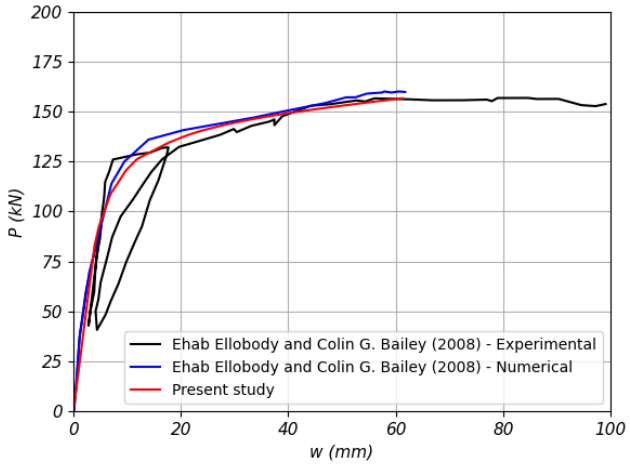


Figure 5. Load-displacement curve: experimental vs numerical

With the model validated, a comparison was made between the two types of prestressing: bonded and unbonded. Figure 6 shows the load-displacement curve obtained for both situations. The essential difference lies in the modeling of the contact between the cable and the surrounding concrete. In the case of perfect bonding, a tie constraint is imposed along the cable's length. The behavior of the curve is consistent with findings by Brenkus et al. [10], demonstrating the consistency of the obtained results.

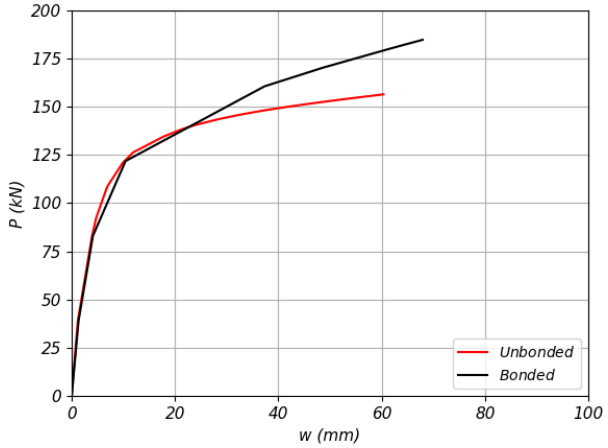
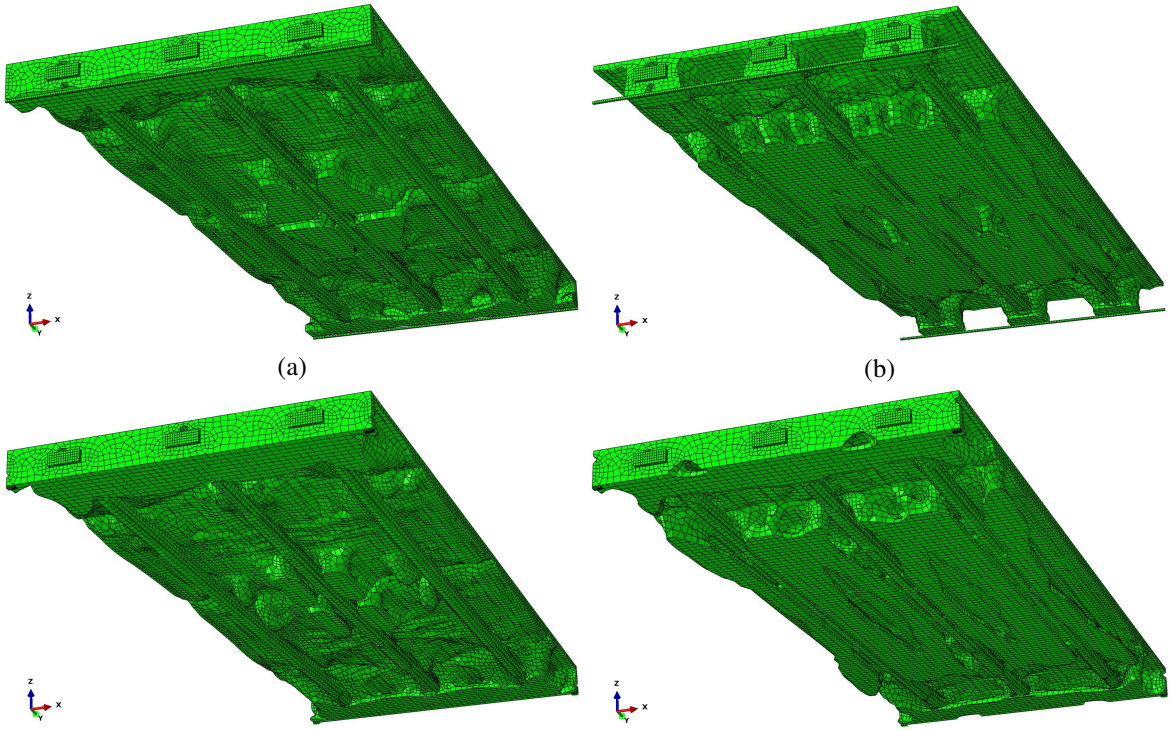


Figure 6. Load-displacement curve: bonded vs unbonded

Optimization was carried out for the model using the MMA algorithm, freezing the top face and anchorage regions, which resulted in different topologies for the respective loading and support cases from Fig. 1, as shown in Fig. 7. The regions around the cables were preserved without the need for freezing. For the TO, distributed and point loads of 4.8 kN/m^2 and 15 kN, respectively, were used.

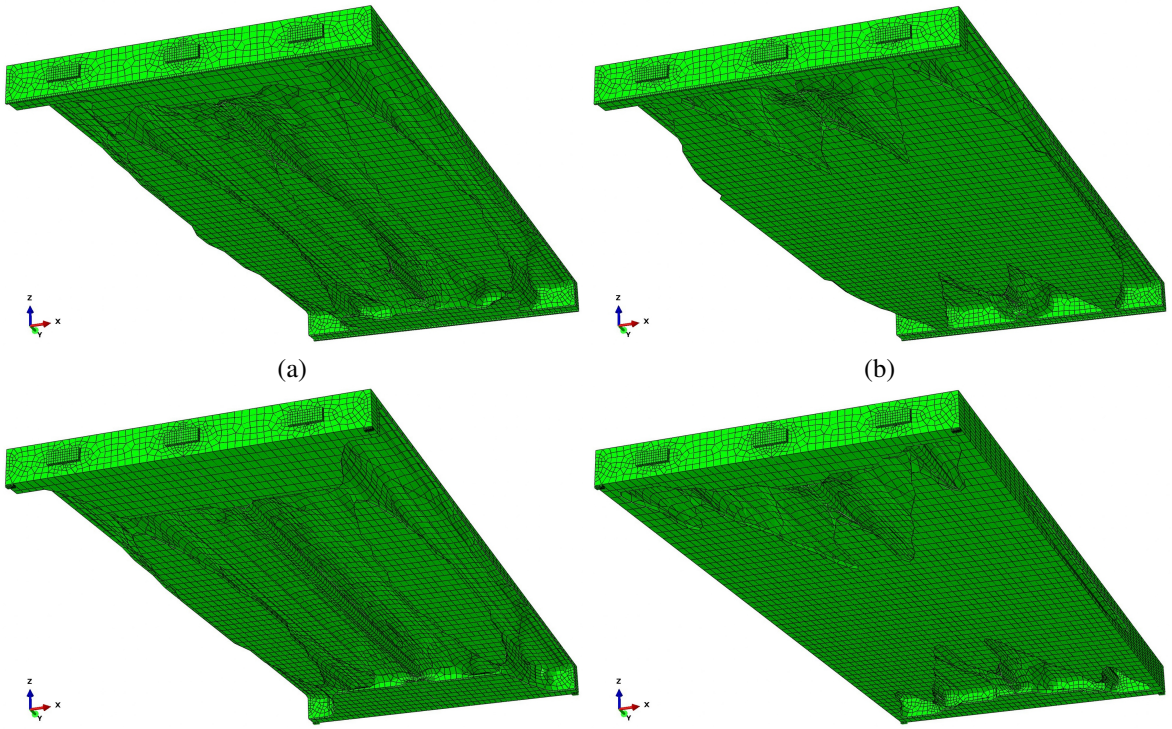


(a) (b)

(c) (d)

Figure 7. Topologies obtained with MMA

Using the same procedure, but now with OC, optimizations were carried out, resulting in the topologies shown in Fig. 8. Unlike the previous solution algorithm, freezing was required in the regions around the cable to ensure cover adequacy.



(a) (b)

(c) (d)

Figure 8. Topologies obtained with OC

6 Conclusion

The proposed analysis model successfully simulated both bonded and unbonded prestressing, with results consistent with the literature [1]. In the context of topology optimization, the results show that the conceptual topologies achieved the objectives, maximizing stiffness without violating the volume constraint for any of the analyzed cases.

The validation of *TOSCA Structure* proved to be both necessary, as demonstrating the proper functioning of the tool is essential to ensure its credibility. Furthermore, the fact that the results obtained were consistent with those found by Ma et al. [9] — even when generated by custom implementations from other authors — reinforces the reliability of the software and highlights its robustness in the context of topology optimization.

The use of the OC algorithm proved to be more computationally efficient, which aligns with what is described in the Abaqus 2024 Documentation [4]. It also has the advantage of controlling the quantity and size of the cycle, which can be set to: very small, small, moderate, medium, large, or dynamic. However, the OC algorithm has limitations: the design responses are limited only to volume and strain energy; the algorithm cannot perform simultaneous multi-material optimizations, requiring one material to be optimized at a time. For this algorithm, freezing was required around the cable and near the supports, since without it, elements were inappropriately removed. The MMA algorithm proved superior in several aspects, such as the variety of design responses, including stress, vibrations, displacements, among others. The algorithm also allows for multi-material optimization. There was no need to freeze the boundary regions around the cable using this algorithm.

The conceptual designs obtained with both algorithms were efficient in meeting the imposed objective. The complex shapes challenge fabrication processes, so the aim was to demonstrate the feasibility of the optimization procedure and present concepts. For the appropriate use of the models, later shape and size optimizations must be considered. The main benefit found from the structure's topology optimization was the clear reduction in concrete volume, which translates to a reduction in material cost. A future study in this regard should address aspects of shape and size optimization, possibly also focusing on improving constructability and incorporating material nonlinearity into the topology optimization process to achieve more realistic and applicable structural designs.

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