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Structural Analysis of Railway Pavements Considering Track Modulus and Rail Effects

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Abstract. One of the main parameters that needs to be understood for the structural behavior of railways is the track modulus (pavement stiffness), to understand how it influences the forces. Estimating vertical displacements is essential both in the design and maintenance stages of the permanent way since the track elements are dimensioned based on the expectation of a global response. In this sense, this research carried out numerical-analytical analyses for a TR – 68 rail profile based on experimental data for different track conditions of the Carajás Railway. The results showed the sensitivity of this parameter to evaluate the quality of the track, where minimum and maximum limits were considered to maintain good elasticity of the system and operational safety in comparison to what is proposed by the AREMA standard.

Keywords: Railways, Modulus Track, Structural Analysis.

1 Introduction

The increasing loads per axle and the frequency of railway operations have intensified the demands on the structural performance of the permanent way, making it essential to adopt more rigorous design and maintenance criteria. Among the various parameters that influence the response of the pavement, the track modulus stands out for condensing, in a single value, the combined stiffness of elements such as rail, sleeper, ballast, sub-ballast, and subgrade. Despite its recurrent use in empirical and semi-empirical evaluations, gaps remain in understanding the direct effects that its variation has on the internal forces in the rails, especially when seeking an analysis aligned with normative safety criteria. In this scenario, computational modeling in conjunction with analytical models emerges as a promising tool to deepen the understanding of the behavior of the railway superstructure under different stiffness conditions.

2 Theoretical Background

2.1 Talbot Model

In 1914, a research group was formed under the leadership of Engineer Arthur Newell Talbot to determine the displacements in permanent ways. The group was funded by ASCE (American Society of Civil Engineers) and AREA (American Railway Engineering Association), in addition to support from the University of Illinois, which used its laboratories for research. In developing the experiments, they concluded, similarly to what Winkler defined, that the model of continuous and elastic supports under the rail was the most representative of the mechanics of the problem. It was also concluded that the hypothesis of considering continuous supports or under sleepers presented a minimal difference, as the sleepers are robust and closely spaced. Thus, in general, the

modulus of elasticity of the track was defined as the ratio between the load to which the rail will be subjected to cause a deflection in the track, according to the following relationship:

$$p = -u \cdot y \tag{1}$$

Where the pressure p will be directly proportional to the deflection of the track y, given the track modulus u as the coefficient that relates them.

Solving the differential problem using successive derivations, Talbot [1] found the following formulations for deflection y and moment M, respectively. In which, P is the load of the wheel, E and I are the modulus of elasticity and the moment of inertia, in the same order with $2{,}1.10^5$ N/mm^2 and $3{,}920.10^7$ mm^4, and λ is the inverse of the elastic length of the rail of the rail, defined as $\lambda = \sqrt[4]{u \,/\, 4 \cdot E \cdot I}$. In this research, the formulations are presented already in a more generalized way considering the superposition of effects, as follows:

$$y(x_i) = -\frac{P_i}{\left(64.E.I.u^3\right)^{1/4}} \sum_{i=1}^{N} e^{-\lambda.x_i}.[\cos(\lambda.x_i) + \sin(\lambda.x_i)] \tag{2}$$

$$M(x_i) = P_i.\left(\frac{E.I}{64.u}\right)^{1/4} .\sum_{i=1}^{N} e^{-\lambda.x_i}.[\cos(\lambda.x_i) - \sin(\lambda.x_i)] \tag{3}$$

Complementing these structural responses, the stress in the lower fiber of the rail skid σ can be considered as a function that depends on the moment M, the cg, which is the center of gravity of the profile, of the moment of inertia I of the rail, and the shear Q , which is also considered, given by:

$$\sigma(x) = \frac{\sum_{i=1}^{N} M(x_i).cg}{I} \tag{4}$$

$$Q(x) = \frac{P_i}{2}.\sum_{i=1}^{N} e^{-\lambda.x_i}.\cos(\lambda.x_i) \tag{5}$$

2.2 Stiffness Considerations in Railway Quality

There are several factors that can interfere with determining the rigidity of a permanent track, such as layer thickness, subgrade quality, ballast, weather conditions, fastenings, and type of sleeper, among others. The track modulus is used as a global parameter that determines the quality of a railway. Generally, it is intuitive to understand that the higher its value, the better the conditions of the railway pavement.

Talbot recorded track modulus limits between 3.7 and 35.7 MPa, according to Bastos [2]. Ahlf [3] considered that values for track modulus below 14 MPa were considered poor; between 14 MPa and 28 MPa would be a track of average quality; and above 28 MPa would be a good track. Alias [4] already defines a poor track with a rigidity of 9.6 MPa; average 28.8 MPa, and good with 57.6 MPa. Hay [5] and Area [6] recommended 14 MPa as the minimum satisfactory value. Raymond [7] suggests a modulus track between 34 and 69 MPa. Stopatto [8] comments that, in Brazilian broad gauge railways with wooden sleepers, track modulus was found in the range of 14 MPa, similar to that found in American railways, and which would be a reasonable value.

Bastos [2] reports on the work carried out by the Instituto de Pesquisas Tecnológicas (IPT) in 1985, at the time of the Companhia Vale do Rio Doce, which determined track modulus with 41.5 and 67.8 MPa, which could correspond to good quality tracks. In 1993, the same Institute took new measurements at different points of the railway and recorded 42 and 37 MPa, which also indicated a good quality railway.

The AREMA Manual [9] also defines some track modulus values, given the conditions presented below:

- Track with wooden sleeper, after compaction → 6.89 MPa;
- Track with wooden sleeper, compacted by traffic → 20.68 MPa;
- Track with plastic sleeper, compacted by traffic → 20.68 MPa;
- Track with concrete sleeper, compacted by traffic → 41.37 MPa;
- Track with wooden sleeper, frozen ballast, and subgrade → 62.05 MPa.

Observing the criteria defined by the authors, none of them suggests that the track modulus should be very high. This is because the track needs to reconcile rigidity with good elasticity for better dissipation of forces.

In terms of the durability of the railway pavement, Lundgren *et al*. [10], in Fig. 1, proposed safety ranges of displacement in mm from the top of the rail based on the useful life of the track that were analyzed in the field. These ranges are defined as follows:

a) A, range in which the track is in very good condition.

b) B, maximum normal deflection desirable for freight railways to obtain a good relationship between flexibility and rigidity of the permanent track.

c) C, maximum desirable deflection limit for light permanent tracks.

d) D, permanent track that is poorly maintained and will deteriorate quickly.

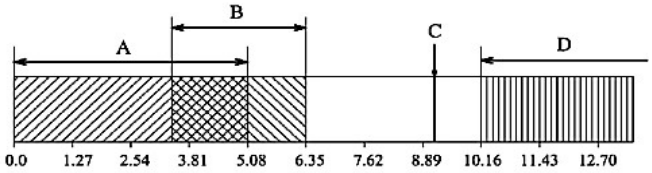


Figure 1. Operational safety ranges as a function of displacement in mm. Source: Lundgren *et al*. [10]

The AREMA manual does not explicitly define safety ranges, but recommends that displacements be in the range between 3.175 and 6.35 mm (equivalent to range B, in Fig 1.). Furthermore, it is worth noting that, in range A, despite being the best range, displacement values below 1.0 mm can make the track extremely rigid, which can consequently have the opposite effect of damaging the permanent way.

3 Methodology

The experimental variations of the track modulus raised by Costa [11] will be considered for the freight car and passenger car cases. The chosen track modulus ranges from a track that registered a minimum value of 9 MPa for freight transport and 5 MPa for passengers, intermediate measurements of tracks that are in good condition, to one that had been recently built at the time with a value of 84 MPa, which was the highest stiffness measured. These values are detailed in Tab. 1 and 2 based on the equivalent stiffness of Teixeira [12], adapted for the Talbot model in Equation 6. In the Fig. 2 illustrates the longitudinal model where *d* was considered for the spring spacing of 600 mm in the numerical analysis, which would be equivalent to the spacing between sleepers.

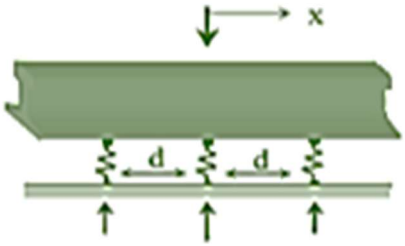


Figure 2. Beam on Elastic Foundation considering equivalent stiffness. Source: Teixeira [12]

$$k_{eq} = \sqrt[3]{\frac{\left(\frac{P}{y_0}\right)^4}{64 \cdot E \cdot I}} \cdot d \tag{6}$$

Where k_{eq} is the equivalent stiffness for a given spacing d, P is the wheel load, y_0 is the displacement generated at the point of application, E and I are the modulus of elasticity and the moment of inertia of the rail. This procedure is used to reduce the number of springs in the model and make it more practical without losing physical representation of the continuum.

The displacement graphs obtained from the SAP2000 simulations are presented in comparison with the analytical results generated in MATLAB. The displacement was selected as the primary parameter in this research, aiming to evaluate the safety limits and performance of the track structure according to the criteria established by Lundgren *et al.* [10], as well as the limits defined by AREMA [9]. The other internal forces will also be presented, but additional details on their numerical analysis can be found in Ribeiro [13] and Ribeiro *et al.* [14]. The track

modulus (stiffness *k*) presented in the second column of Tab. 1 and 2, compared to those obtained by Costa, were recalculated based on eq. 2, considering the applied load and the displacement recorded by Costa, where the maximum is given with the wheel positioned at the zero point. This adjustment was necessary to obtain accuracy in the field monitoring of the displacements. The third column of both tables considers the equivalent stiffness calculated by eq. 6. The loads adopted for the freight wagon and passenger car were 32.5 and 9.3 tons per axle, respectively.

Table 1. Equivalent stiffness used for freight wagon in MPa.

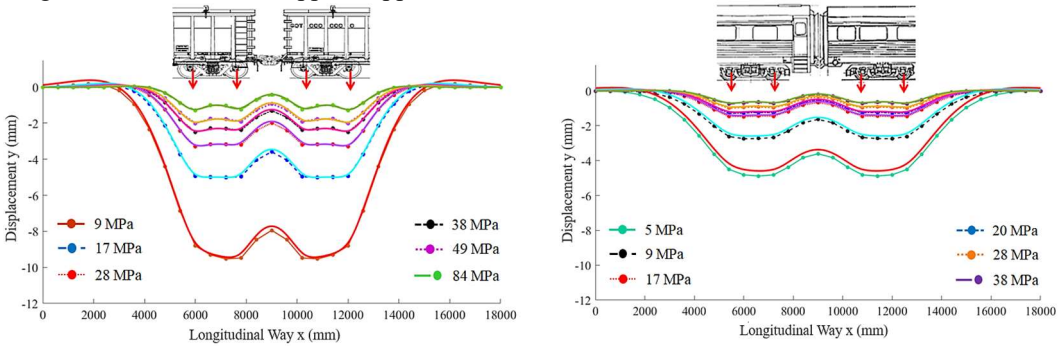
Stiffness Costa (2016)	Stiffness (Authors)	Equivalent Stiffness (Authors)
9	8.82	5.29
17	17.26	10.35
28	27.72	16.63
38	38.33	23.00
49	48.71	29.22
84	83.83	50.30

Table 2. Equivalent stiffness used for passenger car in MPa.

Stiffness Costa (2016)	Stiffness (Authors)	Equivalent Stiffness (Authors)
5	4.68	2.80
9	8.82	5.30
17	17.26	10.35
20	19.90	11.94
28	27.72	16.63
38	38.33	23.00

4 Results

As discussed previously, analytical models based on beam on elastic foundation theory accurately represent longitudinal track behavior. This is confirmed in Fig. 3, where the analytical results (solid lines) closely approximate the numerical results (dotted lines) for each track modulus value. The near-overlap of the pairs of curves (each using the same modulus), validates the analytical model over the numerical one in all stiffness cases, confirming the robustness of the applied approach.



(a) (b)

Figure 3. Displacements varying the track modulus. In (a) – Freight Wagon; (b) – Passenger Car.

Regarding vertical displacements, it is observed that, for low track modulus values, the effect of wheel overlap intensifies, systematically amplifying the structural response, which may be associated with a greater potential for track degradation. As foundation stiffness increases, this effect tends to attenuate, approaching the behavior of each wheel considered individually. Figures 4 and 5 refer to the analytical model, showing both the effect of the individual wheel and the superposition effects, presented here for educational purposes. It is worth noting that, for a numerical model with a 1 mm discretization — which is the closest to the continuous system — the analytical and numerical results practically overlap; further details can be found in Ribeiro [13]. For the present study, only the numerical displacements from SAP2000 with a 600 mm discretization are presented, providing a more practical and optimized model for evaluating global responses in terms of displacements, which are the main

parameters to be measured in this research. For other internal forces (bending moments, stresses, and shear forces), the influence of track modulus variation decreases less with increasing stiffness, indicating less sensitivity to the overall superstructure stiffness. However, displacements remain the quickest indicator of overall stability, and if they exceed safety limits, the system can become operationally critical.

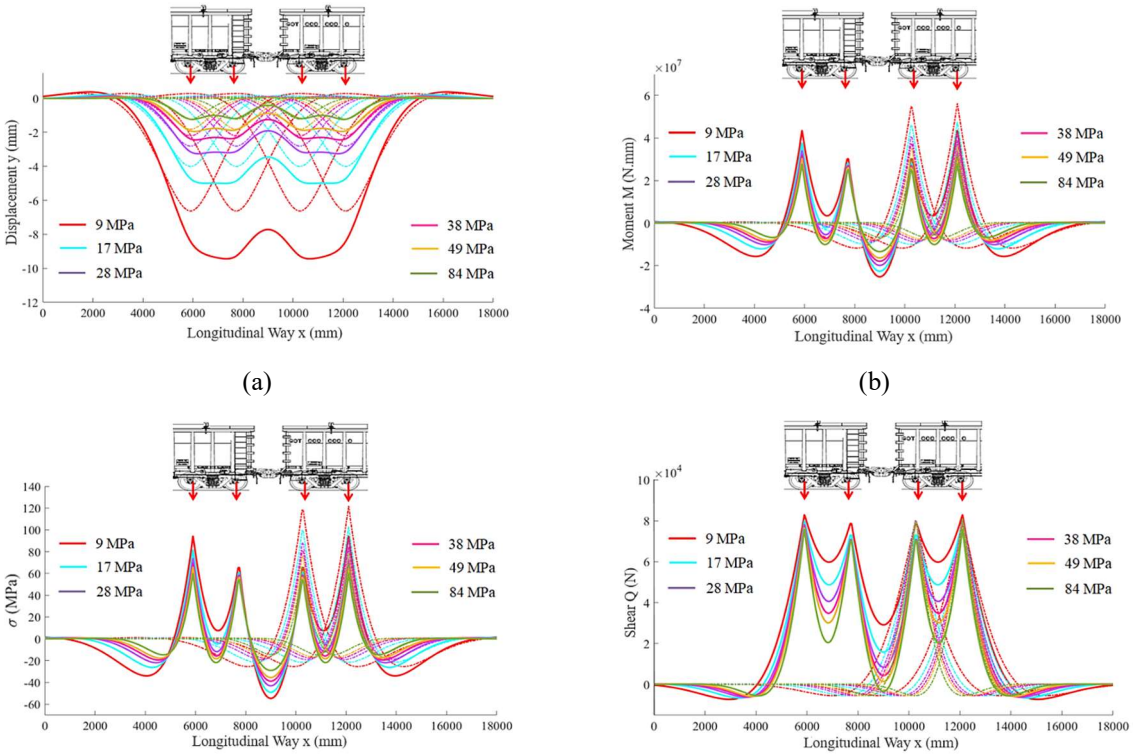


Figure 4. Variation of the track modulus for the case of Freight Wagon. In (a) – Displacements; (b) – Moments; (c) – Stress; (d) – Shear.

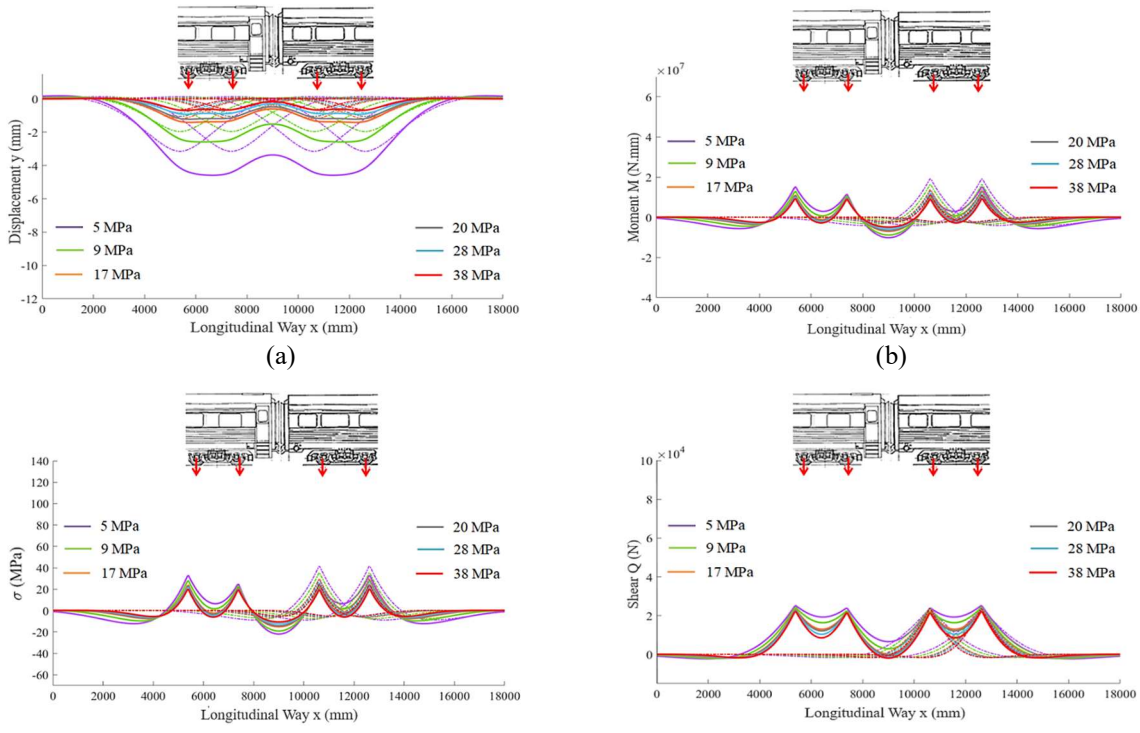


Figure 5. Variation of the track modulus for the case of the Passenger Car. In (a) – Displacements; (b) – Moments; (c) – Stress; (d) – Shear.

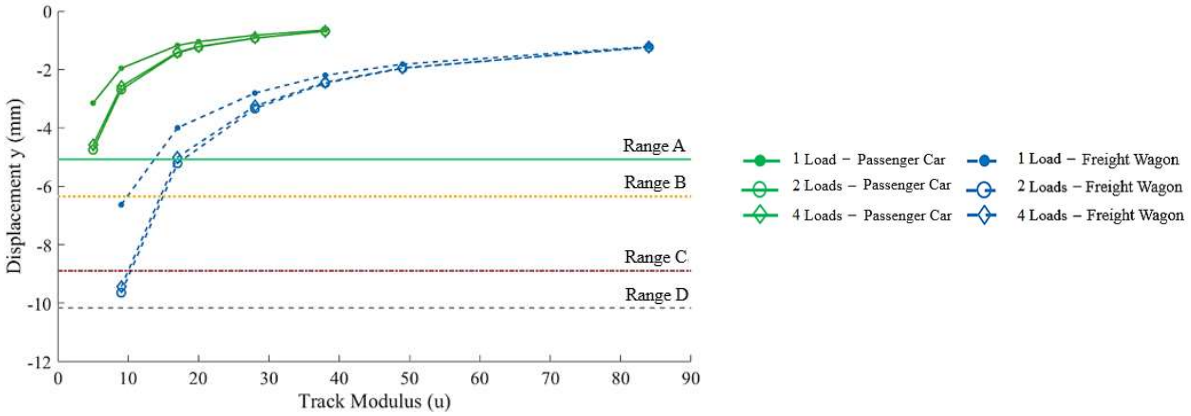


Figure 6. Displacements depending on the track modulus for the case of Passenger Car and Freight Wagon.

For the displacement graph, the operational safety bands A, B, C and D were considered according to the criteria established by Lundgren *et al.* [10] and by the AREMA [9] standard the equivalent for range B. It can be observed that, in the Fig. 6, even under conditions of low track modulus and considering the effects of superimposed loads, the passenger car remained within safety range A, indicating satisfactory structural performance. On the other hand, in the case of the cargo transport vehicle, track modulus values below 17 MPa resulted in displacements that fall within ranges C and D, evidencing a significant amplification due to the superimposed forces. From approximately 50 MPa — and especially above 80 MPa — no relevant gains are observed in terms of displacement reduction, suggesting that intermediate stiffness ranges, between 20 and 60 MPa, represent a more efficient relationship between structural safety and track elasticity. In these ranges, the structure presents favorable behavior analysis for both individual and simultaneous loading of multiple wheels. It is worth noting that, above 9 MPa, the difference in response between scenarios with two or four applied loads becomes practically negligible, reinforcing the suitability of the analysis for both loads.

Table 3 presents the amplification percentages of the superposition effects as a function of track stiffness for freight and passenger transport cases, respectively. For freight cars, it was observed that track modulus values below 10 MPa result in significant amplifications, of the order of 30% or more, especially due to the high axle load, while for modulus values close to 20 MPa, this amplification is of the order of 20%. As stiffness increases, the amplification decreases progressively. For intermediate modulus values, such as 28, 38, and 49 MPa, the percentages fall to approximately 14%, 10% and 6.5%. Above 80 MPa, the superposition effect becomes practically negligible, with amplifications of less than 2%.

In the case of passenger cars, which have a lower axle load, the trend is similar, but with lower magnitudes. For a track modulus of 5 MPa, the amplification reaches 30%, while for 10 MPa it is 24%. The analysis was extended up to 38 MPa, a value at which the observed amplification was approximately 6%. Since this is a parametric analysis considering a light vehicle operating at low speed, it was not considered necessary to explore higher modules, although there are railway tracks with greater rigidity, even for light loads that have specific characteristics in their superstructure, as well as in the frequency of traffic.

Table 3. Amplifications generated by the superposition of effects by varying the track modulus.

Stiffness/ Freight (MPa)	Freight Wagon (%)	Stiffness/ Passengers (MPa)	Passenger Car (%)
9	29.78	5	31.30
17	20.05	9	24.13
28	13.65	17	16.25
38	9.82	20	13.82
49	6.54	28	9.96
84	1.61	38	6.10

5 Conclusions

The track modulus is a fundamental parameter in characterizing the overall stiffness of the permanent track, directly influencing the structural performance of the railway. As evidenced in the results, it is a highly sensitive variable, with a significant impact on the distribution of forces and displacements along the track. For railways intended for the transportation of heavy cargo, an intermediate track modulus in the range of 25 to 70 MPa is recommended as a means of balancing structural performance with economic viability in terms of design, operation, and maintenance. Although values above 80 MPa initially result in smaller displacements, it is essential to consider that greater stiffness can lead to greater reactions in the sleepers, which can accelerate the degradation of adjacent materials, especially ballast. This effect has already been identified in maintenance inspections carried out by companies in the sector.

For low - speed passenger tracks, a modulus of up to 40 MPa presented adequate structural responses, with levels of forces and displacements compatible with safety criteria, proving to be satisfactory for this type of application. Thus, both analytical and numerical models proved capable of determining displacements and internal forces with good agreement for global analyses. The analytical model is highlighted by its classical formulation and efficiency in parametric studies, while the numerical model offers a practical way to apply loads directly to the rail and validate the analytical results. Thus, they are complementary: the analytical model provides simplicity and clarity, and the numerical model adds robustness, together strengthening the understanding of track behavior and supporting design and maintenance decisions.

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