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Set of equations to estimate the modal parameters of the human-footbridge coupled system

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Abstract. Human–Structure Interaction (HSI) refers to the dynamic interplay between pedestrians and footbridge structures, forming a coupled mechanical system that may be represented by a single degree of freedom model (SDoFM) with modal parameters that differ from those of the empty footbridge. This study aims to present fitted equations to estimate the modal parameters of the coupled system, derived from numerical simulations emulating free vibration tests of occupied footbridges. Structural responses are evaluated using three methodologies, under pedestrian induced walking loads: (i) solving the coupled multi-degree-of-freedom model (MDoFM) of the footbridge–pedestrian system; (ii) solving the SDoFM of the coupled system, using the proposed equations; and (iii) solving the SDoFM of the empty footbridge, disregarding HSI effects. Results show neglecting HSI leads to significantly higher vibration amplitudes than those displayed by the coupled systems analyses, while MDoFM and SDoFM yield similar responses, demonstrating the practical relevance of the proposed HSI-SDoFM approach.

Keywords: human-structure interaction, modal parameters, footbridges, crowd loads, coupled system.

1 Introduction

Human–Structure Interaction (HSI) refers to the mutual influence between a footbridge and walking persons, forming a dynamically coupled system with modal parameters differing from those of the empty structure [1]. Codes and Design guidelines [2,3] often disregard HSI, modeling the footbridge as an isolated single-degree-of-freedom system (SDoFM) subjected to walking-induced harmonic loads.

This paper addresses two methodologies to account for the HSI effects in footbridges: (i) a full multi-degree-of-freedom model (HSI-MDoFM) composed of the structure and the walking persons represented by biodynamic models (BM), and (ii) a SDoFM to simulate the coupled system (HSI-SDoFM). The paper proposes the estimation of the modal parameters to be applied to the HSI-SDoFM via a numerical procedure simulating free vibration of the coupled system represented by the HSI-MDoFM.

Previous studies [4-6] have also proposed numerical procedures for estimating the modal parameters of crowd–structure systems. This study further contributes by providing fitted equations for their direct estimation. Based on these parameters, the methodology HSI-SDoFM may be used to design footbridges, in the Serviceability Limit State (SLS), by analyzing the coupled system under code-based walking loads.

The effectiveness of the HSI-SDoFM methodology is assessed by comparing its predicted acceleration responses with those from (i) HSI-MDoFM, and (ii) the procedure adopted in SETRA Guideline [2].

2 Mathematical model of HSI

Walking persons can be modeled as moving dynamic systems, each one represented by a single degree of freedom model (SDoFM), whose modal parameters depend on the physical characteristics and gait of the pedestrian. In this study, such representation is referred to as the biodynamic model (BM). Among the various

BMs proposed in the literature, the formulation adopted herein is that presented by Pfeil *et al*. [7], which was derived under the assumption that the walking person behaves as a SDoF system subjected to base excitation, simulating the vertical displacement imposed by the movement of the heels. This model was chosen for two main reasons: (i) its formulation is consistent with the HSI mathematical model adopted in this study; and (ii) it has shown good agreement with experimental measurements in its validation [7].

Based on the assumption that pedestrians can be modeled as single degree of freedom systems (SDoFM), the resulting HSI mathematical model of the coupled system, composed of the N - walking persons and the SDoFM of the structure, is presented in eq.(1). In this formulation, the amplitude of the mode shape $\phi(x)$ at a given coordinate x is defined as: $\phi(x) = sin(\pi\, x\, /\, L)$, where L denotes the span length of the footbridge.

$$\begin{bmatrix} m_s & 0 & \cdots & 0 \\ 0 & m_{p_1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & m_{p_N} \end{bmatrix} \ddot{Y} + \begin{bmatrix} \left(c_s + \sum_i^N \phi_i^2 c_{p_i}\right) & -\phi_{p_1} c_{p_1} & \cdots & -\phi_{p_N} c_{p_N} \\ -\phi_{p_1} c_{p_1} & c_{p_1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -\phi_{p_N} c_{p_N} & 0 & \cdots & m_{p_N} \end{bmatrix} \dot{Y} + \begin{bmatrix} \left(k_s + \sum_i^N \phi_i^2 k_{p_i}\right) & -\phi_{p_1} k_{p_1} & \cdots & -\phi_{p_N} k_{p_N} \\ -\phi_{p_1} k_{p_1} & k_{p_1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -\phi_{p_N} k_{p_N} & 0 & \cdots & k_{p_N} \end{bmatrix} Y = \begin{Bmatrix} P(t) \\ 0 \\ \vdots \\ 0 \end{Bmatrix} \tag{1}$$

where m, c and k are, respectively, modal mass, modal damping coefficient and modal stiffness. These parameters refer to the SDoFM of the structure, displaying the flexural bending mode, when the subscript is "s", and to the biodynamic model (BM) when the subscript is "p". $Y_{1\,x\,(N+1)}$ is the vector of modal coordinates and $P(t)$ denotes the generalized force induced by the walking persons.

3 Modal parameters of the coupled system

3.1 Proposed procedure

Based on the HSI mathematical model (eq.1), a numerical procedure to estimate the modal parameters of the coupled system was implemented. In this procedure, the modal damping ratio (ξ_{oc}) and natural frequency (f_{oc}) of the coupled system are obtained by simulating a free vibration test. The biodynamic models (BMs) are placed at fixed positions along the structure span, and an initial displacement is applied to the system, which then vibrates freely while the generalized force remains zero over time ($P(t) = 0$), allowing the identification of the desired parameters.

The modal parameters of the coupled system (ξ_{oc} and f_{oc}) are time-variant, as they depend on the number of walking individuals (N), their positions (x), and the dynamic properties of the biodynamic models (BMs) that represent each walking person. To account for this variability, Monte Carlo simulations were employed to generate a wide range of scenarios by sampling the position (x_{pi}), mass (M_i) and walking frequency (f_{pi}) of each pedestrian from the following predefined probability distributions: uniform distribution $\mathcal{U}(0, L)$m for their position, and normal distributions for their mass and walking frequency, with mean and standard deviation given by $M_i \sim \mathcal{N}(750,150)$N and $f_{pi} \sim \mathcal{N}(2.000,0.175)$Hz, respectively.

For each pedestrian density, the cumulative distribution functions (CDFs) of the modal damping ratio and natural frequency of the coupled system are obtained. The modal properties of the occupied structure (ξ_{oc} and f_{oc}) are then defined, by convention, as the median values (50th percentile) of the corresponding distributions.

The validation of the presented methodology is showed in [8] and it was performed by comparing the modal parameters of the coupled system obtained with the numerical implemented tool and those obtained from experimental measurements in a footbridge from the literature.

3.2 Parametric study

In this section, a parametric study is carried out to analyze the variation of the modal parameters of the

coupled system (ξ_{oc} and f_{oc}) in relation of the modal parameters of the isolated structure, — namely, its natural frequency (f_s) and damping ratio (ξ_s) — and of the mass ratio (μ), defined as the ratio between the sum of the mass of the walking persons and the modal mass of the footbridge.

The ratios ξ_{oc}/ξ_s and f_{oc}/f_s are taken herein to represent the relative damping and frequency of the occupied structure compared to the isolated case. For the purpose of this study, a simply supported footbridge is used, with a span length (L) of 50m, width (B) of 2m, and modal mass (m_s), associated with the 1st vertical bending mode, of 12500kg [1].

Fig.1(a)-(h) and Fig.1(i) show, respectively, the relative ratio of ξ_{oc}/ξ_s as function of the mass ratio (μ) for different values of natural frequency (f_s) and damping ratio (ξ_s) of the footbridge and the relative ratio of f_{oc}/f_s as function of f_s for different values μ.

As shown in Fig.1(a-h), there is a clear tendency for the increase of the damping ratio (ξ_{oc}/ξ_s) of the coupled system, relative to that of the isolated structure (ξ_s), as the mass ratio (μ) increases. An exception is observed for the curves with f_s=1.5Hz and ξ_s=2.0% and 3.0%, where the relative damping ratio remains below 1.0, indicating that the damping of the occupied structure is lower than that of the isolated. This behavior can be attributed to the low value of natural frequency of the structure, which is further decreased due to the effect of the pedestrian mass. As a consequence, the biodynamic models (BMs) behave effectively as rigid systems instead of dynamically coupled systems, leading to a reduction in the damping ratio of the equivalent coupled system.

Moreover, it is observed that the curves of damping ratio (ξ_{oc}/ξ_s) as function of mass ratio (μ) exhibit distinct behavior depending on the natural frequency of the structure (f_s). For f_s <3.0 Hz, the curves of ξ_{oc}/ξ_s tend to display an asymptotic variation for increase values of μ. For f_s >3.5 Hz, the curves of ξ_{oc}/ξ_s exhibit a linear and increasing relationship with the mass ratio (μ). For natural frequency values of the structure in the intermediate range ($3.0 \leq f_s \leq 3.5$ Hz), the curves of ξ_{oc}/ξ_s *versus* μ display characteristics of both the previously described asymptotic and linear behaviors, indicating a gradual transition between these two intervals. Regarding the influence of structural damping (ξ_s) on the curves of ξ_{oc}/ξ_s *versus* μ, it is noted that the lower the value of ξ_s, the greater the increase in damping ratio of the coupled system (ξ_{oc}), when compared to ξ_s.

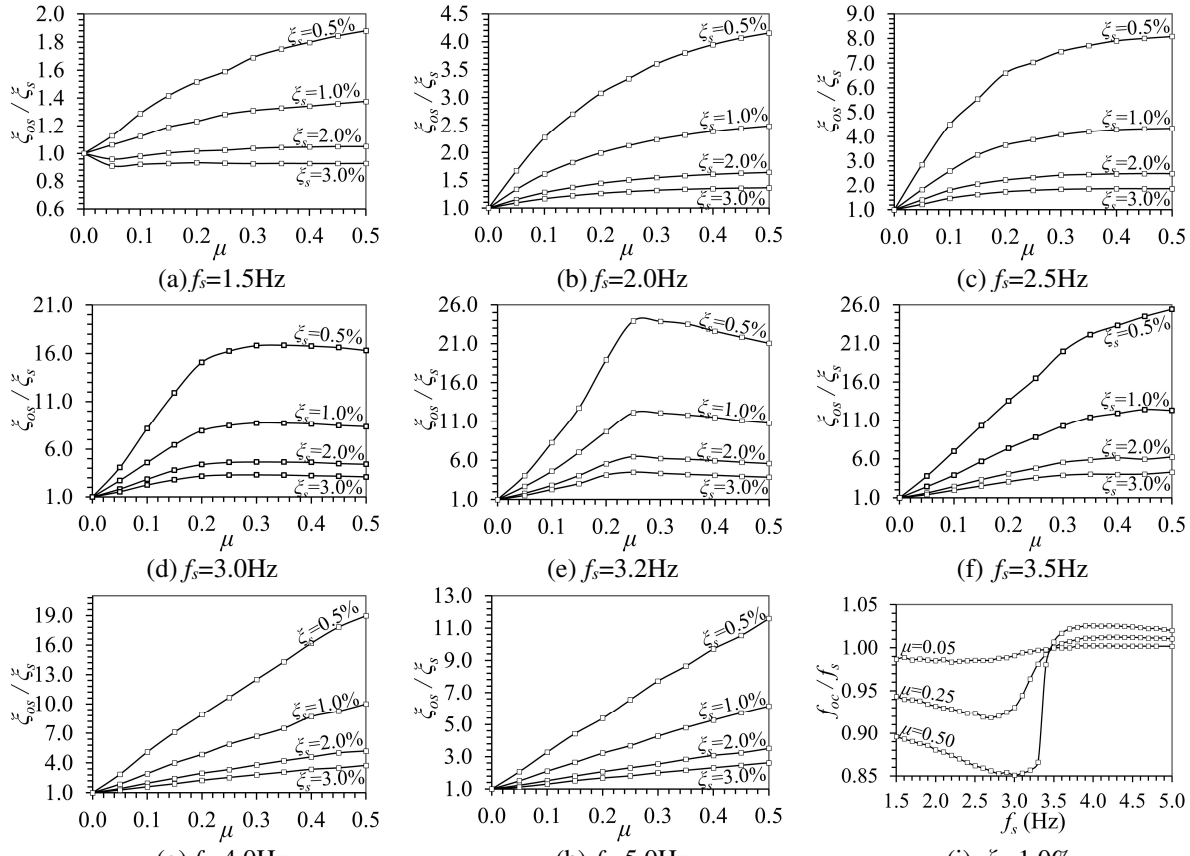


Fig. 1: (a-h) Damping ratio (ξ_{oc}/ξ_s) *versus* mass ratio (μ) for different natural frequencies (f_s) and different damping ratio (ξ_s) of the empty footbridge and (i) ratios of f_{oc}/f_s as function f_s for different values μ and for ξ_s=1.0%.

Regarding the curves of the relative frequency ratio (f_{oc}/f_s) *versus* f_s (Fig.1-i), three aspects should be considered:
(i) the values of the structural damping (ξ_s) do not influence the relative frequency ratio (f_{oc}/f_s). This was checked and verified through data generation for different values of ξ_s beyond those presented in this study and it is expected for systems with low modal damping, since—even though the damping of the occupied footbridge increases due to the presence of pedestrians—the difference between the damped frequency and f_s remains negligible.
(ii) The second aspect concerns the behavior of the relative frequency ratio (f_{oc}/f_s), which tends to be lower than 1.0 for f_s<3.0 Hz, and slightly higher than 1.0 for f_s>3.5Hz. For values of f_s between 3.0 Hz and 3.5 Hz, the frequency of the occupied structure (f_{oc}) is practically equal to that of the isolated structure (f_s).
(iii) The third aspect refers to the dependence of f_{oc}/f_s on the mass ratio (μ). It is verified that the most significant changes in f_{oc}, in relation of f_s, occur for increasing values of μ.

4 Fitting equations

This section presents a set of fitted equations for estimating the relative damping ratio (ξ_{oc}/ξ_s) and the relative frequency ratio (f_{oc}/f_s) as a function of the modal parameters of the empty structure (f_s and ξ_s) and the mass ratio (μ), determined according to the methodology described in previous section. The development of these equations is motivated by the complexity of the previous procedure, which requires multiple steps, including the numerical solution of the differential system of equations presented in eq. (1) and Monte Carlo simulations to obtain the cumulative distribution function. These process demands computational resources and may not be practical for engineering design applications.

The fitting of the curves was based on the behavior described in Section 3.2, where two distinct intervals were identified for the curves of relative damping (ξ_{oc}/ξ_s), depending on the natural frequency of the structure (f_s): (i) in the first interval (f_s<3.0 Hz), the curves of ξ_{oc}/ξ_s *versus* μ display asymptotic behavior, similar to that of critical load *versus* lateral deflection in axially compressed columns with initial geometric imperfections; (ii) in the second interval (f_s >3.5 Hz), the curves of ξ_{oc}/ξ_s *versus* μ exhibit a linear increasing trend with respect to the mass ratio (μ), and can be approximated by linear equations.

Building upon the observed behaviors for the curves of ξ_{oc}/ξ_s *versus* μ, the parameters required for deriving the fitting equations were identified. For the first frequency interval (f_s<3.0Hz), the column analogy was applied and the parameters μ_o and r_{cr} were introduced, which correspond respectively to the initial geometric imperfection and the critical buckling load of a column. In the second interval (f_s >3.5 Hz) the fitting parameters comprise the angular coefficient α and a fixed intercept equal to 1.0, consistent with the linear behavior observed in the curves. These parameters are functions of the modal parameters of the isolated structure, namely f_s and ξ_s. Nonlinear regression was employed to fit these parameters using polynomial, power, and exponential equations. The choice of the fitting equation for each parameter was based on the one that provided the highest coefficient of determination (R^2).

For the curves of frequency ratio (f_{oc}/f_s) *versus* f_s, different behaviors are observed for the defined frequency intervals, as in the curves of the relative damping (ξ_{oc}/ξ_s). In the first interval (f_s<3.0Hz), the frequency of the occupied structure (f_{oc}) decreases almost linearly with respect to f_s, for different values of μ, displaying a behavior that can be reasonably approximated by a straight line. In the second interval (f_s>3.5 Hz), f_{oc} exhibits only a slight increase relative to the f_s, and they are assumed equal, which is justified given that the difference between them is smaller than the typical uncertainty in estimating the natural frequency during the design stage. In the transition interval (3.0 Hz≤f_s≤3.5Hz), variations of f_{oc} relative to f_s are minimal, so f_{oc} is similarly assumed equal to f_s. For the damping ratio curves (ξ_{oc}/ξ_s) *versus* μ, a conservative safety-based criterion is adopted for the transition interval, where the ξ_{oc}/ξ_s value is adopted as the minimum of the fitted equations for the adjacent intervals, as presented in eq. (2).

Figure 2(a-i) show a comparison of the modal parameters of the coupled system (ξ_{oc}/ξ_s e f_{oc}/f_s) for different values of mass ratio (μ), natural frequencies (f_s) and damping ratio (ξ_s) of the structure. The dashed lines represent the results obtained from eq. (2), while the solid lines correspond to the values obtained through the numerical procedure described in Section 3.1, whose results were presented in Section 3.2.

As shown in Figure 2, the curves obtained using the fitted equation — eq. (2) — (dashed lines) provide good agreement with the results from the parametric study (solid lines). Moreover, it is observed that, in general,

$$\frac{\xi_{os}}{\xi_s} = \begin{cases} \dfrac{\dfrac{0.22e^{1.28f_s}}{\xi_s^{0.37f_s}}}{\left[1 - \dfrac{0.17}{\mu . e^{\xi_s\left(2.61 f_s^{-1.62}\right)}}\right]} & , if\ 1.5Hz < f_s < 3.0Hz \\ 1 + \left[\dfrac{507.0\xi_s^{-1.05}}{f_s^{(0.06\xi_s + 2.43)}}\right]\mu & , if\ f_s > 3.5Hz \\ (the\ lower\ of\ the\ two\ values\ above) & , if\ 3.0Hz \le f_s \le 3.5Hz \end{cases} \tag{2}$$

$$\frac{f_{os}}{f_s} = \begin{cases} 1.00 - (0.06 f_s + 0.13)\mu & , if\ \ 1.5Hz < f_s < 3.0Hz \\ 1.00 & , if\ \ f_s \ge 3.0Hz \end{cases}$$

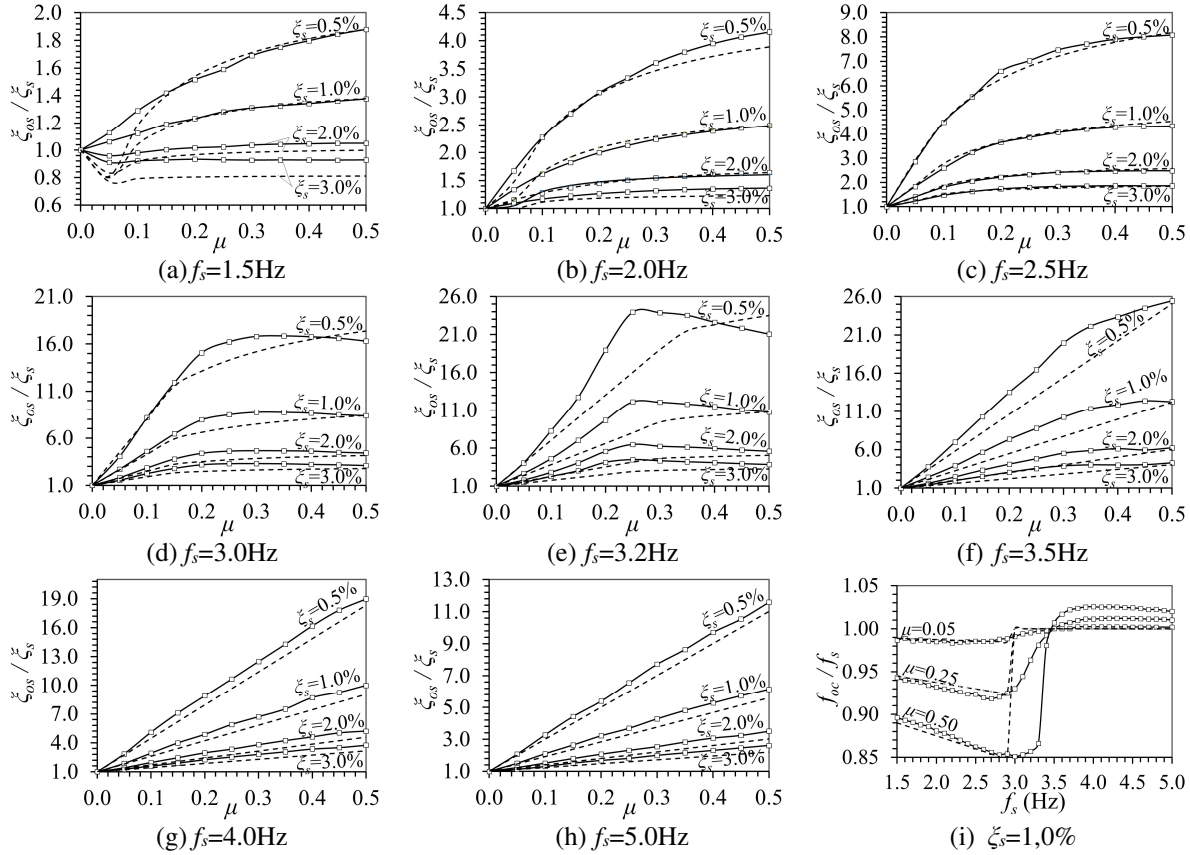


Fig. 2: (a-h) Comparison ξ_{oc}/ξ_s *versus* mass ratio (μ) and (i) f_{oc}/f_s *versus* f_s, for different natural frequencies (f_s) and different damping ratio (ξ_s) of the empty footbridge of the parametric study (solid lines) and determined by eq. (2) (dashed lines).

damping ratio values (ξ_{oc}/ξ_s) obtained through the fitted equations are slightly lower than those derived from the numerical methodology presented in Section 3, leading to more conservative results that are favorable from a structural safety perspective.

5 Response of the structure

The criterion for the Serviceability Limit State (SLS) in footbridges generally consists in limiting the structural acceleration to predefined comfort-based acceleration values [2,3]. Accordingly, it is necessary to determine the response amplitudes through a dynamic analysis, in which the footbridge is modeled as a single-degree-of-freedom model (SDoFM). The vibration mode of interest is that which can be excited by pedestrian-induced loading, whose representative load models are provided by codes and guidelines.

This section presents the acceleration responses obtained using three different methodologies, in which the

force induced by the walking persons, represented here according to the procedure described in the SETRA Guideline [2], is applied to the dynamic system as a generalized force. The methodologies are defined based on the dynamic system where the generalized force is applied, denoted as follows:

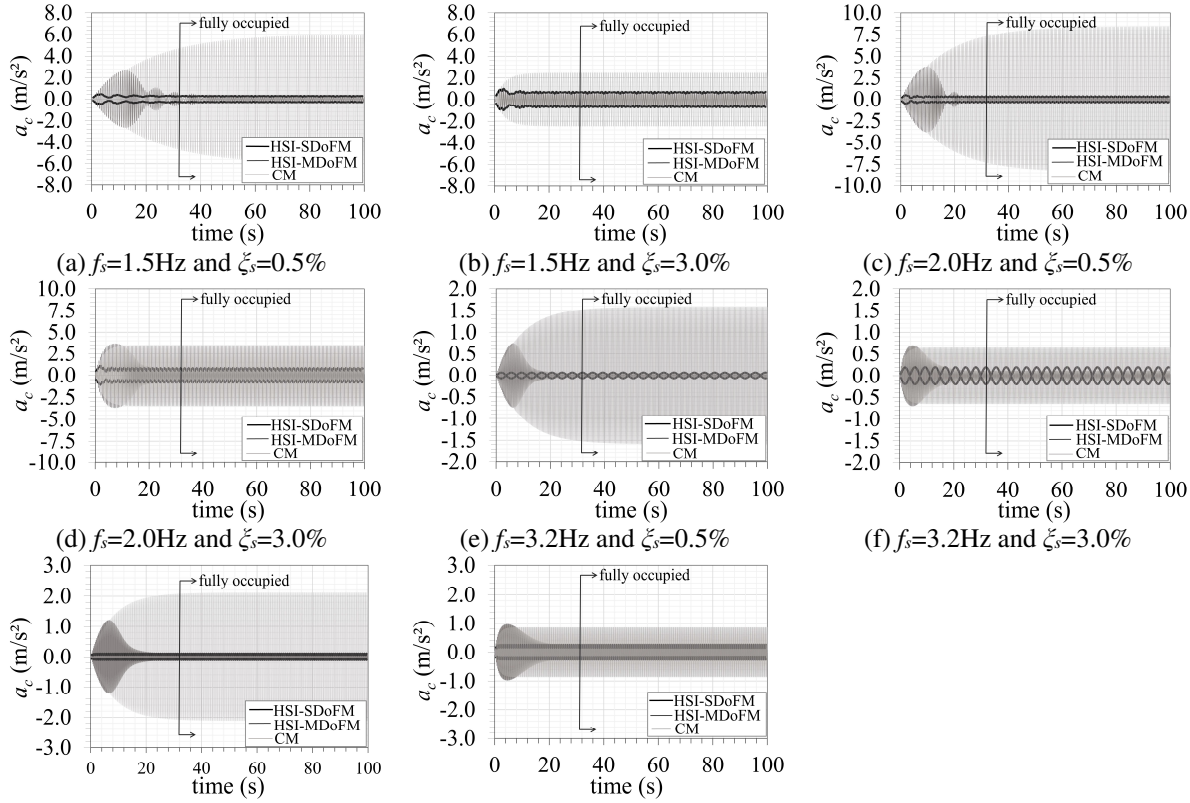
(i) CM (Crowd Model) - corresponds to the single degree of freedom (SDoF) model of the empty structure, thus neglecting human-structure interaction (HSI);

(ii) HSI-SDoFM (Human-Structure Interaction – Single Degree of Freedom Model) - correspond to the equivalent SDoF of the coupled system, composed of the structure with the pedestrians coupled, whose modal parameters are determined by eq. (2);

(iii) HSI-MDoFM (Human-Structure Interaction – Multiple Degree of Freedom Model) - corresponds to the system of equations (1), composed of the SDoFM of the empty structure with the biodynamic models (BMs).

The analysis developed in this section is based on two main assumptions. In the first assumption, the modal parameters of the structure used to calculate the walking-induced force from the SETRA Guide [2] are those of the empty footbridge in all the three methodologies, ensuring consistency in loading, that is: the same force is applied across all approaches, allowing a fair comparison of the resulting acceleration responses. The second assumption, specific to the HSI-MDoFM methodology, ensures consistency with the load model of SETRA [2] by controlling the entrance time of the biodynamic models (BMs) to reproduce a uniformly spaced pedestrian flow. Additionally, each BM was assigned a mass of 70kg and a walking frequency of 2.0Hz, in accordance with the Equivalent Load Procedure defined in the cited guide, which replaces the probabilistic loading by an equivalent deterministic load.

Based on the defined methodologies (CM, HSI-SDoFM, and HSI-MDoFM) and the compatibility assumptions between the pedestrian induced load and each methodology, Fig. 3 presents the time-history acceleration responses of the footbridge (L=50m, B=2m, m_s=12500kg). Two structural damping values were considered: ξ_s=0.5%, representing the lower bound, and ξ_s=3.0%, representing the upper bound values analyzed in Section 3. The natural frequency of the structure was varied among four values: 1.5 Hz (lower bound of the considered range), 2.0 Hz (average walking frequency), 3.2 Hz (associated with one of the f_s values that results in the highest ξ_{oc}), and 4.0 Hz (average value of the second harmonic of walking frequency). In all cases presented, the mass ratio (μ) is set to 0.5 (the upper bound of the range in Section 3), resulting in the largest discrepancies between the multi-degree and single-degree-of-freedom HSI models for the range of μ analyzed.



(a) f_s=1.5Hz and ξ_s=0.5% (b) f_s=1.5Hz and ξ_s=3.0% (c) f_s=2.0Hz and ξ_s=0.5%

(d) f_s=2.0Hz and ξ_s=3.0% (e) f_s=3.2Hz and ξ_s=0.5% (f) f_s=3.2Hz and ξ_s=3.0%

(g) f_s=4.0Hz and ξ_s=0.5% (h) f_s=4.0Hz and ξ_s=3.0%

Fig. 3: Time history acceleration of the structure (a_c) *versus* mass ratio (μ) for different natural frequencies (f_s) and different damping ratio (ξ_s) of the empty footbridge.

In Fig. 3, the vertical reference lines indicate the time instant at which the footbridge is fully occupied. Only from this instant can the structural responses obtained from the HSI-SDoFM and HSI-MDoFM methodologies be compared, as it corresponds to the instant when the damping of the occupied structure—calculated using eq. (2) in the HSI-SDoFM approach—reflects the actual occupancy density of the BMs distributed along the structure in the HSI-MDoFM model.

From the time instant at which the structure is fully occupied, a good agreement is observed between the HSI-SDoFM and HSI-MDoFM methodologies in predicting the dynamic responses of the footbridge. This favorable comparison highlights the relevance of the HSI-SDoFM approach, in which HSI effects can be efficiently accounted for by applying the pedestrian-induced load model of SETRA Guideline [2] directly to the SDoFM of the coupled system, whose modal properties are directly determined using the fitted equations.

Moreover, Fig. 3 shows that the response obtained by applying the CM into the SDoFM of the empty structure, as proposed in [2], results in significantly higher acceleration amplitudes compared to the other methodologies evaluated herein. This occurs because HSI effects are neglected, which leads to a highly conservative approach, as previously reported in the literature [1,5,6].

6 Conclusions

This paper proposed a set of fitted equations to estimate the modal parameters of the coupled pedestrian–footbridge system based on a numerical procedure that emulates free vibration simulations. The fitted equations accurately capture the patterns for the curves presented in the parametric study, enabling direct estimation of the modal parameters of the coupled system. When applied within the denoted HSI-SDoFM methodology, the structural responses under pedestrian-induced loads of a design guide closely match those obtained with the full HSI-MDoFM model, validating the methodology. The HSI-SDoFM offers a practical contribution to engineering design by enabling the inclusion of human–structure interaction effects on the dynamic responses of footbridges through a simple method that remains consistent with more detailed methodologies such as the HSI-MDoFM.

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