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Reliability Analysis of Cold-Formed Steel Columns Subjected to Load Combinations Prescribed by Brazilian and North American Structural Codes

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Abstract. The structural safety of columns made of cold-formed steel sections, when the wind load is considered in the analysis, requires further studies for a more in-depth understanding, as most reliability studies focus on load combinations involving only permanent and live loads. This paper proposes the evaluation of the reliability of columns designed according to the Direct Strength Method (DSM). Using a comprehensive database, a statistical analysis of the variable "model error" was performed, defined as the ratio between the experimentally measured strength and the strength calculated via DSM. Based on this database, a statistical analysis of the test results was carried out to determine the probabilistic distribution that best fits the model error data and its relevant distribution parameters. Variables related to material strength and geometric properties were incorporated into the reliability evaluation. The structural reliability levels were assessed using the FORM (First Order Reliability Method) for columns designed according to the design requirements of the standards NBR 14762 (ABNT, 2010) and AISI S100 (AISI, 2024). The results indicate that both standards provide adequate levels of reliability, with emphasis on the uniformity of the ASCE 7-22 load combinations. The Brazilian standard showed greater sensitivity to variations in loading, especially when wind is the predominant action. Most of the evaluated combinations achieved the target reliability index of $\beta = 2.5$, although in some cases, values below this threshold were observed for the Brazilian standard.

Keywords: reliability; columns; cold-formed; FORM method.

1 Introduction

The safety of cold-formed steel columns has traditionally been assessed based on combinations of gravitational loads, however the consideration of wind load is essential, especially for structures featuring a high strength-to-weight ratio. Although these members offer advantages such as lightness and ease of fabrication, they also pose challenges related to stability.

The FOSM (First Order Second Moment) method is widely used in structural reliability studies of cold-formed steel members due to its simplicity and ease of application. However, its main limitation lies in the assumption of linearity in the performance function and the consideration of only the first two statistical moments (mean and variance), which can lead to inaccurate reliability estimates in systems with nonlinear behavior or non-normal distributions. Given these limitations, the FORM (First Order Reliability Method) emerges as a more robust and accurate alternative, as it allows the linearization of the performance function at the most probable point of failure, providing more representative results in reliability analyses of cold-formed steel members.

When applying the FORM method, in addition to the model error variable, which compares experimental strengths with predicted values, variables such as geometry, material strength, and loading conditions must also

be considered.

The Direct Strength Method (DSM) consists of calibrated curves used to determine the resistance of members subjected to axial compression and/or uniaxial bending, accounting for local, distortional, global buckling modes and their interactions. It is a simple and efficient method, as it provides accurate and safe estimates based on the buckling behavior of the member and the yield stress of the steel used. The DSM curves were calibrated and validated using experimental and numerical results for cold-formed steel members (Schafer [1]). Currently, the DSM is incorporated into the Brazilian code (ABNT NBR 14762:2010 [2]), the American code (ANSI AISI-S100-24 [3]), and the Australian/New Zealand code (AS/NZS 4600:2005 [4]). In this study, the DSM is used to evaluate the performance of columns, with the load combinations involving dead load, live load, and wind obtained from ASCE 7-05 [5], ASCE 7-22 [6], and NBR 8681 (2003) [7].

A comparison between the standards helps identify which one aligns more closely with the reliability indices of LRFD. This study contributes to filling gaps in the literature by providing an in-depth analysis of the reliability of cold-formed steel columns under multiple loads, with a focus on improving design standards and practices for steel structures.

2 Literature Review

2.1 Structural reliability

The safety or failure of a structure has traditionally been quantified by its probability of failure, P_f. The failure function, represented as G(.) = R - S, where R is the resistance and S is the load effect, defines the failure domain as the set of conditions where $G \leq 0$. In this context, the probability of failure (P_f) is the probability that G is less than or equal to zero that is, $P_f = P(G \leq 0)$. In other words, the failure domain corresponds to situations in which the resistance is less than or equal to the demand, resulting in system failure

The reliability of a structure is defined as the complement of the probability of failure P_f, that is, $C = 1 - P_f$, and can be expressed in terms of a reliability index β. The target reliability index is the minimum established value for β, associated with a maximum allowable probability of failure that reflects the acceptable level of risk for the structure. The main criteria for defining an appropriate target reliability index are the consequences of failure and the costs associated with increasing reliability.

The mathematical relationship between β and P_f is defined by eq. (1).

$$\beta = -\Phi^{-1}(P_f). \tag{1}$$

where Φ^{-1} is the inverse of the standard normal cumulative distribution function Φ;

The main idea of FORM is that reliability can be easily obtained by a linear failure function considering the space of statistically independent reduced variables, and by evaluating the shortest distance from the failure function to the origin. In the case of linear limit state functions and uncorrelated normal variables, the result obtained is exact. If the functions are nonlinear, the approximation depends on the curvature of the failure function as well as on the derivatives at the points under analysis to compose the gradient vector. In this method, there are two processes: the transformation of variables from the original space to the standard normal space, and the search for the design point. The transformation of variables is done using equivalent normal distributions, and the design point can be obtained by solving an optimization problem. The most used optimization algorithm is the HLRF developed by Hasofer and Lind [8], and improved by Rackwitz and Fiessler [9].

The failure function G(.) can be rewritten in terms of the nominal resistance (R_n), Nominal values of dead load (D_n), live load (L_n) and wind load (W_n), as well as the dimensionless normalized random variables $\mathbf{R} = R/R_n$, $\mathbf{D} = D/D_n$ e $\mathbf{L} = L/L_n$, $\mathbf{W} = W/W_n$. To avoid compromising the accuracy of the results when using a single variable R in the reliability analysis, $\mathbf{R} = \mathbf{MFP}$ is adopted. For convenience, the normalized random variables are shown in bold. In eq. (2), the variable R depends on uncertainties in material properties, cross-sectional geometry, and model error, which are represented by the dimensionless normalized variables **M** (material factor), **F** (fabrication factor) e **P** (professional factor), respectively.

$$G(.) = R_n \boldsymbol{MFP} - (D_n \boldsymbol{D} + L_n \boldsymbol{L} + W_n \boldsymbol{W}). \tag{2}$$

The normative design criteria using the limit state method can be expressed by eq. (3), when the structural

component is subjected to a combination of loads (D + L+W).

$$\phi R_n \geq \gamma_D D_n + \gamma_L L_n + \gamma_W W_n. \tag{3}$$

Where:

ϕ = Resistence factor;
γD = Dead load factor;
γL = Live load factor;
γW = Wind load factor.

The definition G(.) for a component with known nominal resistance (R_n), involves applying ϕ, γD and γL to transform the limit state design equation into an equality. D_n e L_n are obtained through eq. (4) and the definition of the load ratios $\rho = L_n/D_n$ and $\alpha = W_n/D_n$.

$$D_n = \frac{\phi R_n}{\gamma_D + \rho\gamma_L + \propto \gamma_w}. \tag{4}$$

It is noteworthy that in the Brazilian code format, the resistance factor ϕ is relaced by a partial factor γ greater than one. The resistance factors for the compression limit state are ϕ=0.85 (AISI [3]) and γ=1.2 (NBR [7]).

2.2 On the update of wind load codes

Among the loads acting on a structure, wind load is one of the most impactful for design, especially for structures composed of lightweight profiles. In Brazil, the criteria required for calculating this load are presented in [10]. This standard, which had not been revised since 1988, was updated and brought a series of significant changes. However, no consensus was reached within the Special Study Committees regarding modifications to the isopleth map (contour lines that represent constant wind speed values).

NBR 6123:2023[10] establishes that the characteristic values of variable loads are defined by consensus in specific standards and correspond to values that have a probability of 25% to 35% of being exceeded in 50 years, in the unfavorable direction.

In the North American context, engineers who were familiar with wind speeds from ASCE 7-05 [5] noticed that the wind speeds in ASCE 7-10 [11] were substantially higher. In ASCE 7-05 [5], the wind speed map was developed considering a return period (T) of 50 years. This value was applied universally to all structures, regardless of their importance or risk category. The main change following the revision of this standard was the separation of wind maps by risk categories, eliminating the need for the importance factor. Based on this approach, a typical building went from a design wind load with a 50-year return period [5] to a design wind load with a 700-year return period [11]. However, the wind load factor decreased from 1.6 [5] to 1.0 [11]. Since wind load is a function of the square of the wind speed, a 1.6 factor on wind load is approximately equivalent to a 1.26 factor on wind speed. If you multiply this by the wind speed map from ASCE 7-05 [5], you get wind speeds approximately equal to those in the ASCE 7-10 [11] map. Therefore, performing calculations using ASCE 7-05 [5] or the latest version results in minimal changes in structural design, depending on the intensity of the wind load in comparison to other loads acting on the structure.

In the latest update, ASCE/SEI 7-22 [6] refined the wind maps to account for extreme weather events and their increased frequency, especially in coastal regions. Adjustments were made to aerodynamic coefficients and to the influence of topography on wind speed.

2.3 Load combinations

The history of applying reliability methods for code calibration purposes has shown that, for most design situations, the three load combinations (Ravindra and Galambos [12]) presented in Tab. 1 are sufficient.

NBR 8681 [7] specifies the combination factors for variable actions, which are used to reduce the nominal load based on the probability that their maximum value does not occur simultaneously with other actions. In other words, Ψ serves to reduce the extreme value of a given action to its value "at an arbitrary instant," or similar. These factors reflect the temporal variability of loads and the low statistical correlation between different types of loading, such as wind, live load, among others. Thus, in action combinations, the principal variable load is considered at its nominal factored value, while the other variable actions are reduced by Ψ to represent the lower probability of

simultaneous occurrence of their extreme values. In particular, the expression. $\gamma\Psi_L$ is adopted to represent the reduced variable load in action combinations. This formulation can be written as "$\gamma_{apt}.L_{apt}$", where the subscript "apt" indicates the value of the variable load at an arbitrary instant in time, highlighting the stochastic nature of these actions. This approach is essential to avoid overly conservative designs, ensuring structural safety with probabilistic realism.

Table 1. Load Combinations According to NBR 8681:2003 and ASCE Standards (2005 and 2022)

#	Comb.	NBR 8681:2003 [7]	Note	ASCE 7-05 [5] (LRFD)	ASCE 7-22 [6] (LRFD)
1	D + L	1.25D + 1.5L		1.2D + 1.6L	1.2D + 1.6L
2	D + L + W	1.25D + 1.5L + 0.84W[(a)]	$L_{pricipal}$	1.2D + 1.6W + 1.0L[(c)]	1.2D + 1.0L + 1.0W[(c)]
		1.25D + 0.75L + 1.4W[(b)]	$W_{principal}$	1.2D + 1.6W + 0.5L[(d)]	1.2D + 0.5L + 1.0W[(d)]

(a) $\gamma_w\Psi_L = 0.6x1.4 = 0.84$ (Effective combination value for secondary action) [7];
(b) $\gamma_L\Psi_L = 0.5x1.5 = 0.75$ (Effective combination value for secondary action).[7];
(c) L > 100 pounds force per square [5] [6];
(d) L ≤ 100 pounds force per square [5] [6].

Turkstra's rule (Melchers & Beck [13]) is the simplest and most practical load combination model, and as such serves as the basis for LRFD combinations. Turkstra's rule assumes that when a load reaches its maximum value during the design service life of a structure or structural member (typically 50 years for most occupancy types), other loads are assumed to be at their values at an arbitrary point in time (sometimes also referred to as instantaneous or sustained values).

2.4 Random variables

The statistical data related to the normalized random variables associated with loads (**D**, $\mathbf{L_{apt}}$, $\mathbf{L_{max}}$ e $\mathbf{W_{max}}$) and resistance (**M**, **F**, **P**), which contribute to the performance function, including their distribution types, means (μ), and coefficients of variation (δ), are available in existing literature and are summarized in Tab. 2 and 3.

Table 2. Statistical data of the normalized random variables.

Load	μ	δ	PDF	Reference	Data used in the study
D	$1.05D_n$	0.10	Normal	Ellingwood et al. [14]	NBR [7]/ASCE 7-22 [6]/ASCE 7-05 [5]
$\mathbf{L_{apt}}$	$0.25\ L_n$	0.55	Gamma	[14]	NBR [7]/ASCE 7-22 [6]/ASCE 7-05 [5]
$\mathbf{L_{50}}$	$1.00\ L_n$	0.25	Gumbel	[14]	NBR [7]/ASCE 7-22 [6]/ASCE 7-05 [5]
$\mathbf{W_1}$	$0.33\ W_n$	0.59	Gumbel	[14]	NBR [7]
$\mathbf{W_{50}}$	$0.78\ W_n$	0.37	Gumbel	[14]	NBR [7]/ASCE 7-05 [5]
$\mathbf{W_{700}}$	$0.47\ W_n$	0.47	Gumbel	McAllister, Wang, Ellingwood [15]	ASCE 7-22 [6]

Table 3. Statistical data of variables M, F, P.

Variable	PDF	μ	δ	σ	Reference
M	Lognormal	1.10	0.10	0.11	[14]
F	Lognormal	1.00	0.05	0.05	[14]
P	Lognormal	1.06	0.15	0.016	Freitas et al. [16]

3 Results

As shown in Fig. 1, both reliability curves are very close throughout the entire range of ρ values analyzed, indicating consistency between the safety criteria of the two codes, despite the different partial safety factors applied to permanent and variable loads. For ρ > 1,5, the curve based on the AISI [3]/ASCE [5][6] code presents slightly higher β values, suggesting greater structural reliability when the live load is predominant. However, for ρ < 1.5, this trend is reversed: the curve based on the NBR [7] shows higher β values, indicating better performance of the Brazilian load combination when the dead load becomes more significant.

Another relevant aspect is that both curves remain above the target reliability index ($\beta_0 = 2.5$). The AISI [3]/ASCE [5][6] curve does so throughout the entire range of ρ, while the NBR [7] curve remains above the target only up to approximately ρ = 2. This highlights a greater sensitivity of the NBR [7] formulation to variations in ρ in situations where the variable load is dominant.

The statistics of the random variables were adopted based on well-established references, with emphasis on the significant variability of the variable load, in contrast to the lower variability of the permanent load D.

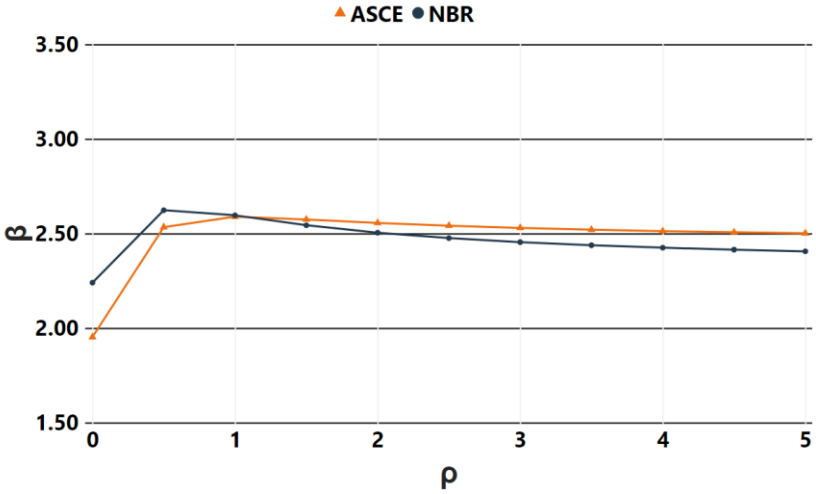


Figure 1. Variation of the reliability index of cold-formed steel columns according to NBR and AISI codes for load combination N.º 1 (D+L).

Figures 2(a) and 2(b) show the variation of the reliability index β as a function of the ratio $\rho = L_n/D_n$, considering different values of the ratio $\alpha = W_n/D_n$, as indicated in the legend. Figures 2(c) and 2(d), in turn, present the variation of β as a function of α, for different values of ρ. In all figures, the load combinations prescribed by ASCE 7-05 [5] (blue curves) and ASCE 7-22 [6] (orange curves) are compared, as well as the combinations defined by NBR 8681:2003 [7] with L as the main load (green curves) and W as the main load (red curves).

It can be observed that, for the ASCE 7-05 [5] and ASCE 7-22 [6] cases (Fig. 2a and 2c), the curve bundles exhibit low spread, indicating uniformity of β values across the variations in ρ and α. This uniformity is desirable from a structural reliability standpoint, as it suggests that changes in the ratio between loads do not lead to critical fluctuations in performance. Moreover, most of the curves remain above the target threshold of $\beta_0 = 2.5$. When comparing ASCE 7-05 [5] and ASCE 7-22 [6], both codes show similar performance, with ASCE 7-22 [6] displaying a slight increase in the average β values, especially for α greater than 2. This suggests a marginal gain in safety with the adoption of the new ASCE 7-22 [6] load combination.

In Fig. 2(b) and 2(d), a more dispersed behavior is observed in the curves associated with the load combinations defined by NBR 8681 [7]. The combination with L as the main load (green curves) presents a more uniform set of curves, generally maintaining β values above 2.5 throughout the variations in ρ and α. In contrast, the combination with W as the main load (red curves) exhibits greater variability; some curves reach β values close to 2.5, while others exceed 3.5 for certain variations of ρ and α. This behavior indicates a higher sensitivity of reliability to the increase in wind load in this case, which may be attributed to the greater statistical variability of this load.

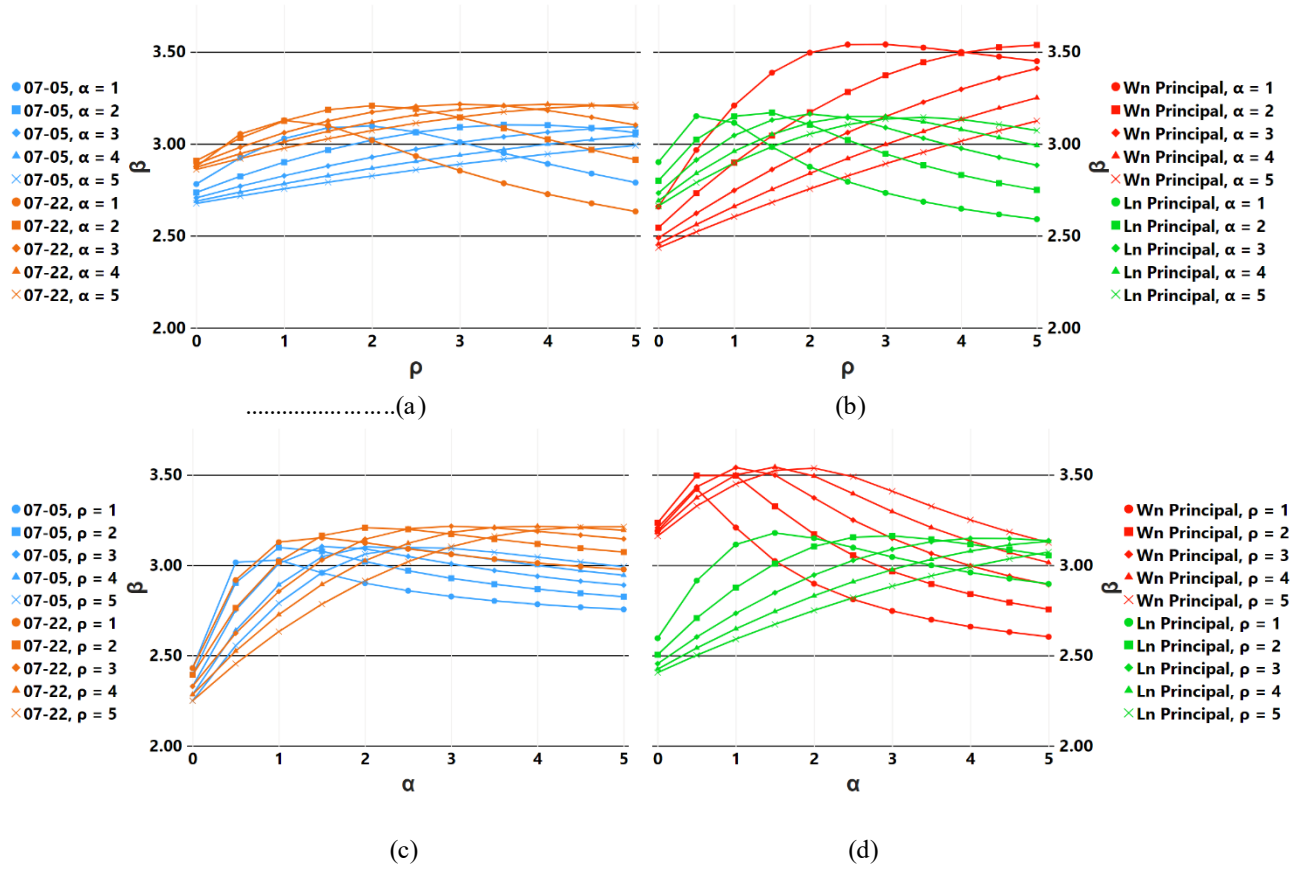


Figure 2. Variation of the reliability index for combination No. 2 (D+L+W): (a) β as a function of ρ for the ASCE7-5 and ASCE7-22 combinations. (b) β as a function of ρ for the NBR 8681 (2003) combinations. (c) β as a function of α for the ASCE7-5 and ASCE7-22 combinations. (d) β as a function of α for the NBR 8681 (2003) combinations.

Considering the pairs of graphs with the same horizontal axis, it is possible to directly compare different codes. It can be observed that the ASCE [5][6] standards exhibit more stabilized curves, with reliability indices generally concentrated around $\beta = 3$. In contrast, the NBR [7] combinations show significantly greater variability in reliability indices for different values of ρ and α, which is even more notable in the curves where wind (W_n) is the main load. This result can be attributed to the differences in load combinations and the statistical characteristics adopted for the variable loads.

4 Conclusions

The main conclusions of the reliability analysis can be summarized as follows:

- Both the NBR [7] and ASCE [5][6] standards provide adequate levels of structural reliability over most of the analyzed range of the ratio ρ, with minor differences depending on the relative predominance between permanent and variable loads.

- Load combinations that promote greater uniformity of reliability indices for variations of ρ and α are desirable, especially when most β values remain above the threshold of 2.5. In this context, the performance of the combinations proposed by the ASCE [5][6] standards stands out, particularly the ASCE 7-22 [6] version.

- The NBR [7] combinations showed greater sensitivity to variations in ρ and α, especially when wind is considered the main load, reflecting a higher dispersion of β values and, in some cases, performance below the desired level.

- Wind load, in particular, proved to be a significant source of uncertainty, more markedly affecting the combinations in which it is predominant.

- When comparing ASCE 7-05 [5] with ASCE 7-22 [6], it was observed that the reliability indices are slightly higher for values of the ratio α greater than 2. It was found that the increase in the return period (T) during the standard update was, to some extent, offset by the change in the wind load weighting factor, which decreased from 1.6 [5] to 1.0.

The results indicate that the formulation of load combinations in the Brazilian standard, especially regarding wind load, can be improved. The adoption of more updated statistical models and reliability-based approaches may make the design criteria more consistent with international practices.

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