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Bridging Structural Reliability and Quality Control: A Risk-Based Optimization Considering Non-technical Factors

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Abstract. In this paper, we explore the interdependence between quality control, represented by inspection frequency, and optimal redundancy in structural systems. A risk-based optimization of a Daniels system is performed, considering inspection intervals throughout the design life. System reliability is evaluated using a model where loads are represented as pulses of random intensity, and failures due to non-technical factors are modeled as pulses governed by a latent hazard rate, λ_L. Both brittle and ductile material behaviors are considered, along with varying correlation coefficients between element strengths. The results reveal a clear dependency between the optimal number of inspections (n_{insp}) and the optimal number of elements (n_c). As λ_L increases, both n_{insp} and n_c increase accordingly. Moreover, the results are strongly influenced by the strength correlation between elements and the material behavior. The findings highlight the role of latent failure probability as a conceptual link between quality control and structural reliability.

Keywords: System reliability, Structural robustness, Redundancy, Quality control, Inspection and maintenance

1 Introduction

Structural reliability theory addresses uncertainties in technical factors - such as loading, material properties, geometry, and engineering models (the known unknowns) - and aims to determine element resistances that ensure adequate safety margins. In contrast, quality control focuses on mitigating the effects of non-technical factors, including gross errors, operational misuse, and unforeseen failure modes (unknown unknowns), on structural safety. Although traditionally treated as independent, these two fields are in fact interdependent, as both technical and non-technical factors affect structural safety. In light of this, early studies have proposed integrating structural reliability and quality control within a broader discipline of Structural Quality Assurance [1, 2].

In this context, the latent failure probability concept was introduced to explicitly account for the effects of non-technical factors in Reliability- and Risk-Based Design Optimization [3, 4]. Recent applications involving progressive collapse [4, 5] have demonstrated that, to enhance robustness against such factors, optimal structural systems should be redundant. However, the interplay between optimal redundancy and quality control remains a largely underexplored research topic.

This paper investigates the interdependence between optimal structural redundancy and quality control, represented by inspection frequency. A risk-based optimization formulation that incorporates the effects of regular inspections and non-technical factors is presented and applied to the optimization of a Daniels Systems. The influence of non-technical factors is modeled through a latent hazard rate, λ_L. A parametric analysis is conducted, considering different material behaviors, correlations between elements strengths, and varying hazard rates. The results show the dependency between quality control and optimal redundancy and highlight how the latent failure probability concept can be the link between these two fields.

2 Problem Formulation

2.1 Daniels Systems

The structure analyzed in this study is the Daniels System [6, 7], presented in Fig. 1. It is composed of n_c elements with same cross-sectional areas A_i, and identically distributed ultimate strengths σ_i, such that the elements' ultimate resistances are $R_i = \sigma_i A_i$. The load S acting quasi-statically on the system is equally distributed between the non-failed components by the horizontal bar. The construction cost for the Daniels system is given simply by the sum of cross-section areas, $A_{total} = \sum_{i=1}^{n_c} A_i = n_c A_i$.

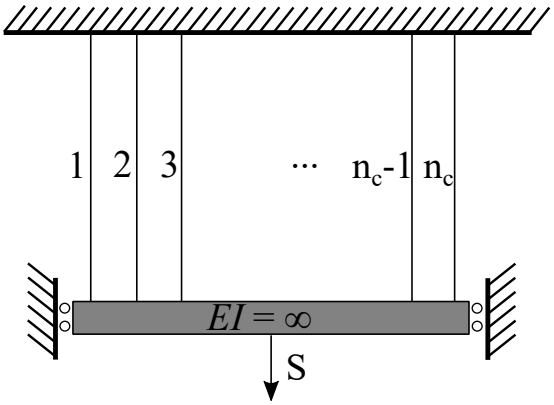


Figure 1. Daniels System

In this study, we address both linear elastic-perfectly plastic (ductile) and perfectly brittle material behaviors, assuming equicorrelated element strengths, characterized by a correlation coefficient $0 \leq \rho < 1$. The resistance of the Daniels system with brittle material behavior is derived as follows. The system's resistance up to the failure of the first element is equivalent to n_c times the resistance of the weakest element. Following this, the system's resistance up to the failure of the second element is reduced to $n_c - 1$ times the resistance of the second weakest element, and this process continues iteratively for each subsequent failure. Under these conditions, the system resistance R_{Sys} is given by:

$$R_{Sys}(A_i, n_c) = \underset{i=1,\ldots,n_c}{max} \left\{ (n_c - i + 1)\widehat{R}_i \right\}. \tag{1}$$

where $\widehat{R}_i$ is the resistance of the i^{th} weakest element, i.e., it is the i^{th}-order statistic of member resistance R_i, such that $\widehat{R}_1 \leq \widehat{R}_2 \leq \cdots \leq \widehat{R}_{n_c}$.

For ideal ductile material behavior, the system resistance corresponds to the sum of the individual component resistances, since each element continues to contribute beyond its yield strength σ_i:

$$R_{Sys}(A_i, n_c) = \sum_{i=1}^{n_c} R_i. \tag{2}$$

The limit state function that defines the failure for both systems is:

$$g_{Sys}(\mathbf{d}, \mathbf{X}) = R_{Sys}(A_i, n_c) - S. \tag{3}$$

where $\mathbf{d}$ is the vector of deterministic design variables (cross-sectional areas A_i), and $\mathbf{X}$ is the vector of random variables (ultimate strengths σ_i and load S), which has a joint probability density function $f_{\mathbf{X}}(\mathbf{x})$. Based on this, the system failure probability, conditional on the arrival of a load pulse S, can be written as:

$$p_{f,Sys} = P[S > R_{Sys}(A_i, n_c)] = \int_{g_{Sys} \leq 0} f_{\mathbf{X}}(\mathbf{x}). \tag{4}$$

2.2 Lifetime System Reliability considering inspections

Eqs. (1) to (4) represent the classical capacity-demand problem commonly addressed in structural reliability analysis. These formulations typically assess small to very small failure probabilities arising from the coincidence of a peak of a conventional load with a low resistance realization. However, real-world structural failures can also be triggered by external factors such as hazards, unexpected failure modes, gross errors in design or construction, and operational abuse [8–12]. These factors can lead to the loss of one or more structural elements and, ultimately, to system collapse. Such external disturbances - referred to as 'shocks' or 'hazards' - can be characterized by a (annual) latent hazard rate, λ_L. As we demonstrate in the following, this concept can be effectively incorporated into system reliability analysis to account for the influence of these factors in structural safety.

Let the load S acting in the Daniels System be represented as a sequence of pulses of random intensity, characterized as a homogeneous Poisson process with (annual) rate of arrival λ_S, as illustrated in Fig. 2. The annual failure rate for the undamaged structural system is:

$$\lambda_{\overline{D}} = \lambda_S P[S > R_{Sys}]. \tag{5}$$

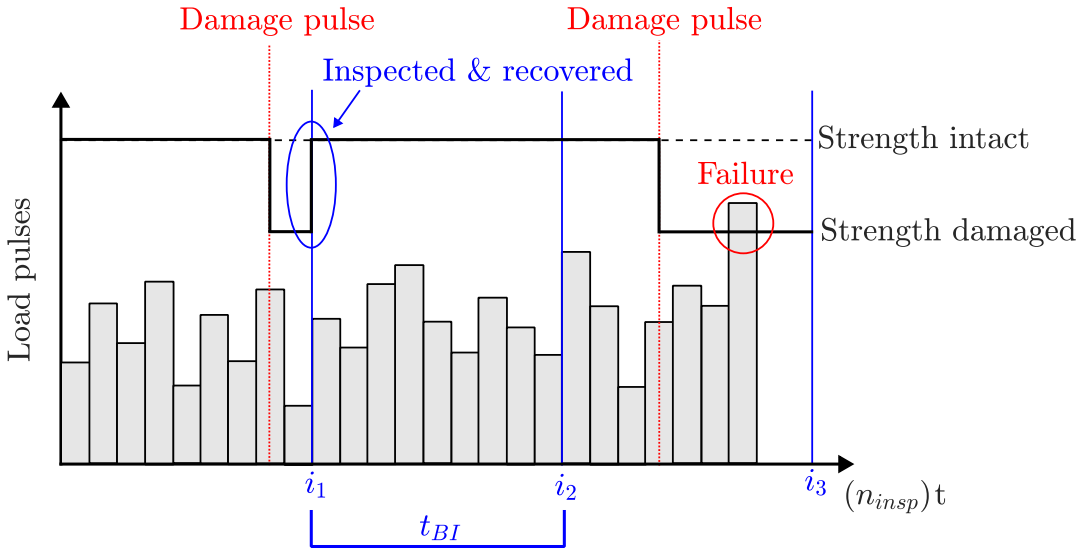


Figure 2. Daniels system subject to load and damage pulses, with scheduled regular inspections

The hazard damage rate is λ_L and represents the loss of any element of the Daniels system. The "loss of component" event is also represented by a Poisson process with rate of arrival $\lambda_{LC} = n_c \lambda_L$. Moreover, we assume that in each inspection an eventual damaged component will be identified and fixed. As a consequence, each "loss of component" pulse has a uniformly distributed duration between 0 and the time between inspections, t_{BI}. Based on these assumptions, the annual failure rate for the damaged system is:

$$\lambda_D = \lambda_S \lambda_{LC} \frac{t_{BI}}{2} P[S > R_{Sys}(A_i, n_c - 1)]. \tag{6}$$

where the term $\lambda_S \lambda_{LC} \frac{t_{BI}}{2}$ represents the rate of coincidence of load and "loss of component" pulses, assuming that the former are of short duration. The term $P[S > R_{Sys}(A_i, n_c - 1)]$ represents the system failure probability of the damaged system, conditional on the arrival of both pulses.

Finally, the system failure probability for a design life t_D can be written as:

$$p_{f,Sys}(t_D) = 1 - exp[-(\lambda_{\overline{D}} + \lambda_D)t_D]. \tag{7}$$

2.3 Optimal Redundancy Allocation Considering Inspections: Risk-based Formulation

The risk-based formulation of the optimal redundancy allocation problem considering inspections addressed in this study can be stated as:

$$\begin{aligned} &\text{given: } \lambda_L \\ &\text{find: } \mathbf{d}^* = \{n_c^*, n_{insp}^*\} \\ &\text{which minimizes: } C_{\text{TE}}(\mathbf{d}, \mathbf{X}, \lambda_L) = C_{const}(\mathbf{d}) + C_{\text{I\&M}}(\mathbf{d}) + C_{EF}(\mathbf{d}, \mathbf{X}, \lambda_L) \\ &\text{subject to: } \mathbf{d} \in \mathcal{D} \end{aligned} \tag{8}$$

In Eq. (8), the design vector is composed of the number of components n_c and the number of inspections n_{insp}; the construction cost is $C_{const}(\mathbf{d}) = c_0 + c_m n_c A_i$, where c_0 is a constant term and c_m is the unit cost of materials; the inspection and maintenance cost is $C_{I\&M}(\mathbf{d}) = (c_{insp,0} + c_{insp,i} n_c) n_{insp}$, where $c_{insp,0}$ is the fixed cost for one inspection campaign, $c_{insp,i}$ is the variable inspection cost; the expected cost of failure is $C_{EF}(\mathbf{d}) = c_F p_{f,Sys}(t_D)$, where c_F is the total cost of system failure.

3 Numerical Results

A parametric analysis is performed to investigate the influence of λ_L and ρ on the optimization results. The parameters evaluated include $\lambda_L \in \{10^{-3}, 10^{-2}, 10^{-1}\}$ and $\rho \in \{0, 0.3, 0.6, 0.9\}$, with ductile and brittle materials. The ultimate strengths are assumed to be lognormally distributed with means $\mu_\sigma = 25$ kN/cm^2 and coefficient of variation $\delta_\sigma = 0.15$. The load pulses S are represented with a Gumbel distribution, with $\mu_S = 100$ kN and $\delta_S = 0.30$. The total cross-sectional area of the Daniels system is calibrated to achieve an annual reliability index of 3.0 for the undamaged condition. The reliabilities conditional on the arrival of the load pulses are computed using FORM. Moreover, the unitary costs considered in the optimization are presented in Table 1.

Table 1. Unitary costs for risk-based optimization

Cost	Symbol	Value
Fixed construction cost	c_0	1000
Unit cost of materials	c_m	100
Fixed inspection cost	$c_{\text{insp},0}$	100
Unit cost of inspection	$c_{\text{insp},i}$	10
Cost of system failure	c_F	10^5

Results for the Daniels system with brittle behavior are presented in Fig.3. It can be observed that the optimal number of components and the optimal number of inspections increase with increasing λ_L. The total expected cost also increases accordingly. The correlation between elements strengths has a smaller effect, unless it goes to $\rho = 0.90$; in this case, the optimal number of inspections decreases and the optimal number of components increases.

Fig.4 shows similar results for the Daniels system with ductile material behavior. In this case, as λ_L increases, the optimal number of components is reduced, but the optimal number of inspections increases. As the correlation coefficient gets larger, optimal n_c reduces and optimal n_{insp} increases, a behavior opposite to that observed for the brittle system with $\rho = 0.90$. Compared to the brittle systems, optimal ductile systems have higher redundancy and fewer inspections, due to the system reliability gains from ductile failures.

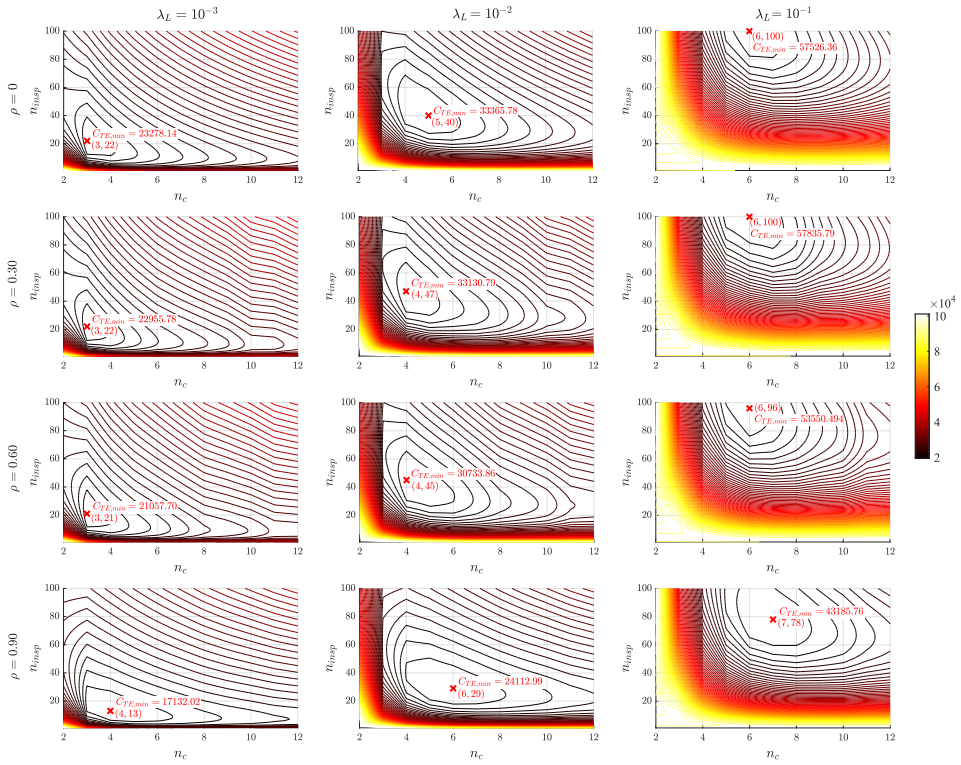


Figure 3. Total expected costs as a function of n_{insp} and n_c, assuming brittle material behavior. The results are evaluated for different values of λ_L and ρ, with $\delta_\sigma = 0.15$. The red cross marks the minimum point and its objective function value.

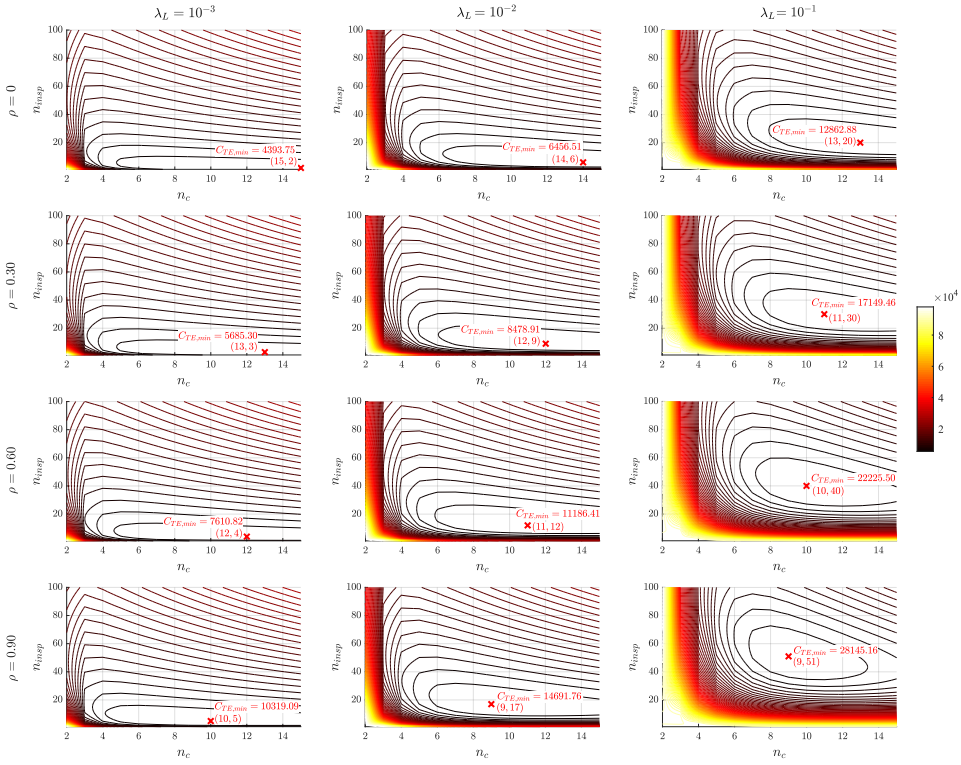


Figure 4. Total expected costs as a function of n_{insp} and n_c, assuming ductile material behavior. The results are evaluated for different values of λ_L and ρ, with $\delta_\sigma = 0.15$. The red cross marks the minimum point and its objective function value.

4 Conclusions

This paper has explored the optimal redundancy allocation in structural systems subjected to external shocks - such as abnormal loading events, unforeseen failure modes, gross errors in design, construction or operation, and misuse - factors historically associated to structural collapses. These events are typically disregarded in conventional structural reliability theory, as it is assumed they can be managed through quality control. However, structural reliability should be viewed within the broader discipline of *Structural Quality Assurance*, which integrates reliability analysis and quality control. This study contributes to this perspective by revealing an intricate interaction between optimal redundancy and quality control, represented by the frequency of inspections. Furthermore, the findings demonstrate that the optimal system redundancy is directly related to the likelihood of abnormal events causing component failures - an often overlooked form of epistemic uncertainty in structural reliability literature. Overall, this study offers new insights into the robustness and risk-based optimization of engineering structures.

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