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Probabilistic approach based on design response spectra to evaluate the human comfort of pedestrian footbridges

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Abstract: Nowadays, the structural project of pedestrian footbridges has been conceived based on the use of lightweight structures, with low natural frequencies and structural damping ratios. These facts have generated slender footbridges, sensitive to human dynamic excitations, and consequently modified the design serviceability limit states. Thus, current design codes and technical guides recommend the use of deterministic models to evaluate the footbridges dynamic structural behaviour. On the other hand, pedestrian walking is related to a stochastic phenomenon and the dynamic force generated at each step depends on the weight, frequency and length of each pedestrian's step. This way, this research work aims to propose a probabilistic approach to assess the dynamic behaviour of pedestrian footbridges (steel; steel-concrete; and concrete footbridges), based on the use of design response spectra. The proposed analysis methodology considered the stochastic nature of pedestrian walking aiming to evaluate the structural response taking into account excessive vibrations that can induce human discomfort. Based on the use of a probabilistic approach, it is possible to determine the probability of the peak acceleration values of the footbridge exceeding or not the human comfort criteria. Therefore, it was possible to conclude that the 95% percentile values of accelerations calculated using the proposed design response spectra are lower when compared to the experimental tests peak accelerations. On the other hand, the results obtained in this study indicated that the peak acceleration values calculated through the deterministic methods can be overestimated in project situations.

Keywords: Pedestrian Footbridges; Design Response Spectra; Human Comfort Criteria.

1 Introduction

The pedestrian footbridges are more and more becoming the modern landmarks of urban areas. This way, the designers seem to continuously move the safety border, aiming to increase slenderness and lightness of the pedestrian footbridges. However, more and more footbridges are carried out as light weight structures with low frequencies and low damping [1-5]. These facts have generated very slender footbridges, sensitive to dynamic excitations, and consequently changed the serviceability limit states associated to the design.

The current design codes and project guides recommend the use of deterministic models to assess the dynamic behaviour of footbridges [2-3]. On the other hand, the effect of the uncertainties in mass, stiffness and damping of the investigated structure are relevant and lead to uncertainties on the values of the footbridges natural frequencies. Nevertheless, the human walking is a stochastic phenomenon and the dynamic force generated at each step depends of the weight, the step frequency and the step length of each pedestrian. This way, opposite to the deterministic approach, this work aims to present a probabilistic model to assess the dynamic behaviour of footbridges, when subjected to pedestrian walking, considering the stochastic nature of the human walking [1].

The probabilistic dynamic analysis methodology considered the uncertainty related to the pedestrian weight

[W (N)], pedestrian step frequency [f_p (Hz)], and pedestrian step length [l_p (m)]. The proposed probabilistic model takes into consideration both inter-subject [Inter] and intra-subject [Intra] step variability [1]. These uncertainties in pedestrian weight, step frequency, step length, inter-subject and intra-subject step variability were modeled as random variables and included in the Monte Carlo simulations. The proposed probabilistic methodology is used to assess the dynamic response of steel and steel-concrete composite footbridges located at Rio de Janeiro/RJ, Brazil [1]. The results are presented and discussed based on design response spectra that allow the assessment of the vibration serviceability of footbridges. These results have demonstrated that the peak accelerations calculated using the deterministic methods [2-3] may be overestimated.

2 Maximum accelerations calculated by analytic methods

The human walking generates periodic loadings on the footbridge slabs and the two feet move alternately from one position to another and do not simultaneously leave the surface. The dynamic part of the load-time history of a single pedestrian can be modeled based on the Fourier series expansion that represents a combination of sinusoidal forces, whose frequencies are multiples of the step frequency, as presented in eq. (1).

$$F(t) = P\left[1 + \sum \alpha_i \cos(2\pi i\, f_p\, t + \phi_i\right] \tag{1}$$

Where:
$F(t)$: dynamic loading function (N);
P : pedestrian weight (N);
α_i : dynamic coefficient of the harmonic force;
i : harmonic multiple (1, 2, 3, etc.);
f_p : pedestrian step frequency (Hz);
t : time (s);
ϕ_i : phase angle to the harmonic i (rad).

In this research work the analytical solution proposed by Živanović [4] was considered for investigate the pedestrian footbridges peak acceleration, having in mind that for this purpose the first vibration mode of the structure can be represented by a half sine wave function sen ($\pi v_p t/L$). This way, the equation that governs the first vibration mode of the investigated footbridge is presented in eq. (2).

$$a(t) + 2\xi\omega\, v(t) + \omega^2\, u(t) = \frac{1}{m}\alpha\, P\, sen(2\pi\, f_p\, t)\, sen(\frac{\pi\, v_p}{L} t) \tag{2}$$

Where:
$a(t)$: peak acceleration (m/s²);
ξ : damping coefficient;
m : footbridge modal mass (kg);
α : dynamic coefficient;
P : pedestrian weight (N);
f_p : pedestrian step frequency (Hz);
l_p : pedestrian step length (m);
v_p : pedestrian step velocity [$v_p = f_p\, l_p$] (m/s);
L : footbridge span (m).

3 Probabilistic analysis methodology: response spectra

In this research work, the proposed probabilistic dynamic analysis methodology assumes that the independent random variables associated to the pedestrian weight [W (N)], pedestrian step frequency [fp (Hz)], and pedestrian step length [lp (m)] follow a normal distribution [1]. The developed analysis methodology was computationally implemented and the generated design response spectra allow structural engineers the vibration serviceability assessment of pedestrian footbridges.

Hausdorff et al. [5] have demonstrated that the step frequency is not constant during walking. Each pedestrian walks freely with a preferential step frequency. However, at each step, the frequency varies because the pedestrian cannot repeat the same motion with the same frequency. This step-by-step variation is called the intra-subject step variability, whereas the step frequency variation between different people represents the inter-subject step variability.

In this study, the proposed probabilistic model takes into consideration both inter-subject [Inter] and intra-subject [Intra] step variability [1]. Table 1 shows the mean values (μ) and respective standard deviations (σ) of the random variables adopted in the Monte Carlo simulations [2,4,6,7] to assess the dynamic structural behaviour of simply supported footbridges.

Table 1. Statistical parameters used to simulate the pedestrian walking

Variable	Units	Mean (μ)	Standard Deviation (σ)	References
Weight	N	727	145	Ingólfsson [6]
f_p	Hz	2,00	0,2	Bachmann [2]
l_p	m	0,71	0,071	Zivanovic [4]
Intra	Hz	-	3,0% de μ	Brownjohn [7]

Thus, for each pedestrian, a preferential step frequency was randomly generated. At each step period, the step frequency varied by 3% around the preferential pedestrian's step frequency to account the intra-subject step variability [7]. The dynamic load factors were calculated using the mathematical formulations proposed by Rainer et al. [8] and Young apud Hauksson [9].

In sequence, Fig. 1 presents the developed analysis methodology in details. It must be emphasized that there are significant differences between the analysis methodologies I [4] and II [1]. In fact, analysing the Fig. 1 it is possible to conclude that when the methodology I (M-I) is utilised in this investigation, the effect of the intravariability is considered only to the calculation of the dynamic coefficients (αi) [Step 3: see eq. (1)], and used to determine the peak accelerations [Step 4: see Fig. 1]. On the other hand, when the analysis methodology II (M-II) is investigated, the intravariability is considered on the dynamic coefficient (αi) calculations and on the sinusoidal harmonic function sen (2πfp) [Step 3 and Step 4: see eq. (1)]. After that the peak acceleration percentile 95% (a95%) is determined, based on Step 5, as illustrated in Fig. 1.

4 Investigated pedestrian footbridges

In this research work two pedestrian footbridges located at the Rio de Janeiro city (Rio de Janeiro/RJ, Brazil) were investigated, aiming to compare the results calculated based on the use of the proposed analysis methodologies I and II (see Fig. 1), and the results determined based on experimental tests developed on the analysed pedestrian footbridges [1,10].

The first structural model analyzed (FB-I: m = 30,869 kg; f_{01} = 4.85 Hz; ξ = 1.7% [1]) is located on the 5th floor of the State University of Rio de Janeiro (UERJ), between Blocks A and B. The footbridge is made of a concrete structural system, with a 24.4m long and 3.4m wide span, as presented in Fig. 2a.



a) Concrete footbridge: FB-I b) Steel footbridge: FB-II

Figure 2. Investigated pedestrian footbridges (FB-I: L = 24.4m; FB-II: L = 68.6m)

The second analysed footbridge (FB-II: m = 34,490 kg; f_{01} = 1.85 Hz; ξ = 0.23% [10]) is associated to a

simply-supported inverted-queen-post-truss steel footbridge spanning 68.6 m, with all beams and columns made of steel [10]. The deck floor is made of precast concrete slabs 7 cm thick, and the two parallel trusses are braced at their top and bottom. The cross-sections of the truss members, the reinforcement beams and the primary members of the bracing systems are made of steel with double-U profiles welded at theirs flanges to form a box section [10], as illustrated in Fig. 2b.

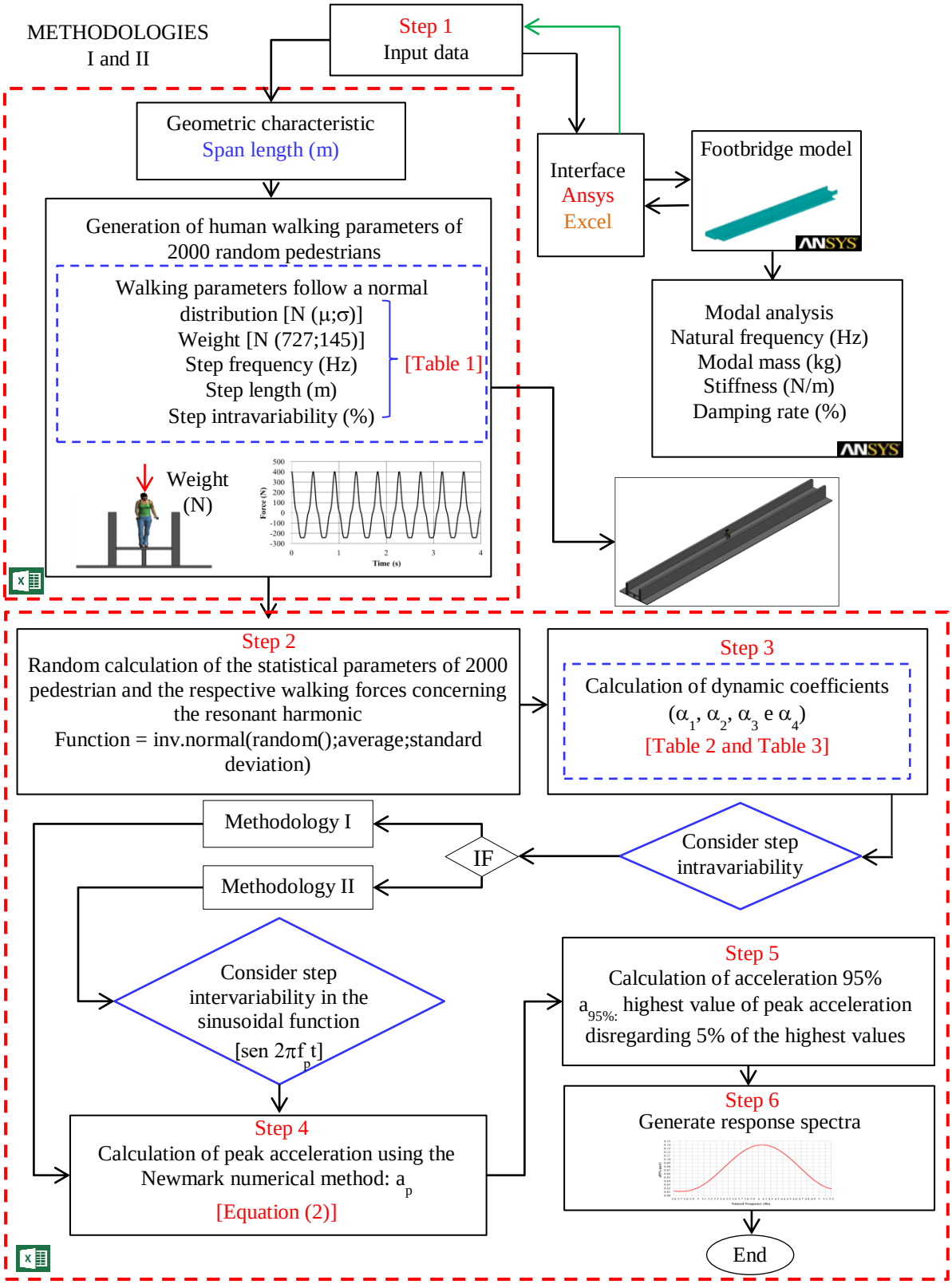


Figure 1. Proposed analysis methodology to generate the project response spectra

5 Probabilistic design response spectra

Initially, it is worth highlighting that the results obtained based on the proposed probabilistic analysis methodology was calculated from 2,000 analyses associated with pedestrian crossings. Therefore, the probabilistic response spectra were constructed based on an extensive parametric study, based on the calculation of maximum acceleration values (peak accelerations), corresponding to each pedestrian crossing, varying the fundamental frequency in the vertical transverse direction in the range of 0.5 Hz to 10 Hz of the reference footbridges, FBI and FB-II (see Figs. 2a and 2b) and the structural damping (ξ) of 0.5% to 2.0%.

The peak acceleration values were determined using analysis methodologies I and II (M-I [4] and M-II [1]), see Fig. 1. The calculation of the dynamic coefficients (α_i), see eq. (1), was considered based on the use of the mathematical formulations proposed by Rainer et al. [8] and Young apud Hauksson [9], presented in Tab. 3 and Tab. 4. The 95% percentile values ($a_{95\%}$) were determined from the calculated probabilistic peak accelerations. The 95% percentile corresponds to the value with a 95% probability of occurrence, that is, by calculating the maximum structure dynamic response 100 times, it is expected that in 95 cases the peak acceleration values will be equal to or lower than the representative value of the 95% percentile ($a_{95\%}$).

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Table 2. Dynamic coefficients proposed by Rainer et al. [8]: footbridge vertical direction

Dynamic coefficients (α_i)
$\alpha_1 = -0.22169f_p^3 + 1.11946\ f_p^{\ 2} - 1.44748\ f_p + 0.5967$
$\alpha_2 = -0.012037(2\ f_p)^3 + 0.1494(2\ f_p)^2 - 0.53146(2\ f_p) + 0.6285$
$\alpha_3 = 0.00009068(3f_p)^5 - 0.0021066(3fp)^4 + 0.018364(3f_p)^3 - 0.077278(3f_p)^2 + 0.17593(3f_p) - 0.1477$
$\alpha_4 = 0.00051715(4\ f_p)^4 - 0.014388(4\ f_p)^3 + 0.14562(4\ f_p)^2 - 0.6018469$

Table 3. Dynamic coefficients proposed by Young apud Hauksson [9]: footbridge vertical direction

Dynamic coefficients (α_i)
$\alpha_1 = 0.37\ (f_p - 0.92)$
$\alpha_2 = 0.054 + 0.0044f_p$
$\alpha_3 = 0.026 + 0.0050f_p$
$\alpha_4 = 0.010 + 0.0051f_p$

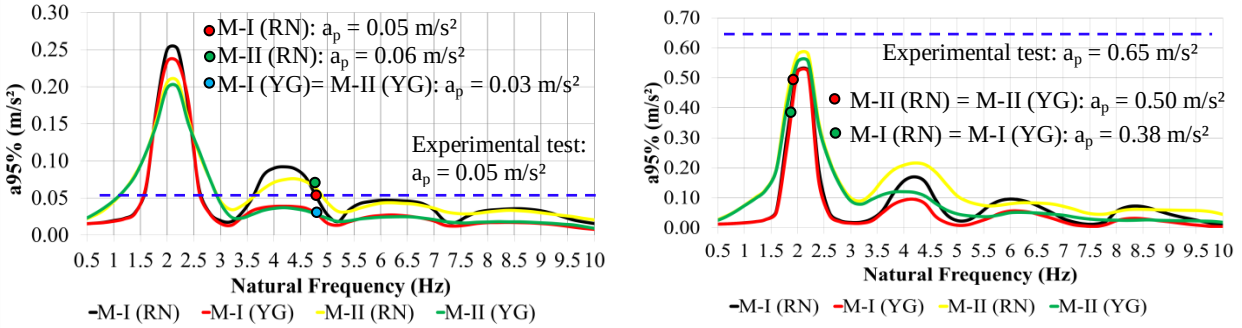
6 Analysis and discussion results

Having in mind the assessment of the 95% percentile values ($a_{95\%}$), determined based on the use of methodologies I and II (M-I [4] and M-II [1]) using the mathematical formulations proposed by Rainer et al. [8] and Young apud Hauksson [9], the experimental results are presented and delimited by dashed lines (see Figs. 3a and 3b). Following the investigation, the results comparison is presented considering the damping coefficient (ξ) variation from 0.5% to 2.0%, as well as the analysis of the human comfort criterion recommended by design standards for external footbridges (External footbridge: $a_{lim} = 0.49$ m/s²; Internal footbridge: $a_{lim} = 0.15$m/s² [3]) (see Figs. 4a and 4b).

Based on the results presented in Fig. 3a, it's possible to conclude that the 95th percentile values (a95%), determined using the proposed design response spectra (M-I [4] and M-II [1]), are lower when employing Methodology I with the dynamic coefficient formulation proposed by Young apud Hauksson compared to the peak accelerations from experimental tests. However, when evaluating Fig. 3b, it's noted that the 95th percentile values (a95%) are lower for both response spectra (M-I [4] and M-II [1]) when compared to the peak accelerations from experimental tests.

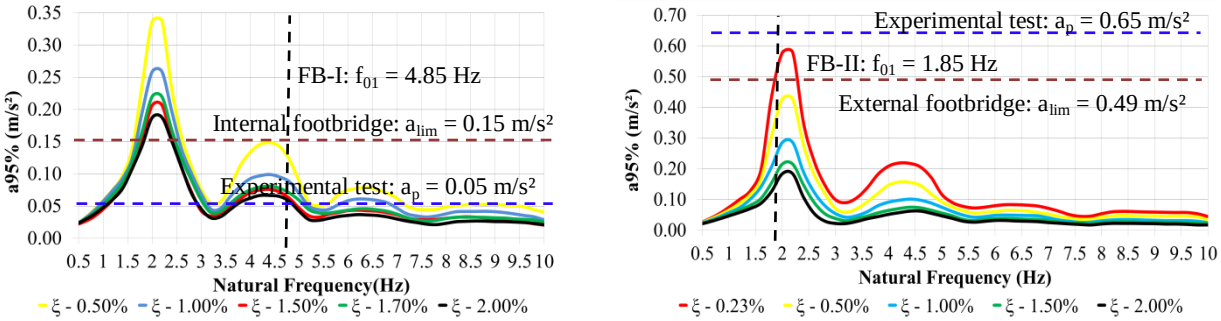
It's important to highlight that when considering FB-I (harmonic resonance: 2nd; see Figure 2a) in the investigation, the a95% values determined using the design spectra were the same for both analysis methodologies (M-I [4] and M-II [1]) when Young apud Hauksson is dynamic coefficient formulation [9] was considered. However, accelerations showed different values when methodology II (M-I [4]) and Rainer et al.'s dynamic coefficient mathematical formulation [8] were adopted, as seen in Fig. 3a.

On the other hand, when analysing the FB-II (Harmonic resonant: 1st; see Fig. 2b) the determined $a_{95\%}$ values considering the M-II [1] were greater than those obtained based on the use of M-I [4], to the both dynamic coefficients mathematical formulation [8,9]. These observations can be verified in Figs. 3a and 3b, looking at design spectra behaviour, when assessing the first and second harmonic.



(a) FB-I: L = 24.4 m; ξ = 1.70% (b) FB-II: L = 68.6 m; ξ = 0.23%

Figure 3. Response spectrum for peak accelerations



(a) L = 24.4m; ξ = 0.5% to 2.0% (b) L = 68.6 m; ξ = 0.5% to 2.0%

Figure 4. Response spectrum for peak accelerations

Having in mind the variation in structural damping related to the investigated pedestrian footbridges (see Figs. 4a and 4b), when the reference footbridge FB-I (Harmonic resonant: 2nd; see Fig. 2a) was investigated, it's possible to observe that the a95% values are higher than the experimental accelerations (a_{exp}=0.05 m/s²), for all damping ratios between 0.5% and 2.0% ξ=0.5% and 2.0%). On the other hand, all a95% values are lower than the design standard limit acceleration (Internal footbridges: a_{lim}=0.15 m/s² [3]) for all investigated damping ratios (ξ=0.5% to 2.0%), as illustrated in Fig. 4a.

Considering the design response spectrum of the reference footbridge FB-II (Harmonic resonant: 1st; see Figure 2b) illustrated in Fig. 4b, it is noted that the a95% values are lower than the experimental accelerations (a_{exp} = 0.65 m/s²) to all investigated structural damping rates (ξ = 0.23% to 2.0%). It was verified that the a95% values slightly surpass the design standard limit acceleration (a_{lim} = 0.49 m/s² [3]) to the damping rate ξ = 0.23% (damping experimental value [10]). However, all the a95% values are lower the recommended design limit to external footbridges when the structural damping rates are equal to 0.5%, 1.0%, 1.5% and 2.0% (ξ = 0.5%, 1.0%, 1.5% and 2.0%), as presented in Fig. 4b.

7 Conclusions

In this research work, design spectra response are proposed, based on development of a probabilistic analysis methodology, aiming to assess the dynamic structural behaviour of pedestrian footbridges when subjected to human induced loadings considering the stochastic nature of the walking. This way, the following conclusions can be drawn from the results presented in this investigation:

1. The proposed probabilistic design response spectra demonstrated that the dynamic response of simply supported footbridges [95% percentile ($a_{95\%}$)], when subjected to pedestrian walking, can be modified based on the effect of the step intravariability, as well as the randomness of the other variables used in the probabilistic analysis methodology.

2. Considering a quantitative analysis based on 2,000 pedestrians crossing the investigated footbridges, the results calculated using the proposed probabilistic methodology indicate that the first footbridge studied (FB-I) meets the human comfort criterion (alim = 0.15 m/s²) for all damping rates analyzed (ξ = 0.5% to 2.0%). The acceleration values obtained were 0.12 m/s², 0.09 m/s², 0.07 m/s², 0.06 m/s², and 0.06 m/s² for damping rates of 0.5%, 1.0%, 1.5%, 1.7%, and 2.0%, respectively.On the other hand, the second footbridge (FB-II) showed an a95% value slightly above the limit recommended by design standards (alim = 0.49 m/s²) when considering the experimental damping rate of ξ = 0.23%, with an acceleration of 0.5 m/s². However, for structural damping rates between 0.5% and 2.0% (ξ = 0.5%, 1.0%, 1.5%, and 2.0%), the acceleration values were below this limit, with results of 0.3 m/s², 0.2 m/s², 0.15 m/s², and 0.13 m/s², respectively.

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References

[1] P.O.B. Diniz, Human comfort assessment of pedestrian footbridges based on the development of a probabilistic analysis methodology (In Portuguese). MSc Dissertation. Postgraduate Programme in Civil Engineering (PGECIV). State University of Rio de Janeiro (UERJ). Rio de Janeiro/RJ, Brazil, (2024).
[2] H. Bachman and W. Amman, Vibration in Structures Induced by Man and Machine, Structural Engineering Document 3, International Association for Bridge and Structural Engineering (IABSE), Zurich, Switzerland, (1987).
[3] T.M. Murray, D.E. Allen, E.E. Ungar and D.B. Davis, Steel Design Guide Series Nbr. 11: Vibrations of Steel-framed Structural Systems due to Human Activity. American Institute of Steel Construction (AISC), 2nd Edition, Chicago, USA, (2016).
[4] S. Živanović, Probability-based estimation of vibration for pedestrian structures due to walking. PhD Thesis. Department of Civil and Structural Engineering. University of Sheffield. England, UK, (2006).
[5] J.M. Hausdorff, L. Zemany, C.K. Peng, Goldberger A.L., Maturation of gait dynamics: stride-to-stride variability and its temporal organization in children. Journal of Applied Physiology, Vol. 86(3), pp. 1040-1047, (1999).
[6] E.T. Ingólfsson, Pedestrian-induced lateral vibrations of footbridges: experimental studies and probabilistic modelling. PhD Thesis. Department of Civil Engineering. Technical University of Denmark, (2011).
[7] J.M.W Brownjohn, A Pavic, P. Omenzetter. A spectral density approach for modelling continuous vertical forces on pedestrian structures due to walking. Canadian Journal of Civil Engineering 2004.
[8] Rainer J.H., Pernica G., Allen D.E., Dynamic loading and response of footbridges, Canadian Journal of Civil Engineering, Vol. 15(1), pp. 66-71, (1988).
[9] F Hauksson,. Dynamic behaviour of footbridges subjected to pedestrian induced vibrations. MSc Dissertation. Lund University. Sweden, (2005).
[10] F.A. Rezende, W.D. Varela, E.M.L. Carvalho and A.M.B. Pereira, Vibration performance assessment of a long-span steel footbridge. Proceedings of the Institution of Civil Engineers - Structures and Buildings, Vol. 175(6), pp. 500-512, (2022).