XLVI CILAMCE
VITÓRIA - ES - BRAZIL - 2025
XLVI IBERO-LATIN AMERICAN CONGRESS ON COMPUTATIONAL METHODS IN ENGINEERING

Application of Random Field Finite Element Analysis to Earthfill Dams

André Luis Meier[1], Gracieli Dienstmann[1], Albert Luiz Follmann[1]

[1]*Dept. of Civil Engineering, Federal University of Santa Catarina*
R. João Pio Duarte Silva, 151-179, 88037-001, Santa Catarina, Brazil
de.luis.meier@gmail.com, g.dienstmann@ufsc.br, albert.follmann@gmail.com

Abstract. The safety assessment of earthfill dams has traditionally relied on limit equilibrium methods, which require the predefinition of a failure surface and provide limited insight into stress-strain behavior and material variability. This study presents a simplified application of Random Field Finite Element Method (RFEM) to model the spatial variability of geomechanical parameters in an earthfill dam. A two-dimensional stress-strain simulation of a clay-core rockfill dam section is performed, incorporating random fields to represent material heterogeneity. The model also implements the shear strength reduction method to estimate reliability metrics. The results are directly compared to those obtained using limit equilibrium analysis to evaluate the observed failure mechanisms and reduction factors. Additionally, a probabilistic analysis considering a homogeneous section was carried out for comparison, allowing the evaluation of the influence of spatial variability on the dam's safety assessment.
Keywords: Dam Safety, Rockfill Dam, Random Fields, Shear Strength Reduction, Strain.

1 Introduction

The safety of large civil structures like embankment dams is a primary concern in geotechnical engineering. For decades, the standard approach for stability assessment has been the limit equilibrium method (LEM) as shown by Khran [1]. This method calculates a Factor of Safety (FS) by analyzing the equilibrium of a potential sliding mass, but it has inherent limitations. LEM requires the engineer to assume the shape and location of the failure surface, it simplifies complex stress states, and it struggles to properly account for the natural spatial variability of soil properties. Consequently, a high FS from a deterministic LEM analysis may not accurately represent the true safety margin of the structure (Silva, Lambe and Marr [2]).

To overcome these limitations, the Random Field Finite Element Method (RFEM) has emerged as a powerful alternative. RFEM integrates statistical random field theory with the robust stress-strain analysis capabilities of the finite element method (FEM). This allows for the explicit modeling of material heterogeneity, where soil properties like friction angle and stiffness can vary spatially, mimicking real-world conditions. By doing so, failure mechanisms are allowed to develop naturally through the weakest paths in the soil mass, providing a more realistic depiction of potential failure modes (Fenton and Griffiths [3]).

In the past years, many authors had used probabilistic analyses with random fields to evaluate geotechnical problems. For example, Mouyeaux, Carvajal, Bressolette, Peyras, Breul and Bacconnet [4] compared probabilistic methods (random field and random variable) using LEM and RFEM in an earth dam stability analysis. Guo [5] checked the influence of different kinds of random fields over the reliability index of an earth dam. Siacara, Napa-García, Beck and Futai [6] used First Order Reliability Method (FORM), Second Order Reliability Method (SORM) and Monte Carlo with importance sampling to compare the reliability index of a dam over different pool levels.

This paper presents a simplified application of RFEM and Random Variable (RV) to assess the stability of a clay-core rockfill dam. A two-dimensional stress-strain simulation that incorporates random fields to represent the heterogeneity of the dam's clay core. The shear strength reduction (SSR) technique is implemented within the RFEM framework to compute a distribution of Strength Reduction Factors (SRF), which serves as a reliability metric analogous to the FS. Results from the probabilistic RFEM and RV analysis are compared with those from

a conventional LEM analysis.

2 Random Fields: Randomization method

As defined by Vanmarcke [7], a random field is a collection of random variables indexed by spatial coordinates in an n-dimensional space, where the values at different points are related by a covariance function. For simplicity in geotechnical applications, it is common to assume the field is stationary (statistical properties do not change with location) and to adopt Gaussian random fields (Fenton and Griffiths [3]). Several methods exist to generate these fields, including Karhunen-Loève expansion, Kriging (for conditional random fields), and the Local Average Subdivision (LAS) method (Liu [8]). This study employs the Randomization Method, defined by Haße [9].
As shown by Haße [9], the Randomization Method is an efficient technique derived from the Fast Fourier Transform (FFT). It begins with the integral representation of a stochastic process u(x) based on its power spectrum S(k), as shown in eq. (1):

$$u(x) = \int_{-\infty}^{\infty} e^{-2\pi i k x} \sqrt{S(k)}\ dW(k) \tag{1}$$

Using Euler's theorem, this integral can be expressed as the sum of sine and cosine terms. By leveraging the properties of the power spectrum and the Weiner process, the expression can be simplified and approximated using a Monte Carlo integration scheme. For a two-dimensional, zero-mean, stationary Gaussian random field, the process at a point x is generated by the following summation (eq. (2)).

$$u(x)_{rand} = \sqrt{\frac{2\sigma^2}{N}} \left(\sum_{i=1}^{N} Z_i^1 \cos(2\pi \boldsymbol{k_i x}) + \sum_{i=1}^{N} Z_i^2 \operatorname{sen}(2\pi \boldsymbol{k_i x})\right) \tag{2}$$

where:

- Z_i^1 and Z_i^2 are independent random numbers from a standard normal distribution;
- σ^2 is the variance of the field;
- N is the number of terms in the summation, which defines the resolution of the field;
- x is the coordinate vector (x,y);
- k_i is a vector of random frequencies generated according to a specified covariance model (e.g., Gaussian, exponential), which incorporates the autocorrelation length.

This method is implemented in the open-source Python library GSTools and was used to generate the random fields for the dam's clay core in this study.

3 Materials and Method

3.1 The Studied Dam and Material Properties

The subject of this study is a 48-meter-high rockfill dam with a central clay core, located in Brazil. A typical cross-section is shown in Fig. 1, detailing its internal zoning, which includes the clay core (1), fine (3, 3C) and coarse (4A) transitions, rockfill shells (4, 4B), rip-rap (5) and rock foundation (6). For the numerical model, some zones were simplified; for example, the various rockfill and transition zones were grouped into single "Rockfill" and "Transition" materials, respectively.

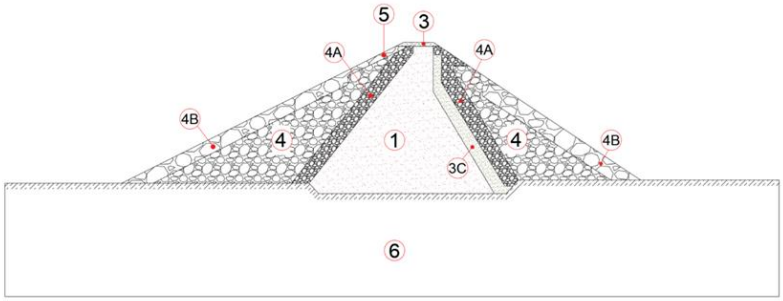


Figure 1. Rockfill dam cross-section

The core material is a silt-rich saprolite of gneiss, for which extensive laboratory data was available from 71 triaxial tests (CU and CD). A statistical analysis of this data was performed to generate the random fields and to obtain random variables, using RStudio EnvStats library for Shapiro-Wilk hypothesis test, which is known to be robust for small data sets (Mishra, Pandey, Singh, Gupta, Sahu, Keshri [10]), and fitdstrplus library for chart plots (density function, histogram, quantile-quantile, percentile-percentile). In the absence of spatial data and to ensure the random field generation, correlation length was evaluated throughout a sensitivity test. Isotropic and anisotropic correlation lengths were tested, considering, in the anisotropic case, 13.0m and 1.5m for the correlation lengths in both directions. Then, the random fields were generated using the Python Library GSTools.

3.2 Analysis workflow

A Monte Carlo simulation framework was established using ABAQUS software. To check the number of necessary analyses a convergence chart was made, being verified that 250 individual runs were enough to ensure the stabilization in around mean SRF values. Although, 500 analyses were carried out considering random variables and random fields.
In each run for the random field model (RF) a new pair of random fields for friction angle and elastic modulus was generated and assigned to the core elements in the ABAQUS model. For the Random Variable model (RV), a pair of values were generated instead. The simulation proceeded through four steps: 1) Geostatic initialization (necessary to compute porepressures properly), 2) Gravity load, 3) Steady-state seepage analysis to establish operational pore pressures, and 4) Shear Strength Reduction (SSR). In the SSR step, both the elastic modulus and effective friction angle were incrementally reduced by a factor (SRF) until the analysis failed to converge, which was taken as the point of structural failure.
The results from this probabilistic analysis were compared against a conventional LEM analysis performed with Rocscience Slide software, using Spencer's method and the mean parameters obtained in statistical analysis for each material. The comparison focused on evaluating the differences in the calculated stability factor (FS vs. SRF) and the predicted failure mechanisms for LEM, RF and RV models.

4 Results and discussion

4.1 Statistical Analyses and Random Field Generation

The statistical characterization of geomechanical parameters was a fundamental step in generating the random fields, focusing on the dam's clay core material. The analysis targeted the drained Young's modulus (E′) and the effective friction angle (ϕ′), as the effective cohesion (c′) was treated deterministically due to its value being null in 77% of the available test data. To determine the best-fit probability distribution for the data, the Normal, Lognormal, Gamma, and Weibull distributions were tested. Goodness-of-fit was evaluated both visually, using histograms and quantile-quantile (Q-Q) plots, and statistically, using the Shapiro-Wilk hypothesis test. Figure 2 shows the probability density function for the mentioned distributions considering both parameters.

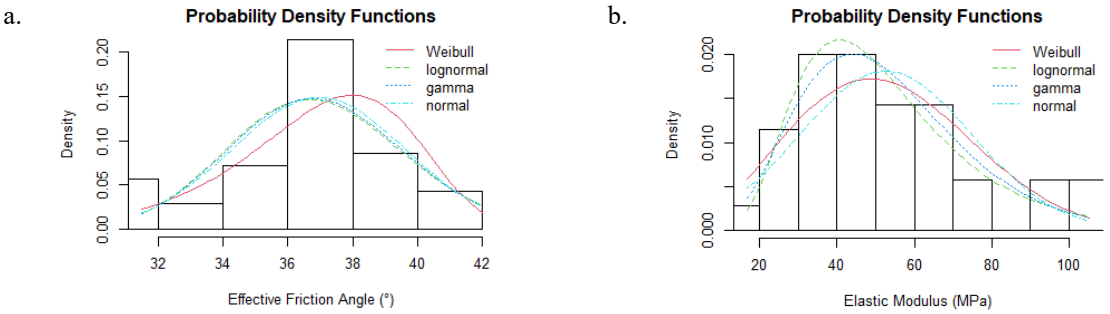


Figure 2.Probabilty Density functions for a) effective friction angle and b) elastic modulus

Considering a significance level of 5%, the Shapiro-Wilk test results indicated that the Young's modulus followed a Lognormal distribution, which yielded the highest p-value (0.7543) among the tested models. For the friction angle, the test did not reject the null hypothesis for normality (p-value = 0.1623), and the Normal distribution was therefore adopted for this parameter. Based on these findings, the statistical parameters used to generate the random fields were established as follows: a mean of 52.81 MPa and a Coefficient of Variation (COV) of 44.13% for the Young's Modulus (E′), and a mean of 36.96° and a COV of 7.25% for the effective friction angle (ϕ'). Table 1 shows the results and adopted parameters.

Table 1. Geomechanical parameters

Material	γ (kN/m³)	c' (kPA)	Φ'(°)		E (MPa)[1]		v	k (cm/s)	Distribution
			Mean	Std. Dev.	Mean	Std. Dev.			
Core	18,35	10	36,9571	2,6789	52,8102	23,3061	0,285	$6x10^{-5}$	Normal (ϕ') e Lognormal (E)
Transitions[2]	19	0	36	-	75	-	0,292	1	Deterministic
Rockfill	21	0	37	-	107	-	0,285	1	Deterministic
Foundation[3]	26	100	35	-	1000	-	0,299	$1x10^{-5}$	Deterministic

[1] Moduli obtained from Lambe and Whitman [11], Cruz [12], and Cruz, Materón and Freitas [13], except for the core and rockfill materials.
[2] M15% envelope for transitions, disregarding the cohesion intercept. Obtained from Espindola [14]
[3] Wines e Lily [15].
[4] COV Φ' = 7,2487% e COV E = 44,1318%

The sensitivity analysis made for the correlation length showed that for the data used in this research, there were no significant differences in the values of SRF. In this way, the random field was generated considering an isotropic scenario.

4.2 LEM, RFEM and Random Variable comparison

For the probabilistic analyses for the dam under consideration, a preliminary set of deterministic analyses was performed considering random fields. The finite element model was assessed deterministically for comparison with the limit equilibrium method by applying the SSR technique in three distinct scenarios: reducing only the friction angle, reducing only the Young's modulus, and reducing both parameters simultaneously. All analyses utilized the mean parameters summarized in Tab. 1. The reduction of strength by decreasing the friction angle alone resulted in an SRF of 1.444 and indicated the development of a plastic strain zone downstream of the dam's core, passing through the downstream toe and tangentially touching the transition-rockfill contact (Fig. 3).

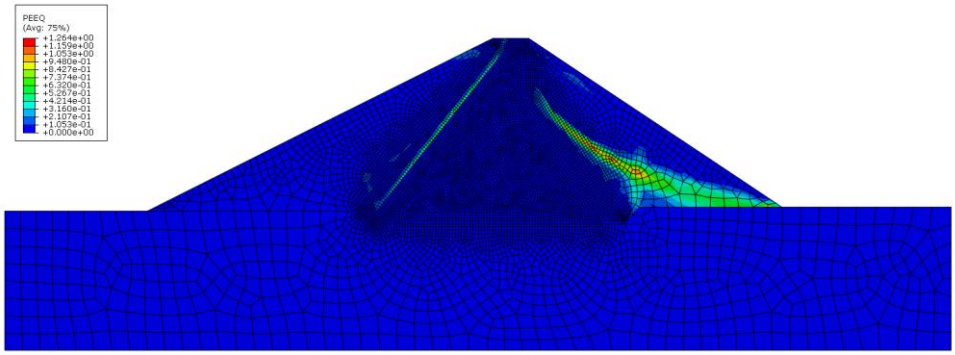


Figure 3. Plastic Equivalent Strain for friction angle reduction

When only the Young's modulus was reduced, or when both the modulus and the friction angle were reduced concurrently, a different behavior was observed. In both cases, a minor plastic strain zone initiated at the lower interface of the downstream transition and rockfill, extending upwards towards the slope without forming a continuous failure surface, yielding a lower SRF of 1.074. Khitrov, Andronov, Martynov and Spiridonov [16] established a direct relationship between the elastic modulus and strength parameters like the friction angle, so, reduction in one would imply a reduction in the other. Therefore, the simultaneous reduction of both the friction angle and Young's modulus by the same factor was adopted as the most critical scenario for the subsequent probabilistic analyses (Fig. 4).

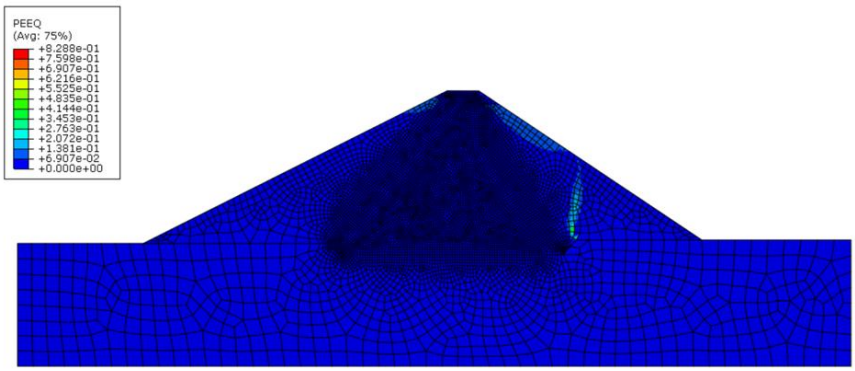


Figure 4. Plastic equivalent strain for moduli reduction or for both parameter reduction (modulus and effective friction angle)

For a comparative assessment, stress-strain results were benchmarked against limit equilibrium analyses conducted with Rocscience's Slide software, using the Spencer method. The upstream slope analysis yielded a high Factor of Safety (FS) of 1.58, a result attributed to the stabilizing effect of the reservoir pressure. Conversely, the analysis of the downstream slope indicated a significantly lower FS of 1.19. This lower value is associated with shallow failure surfaces, which are typical in limit equilibrium analyses for materials with zero cohesion and low friction angles. It is noteworthy, however, that deeper failure surfaces in the limit equilibrium model, which more closely resemble the failure mechanism observed in the finite element analysis, yielded an FS of approximately 1.40. Figure 5 shows the LEM analysis results mentioned above.

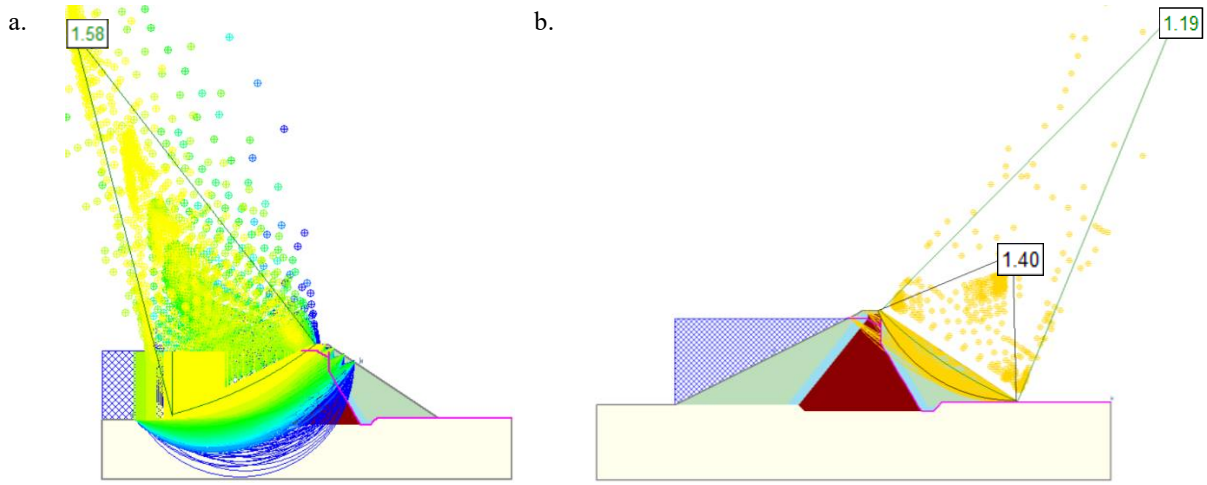


Figure 5. LEM Analysis results. a) Upstream slope stability. b) downstream slope stability

This difference between the SRF and FS values aligns with findings from researchers like Cheng, Lansivaara, and Wei [17], who noted that the difference between these factors is influenced by the soil's cohesion and friction angle differences. A key limitation of the performed analyses is the use of the linear Mohr-Coulomb failure criterion for the rockfill, which does not accurately represent its behavior, as rockfill exhibits high friction angles at low stress levels and lower angles at high stress levels. For more precise limit equilibrium analyses of structures involving rockfill, the use of curved strength envelopes, such as those proposed by Leps [18] or Espindola [14], is recommended.

The RFEM model and the RV model had shown a similar behavior for the failure surface. In both cases, there is a small plastic strain upstream, in the contact between transition and core materials. This small strain cannot be attributed to a structural failure directly, although, indicates that plastic strain occurs in this place, resulting in deformations that could lead to cracks in the dam crest. Dyson and Tolooiyan [19] mention that structural failure in SSR models need tree factors to occur: plastic zones from toe to crest of the slope, huge deformations and solution non-convergence. In these models, only the last one is observed, sustaining the behavior mentioned above. Figure 6 shows an example for the zone with largest strain observed in the models.

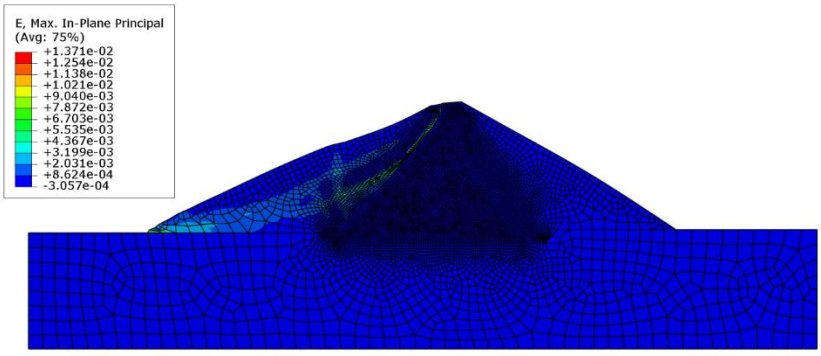


Figure 6. Maximum strain in principal stress plane

As the result of the probabilistic analyses, it was verified that RFEM model presented a mean SRF of 1.1038 with a standard deviation of 0.0143, while, RV model presented a mean value of 1.1022 and a standard deviation of 0.0142. No RFEM models failed, while 6 models for RV failed, representing 1,2% of probability of failure in this case. Although, for a reliable estimation of the probability of failure, a large number of analyses would be necessary. Figure 7 shows the histograms for SRF values in both cases, being possible to observe that RFEM and RV analyses had very similar distributions for the same number of evaluations, and also, had very similar mean SRF values.

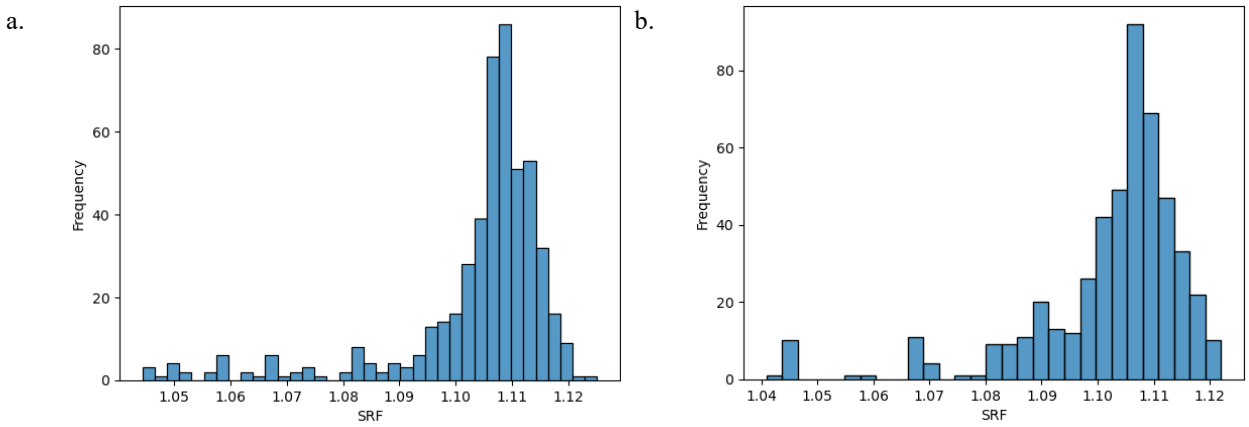


Figure 7. Histograms for Strength Reduction Factor a) RFEM and b) Random Variable (RV)

5 Conclusions

The probabilistic analysis revealed a distinct failure mechanism not identified by conventional deterministic methods. By implementing a more complex approach involving the simultaneous reduction of both the friction angle and the elastic modulus, the models predicted failure initiation as a concentration of plastic strain at the upstream core-transition interface. This resulted in a mean SRF of approximately 1.10, indicating a lower stability margin than suggested by either the Limit Equilibrium Method for downstream failure (FS = 1.19) or the deterministic FEM analysis that reduced only the friction angle (SRF = 1.44). This behavior points towards a potential for localized deformation and cracking rather than a complete slope collapse.

A key finding from the probabilistic investigation was the remarkable similarity between the Random Field (RFEM) and Random Variable (RV) models. Both methods produced nearly identical SRF distributions and mean values, suggesting that for estimating the mean stability response under these conditions, the computationally less demanding RV approach was as effective as the more rigorous RFEM, which explicitly models spatial correlation. Although, this behavior could be different in a dam where spatialized data would be available. The adoption of a linear Mohr-Coulomb criterion for rockfill is a significant simplification, and more advanced non-linear strength envelopes are recommended. Furthermore, a substantial increase in the number of simulations is necessary to move beyond mean SRF values and reliably quantify the probability of failure for a comprehensive risk assessment.

Acknowledgements. This work was supported by the Fundação de Amparo à Pesquisa do Estado de Santa Catarina (FAPESC). The authors also wish to thank the PostGraduate Program in Civil Engineering (PPGEC) at the Universidade Federal de Santa Catarina (UFSC).

Authorship statement. The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

References

[1] J. Khran. The 2001 R.M. Hardy Lecture: The limits of limit equilibrium analysis. *Canadian Geotechnical Journal 40*, pp. 643-660, 2003.
[2] F. Silva, T.W. Lambe, W. Marr. A. Probability and Risk of Slope Failure. *Journal of Geotechnical and Geoenvironmental Engineering, 134*, pp. 1691–1699, 2008.
[3] G.A. Fenton; D.V. Griffiths. *Risk Assessment in Geotechnical Engineering. Hoboken*: John Wiley & Sons, 2008.
[4] A. Mouyeaux, C. Carvajal, P. Bressolette, L. Peyras, P. Breul, C. Bacconnet. Probabilistic stability analysis of an earth dam by Stochastic Finite Element Method based on field data. *Computers and Geotechnics 101*, Elsevier, pp.34-47, 2018.
[5] X. Guo. *Probabilistic stability analysis of and earth dam using field data*. PhD Thesis, Université Grenoble Alpes, 2020.
[6] A.T. Siacara, G.F. Napa-García, A.T. Beck, M.M. Futai. Reliability analysis of earth dams using direct coupling. *Journal of Rock Mechanics and Geotechnical Engineering 12*, pp.366-380, 2020.

[7] E. Vanmarcke. Random Fields: Analysis and Synthesis. Cambridge: MIT Press, 1983.
[8] Y. Liu, J. Li, S. Sun, B. Yu. Advances in Gaussian random field generation: a review. Computational Geosciences 23, p.1011–1047, 2019.
[9] F Haße, V. Prykhodko, S. Schlüter, S. Attinger. Generating random fields with a truncated power-law variogram: A comparison of several numerical methods. *Environmental Modelling & Software 55*, pg. 32-48, 2014.
[10] P. Mishra, C. M. Pandey, U. Singh, A. Gupta, C. Sahu, A. Keshri. Descriptive Statistics and Normality Test for Statistical Data. *Annals of Cardiac Anaesthesia 22*, p. 67-72, 2019.
[11] T.W. Lambe, R.V. Withman. *Soil Mechanics*. New York: John Willey & Sons, 1969.
[12] P.T., Cruz. *100 Barragens Brasileiras: casos históricos, materiais de construção, projeto*. São Paulo: Oficina de Textos, 1996.
[13] P.T. Cruz, B. Materón, M. Freitas. *Barragens de enrocamento com face de concreto*. 2. ed. São Paulo: Oficina de Textos, 2014. 359 p.
[14] M. Espíndola. *Ensaios triaxiais de grande escala em amostras de enrocamento da UHE Machadinho*. PhD Thesis, Federal University of Santa Catarina. Civil Engineering Department, 2016.
[15] D. R. Wines, P.A. Lilly. Estimates of rock joint shear strength in part of the Fimiston open pit operation in Western Australia. *International Journal of Rock Mechanics and Mining Sciences 40*, pp. 929-937, 2003.
[16] E.G. Khitrov, A.V. Andronov, B.G. Martynov, S.V. Spiridonov. Interrelations of various soil types mechanical properties. *International conferences Series: Journal of Physics: Conf. Series 177*, 7 p., 2019.
[17] Y. M. Cheng, T. Lansivaara, W.B Wei. Two-dimensional slope stability analysis by limit equilibrium and strength reduction methods. *Computers and Geotechnics 34*, p. 137-150, 2007.
[18] T. M. Leps. *Review of Shearing Strength of Rockfill. Journal of the Soil Mechanics and Foundation Division*, ASCE, vol. 96, no. SM4, Proc. paper 7394, july, pp. 1159 1170, 1970.
[19] A. P. Dyson, A. Tolooiyan. Optimization of strength reduction finite element method codes for slope stability analysis. *Innovative Infrastructure Solutions,* v. 3, 2018.