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Topology Optimization of Three-Dimensional Trusses Using the Progressive Directional Selection Method

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Abstract. The pursuit of designing structures that are more efficient in supporting the applied loads while also achieving greater material savings has led many designers to turn to structural optimization methods. Topological structural optimization has been gaining increasing importance in the early stages of design, as it allows for the identification of the optimized material distribution so that the structure can support the applied loads. However, many topological optimization methods rely on mathematically complex equations, making them difficult to implement numerically and even harder for many design engineers to understand. Consequently, heuristic topological optimization methods—which do not depend on mathematical gradients—are a good alternative for achieving optimized solutions with easier implementation. One such method is the Progressive Directional Selection (PDS) method, a purely heuristic evolutionary approach. The PDS method is inspired by evolutionary principles—particularly the idea of directional selection. It aims to optimize a structure by selecting the elements that contribute most to bearing the applied loads and eliminating those that contribute the least, through distinct removal stages. This study aims to implement the PDS method for optimizing three-dimensional truss structures using the ground structure approach. The results demonstrate that PDS can find optimized solutions for three-dimensional truss structures.

Keywords: topology optimization, Progressive Directional Selection, ground structure, tridimensional trusses.

1 Introduction

The main challenge for design engineers is to design structures or structural elements that are more efficient, cost-effective, and within acceptable safety limits. An important tool for achieving this goal, and one that is already a reality for many designers, is the topological structural optimization technique. This type of structural optimization seeks to distribute material within an initial design domain to achieve the best arrangement for a structure when subjected to loads and support conditions [1].

According to Querin et al. [2], there are several topology optimization methods, which can be grouped into two categories: Optimality Criteria methods and Heuristic or intuitive methods. Optimality Criteria methods, based on mathematical gradients, are more rigorous and are applied to problems with a large number of design variables and few constraints. Examples of this category include the Homogenization method [3] and the Solid Isotropic Material with Penalization (SIMP) [4]. Heuristic methods, on the other hand, are derived from intuition, observations of engineering processes, or observations of biological systems. These methods cannot always guarantee optimality, but they can provide efficient and feasible solutions. The Evolutionary Structure Optimization (ESO) and Bidirectional Evolutionary Structural Optimization (BESO) methods [6] can be cited as examples of evolutionary heuristic methods

Heuristic methods offer the computational advantage of being simpler to implement than optimality criterion methods, primarily due to the latter's rigorous mathematical gradient approach. Motivated by this, an evolutionary heuristic method for topology optimization was proposed by Veras and Cavalcante [7], inspired by Darwin's theory of natural selection and, more specifically, by the concept of directional selection as formalized in modern evolutionary biology.

The Progressive Directional Selection (PDS) method, like other topological optimization methods, seeks to find the best material arrangement in a structure subject to specific loads and boundary conditions. However, what differentiates the PDS optimization process from other evolutionary heuristic methods, such as ESO/BESO, is that the element selection process for the structure is divided into different scenarios or selection stages. In each stage, the original population of elements (initial domain) undergoes a distinct element removal process through removal steps until the number of elements pre-established by the user is reached. At the end of each stage, the selected population of elements may vary because the element removal process differs for each selection stage. The algorithm stops when the same group of selected elements is repeated in consecutive selection stages, thus achieving convergence.

The PDS method has already been employed for topology optimization of planar trusses within the ground-structure approach, as shown by Cavalcante et al. [8]. This work extends the Progressive Directional Selection (PDS) method to 3D truss topology optimization under the ground-structure approach, integrating structural design limit-state checks—strength and stability (member stress and buckling/slenderness) and serviceability (top-deflection)—to make the procedure closer to engineering practice. A lattice wind-turbine tower serves as the case study.

2 Progressive Directional Selection

The initial design domain is considered the starting population of individuals, with each element in that domain being treated as an individual. When the acting loads and constraints are imposed, each element must contribute to support the structure under these loads. The elements that contribute the least to the structure are gradually eliminated, similar to what occurs in nature. This process results in the structure evolving into a configuration that efficiently supports the loads.

Consider this as an illustrative example of this process. Suppose a building floor where the designer needs to define the position of ten columns, knowing there are twenty possibilities for their location. In this scenario, the initial population of twenty columns will undergo a progressive directional selection. Once the loads are applied to the structure, the first selection stage consists of removing the ten least contributing columns in a single step to obtain the ten selected columns.

At each selection stage, the goal is to arrive at the selected population, with the number of elements predefined by the user, and this process is applied differently at each stage. Thus, the selected population is achieved differently at each stage, which may or may not result in the same group of elements.

At the beginning of each selection stage, the initial population will undergo a process of removing elements differently through pruning steps. The number of individuals removed is partitioned differently for each stage. For instance, the second stage consists of two steps: the load is applied, and the five least contributing columns are removed. Then, the load is applied again to the remaining fifteen elements, and the five least contributing columns are removed, resulting in ten columns at the end of the stage. The third stage involves three steps for removing elements, with 4, 3, and 3 columns removed in each subsequent step.

The final population of columns in each stage can differ due to the rearrangement of internal forces. The

maximum number of selection stages equals the total number of elements to be removed, which is ten in the case of the building floor with twenty columns. The process continues until the selected elements in the last stage match those in the previous stage, indicating convergence. In the case of the building floor, the positions of the columns at the end of the third stage match those in the second stage, demonstrating convergence. Generally, the user determines the number of consecutive stages (n) with repetition of the selected elements by applying the convergence criterion C_n, which defines when the process must stop.

During the optimization process, it is possible to apply a penalty factor (PF) to the stiffness of eliminated elements from the discretized analysis domain, thus avoiding remeshing and the singularity of the global stiffness matrix. Special attention must be given to the population ranking. To address numerical accuracy and symmetry issues during the selection process, a selection tolerance (ST) must be applied for each removal step, allowing the addition of elements with a performance criterion value very close to the last selected element in the ranked vector.

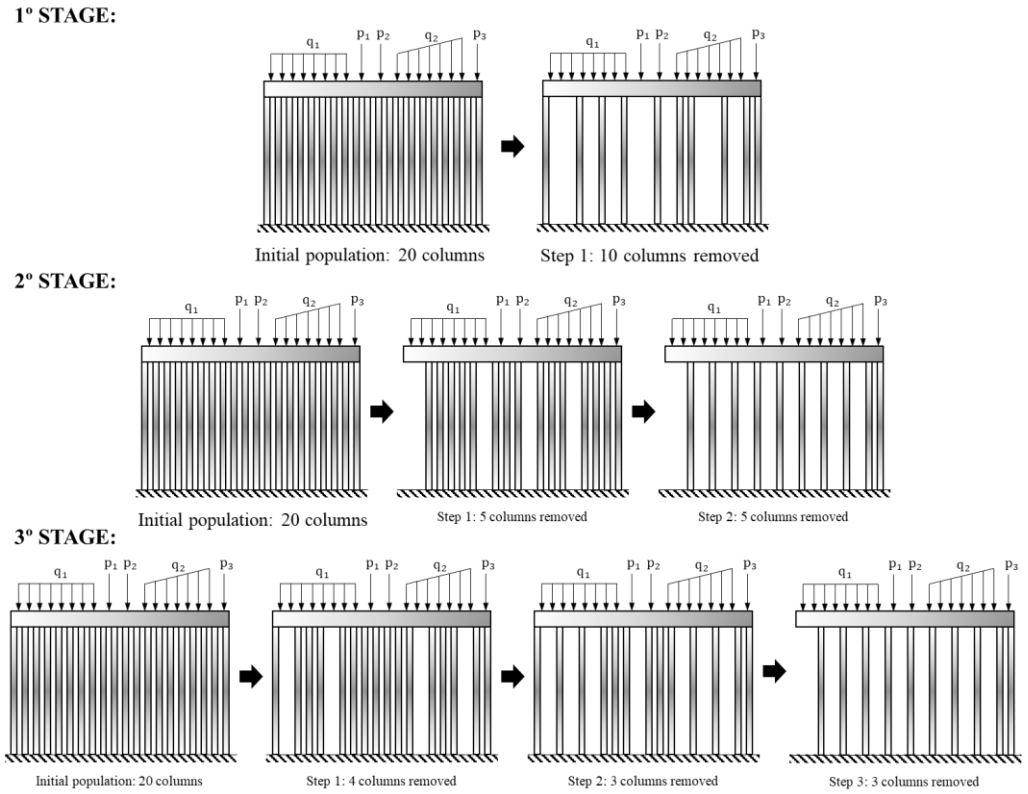


Figure 1. Hypothetical example of the PDS application on a building floor with an initial population of twenty columns.

3 Methodology

The PDS method is used in this work for the topology optimization process in three-dimensional trusses using the ground structure approach, applied to an example of a lattice tower of a wind turbine. The methodology of this work can be represented by the flowchart in Figure 2.

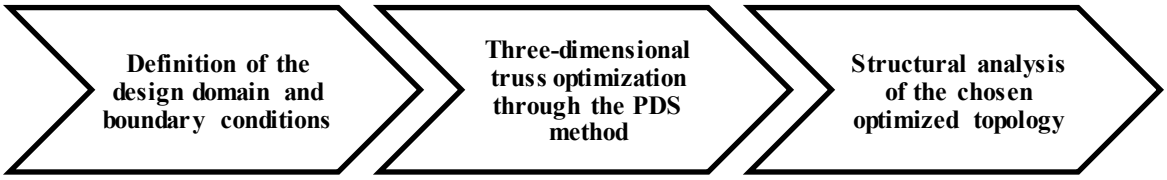


Figure 2. Work methodology flowchart.

3.1 Definition of the design domain and boundary conditions

The initial design domain was established using a three-dimensional ground structure with a square base, measuring 1.5 m in both width and length. This domain was discretized into 5 elements along the width (x-direction) and 5 elements along the length (y-direction), resulting in individual elements with a dimension of 0.3 m. The tower's height was set to 12 m. For standardization, all ground structure configurations are set to use 6 elements along the height, maintaining uniformity in the vertical discretization, analogous to what has already been established for the x and y directions. All truss elements have a solid circular cross-section of 5 cm in diameter and are assumed to be made of steel with a Young's modulus of 220 GPa.

For this work, two levels of ground-structure connectivity are adopted: *Connectivity level 1*, in which each node is connected only to its immediate neighboring nodes; and *Connectivity level 3*, in which each node is connected to all other nodes located within three adjacency levels in the nodal network.

Based on Gama Jr [9], the loads considered in the analysis can be seen in Figure 3, which consists of the weight of the nacelle-rotor assembly, w, and the drag force generated by the wind on the blades, F_w. Both loads were applied concentrically and equally distributed among the four central nodes of the upper plane of the domain, which represents the wind turbine attachment point. No eccentricities were considered for these forces (implying zero moments), nor were the self-weight of the truss elements or the direct action of the wind on them. Since the turbine can rotate 360° around the z-axis due to the wind direction, the load F_w varies in direction as a function of angle theta. Due to the dynamism of F_w, it is necessary to determine the maximum stress envelope that each bar element can develop when the load direction varies.

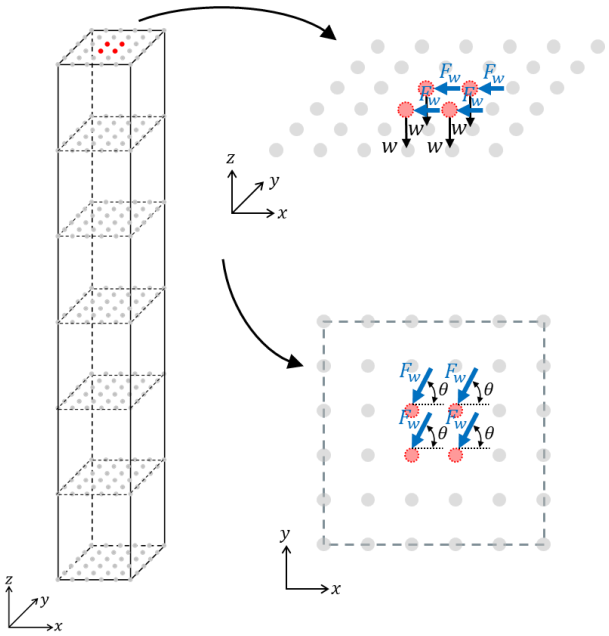


Figure 3. Acting loads considered in the design domain.

To determine the loads applied to the structure, the following input parameters were considered, drawing from Gama Jr [9] for the Skystream 3.7 by Xzeres wind turbine specifications and the structural analysis guidelines for environmental factors: a nacelle-rotor assembly mass of 77 kg, a rotor diameter of 3.72 m with 3 blades, an air density of 1.131 kg/m³, a basic wind speed of 30.00 m/s for the location, and a design wind speed of 13.00 m/s for the wind turbine. A characteristic wind speed of 31.80 m/s was used (obtained from the basic wind speed factored by S1, S2, and S3 according to NBR 6123 [10]) for the Ultimate Limit State (ULS), while the design wind speed was adopted in the Serviceability Limit State (SLS).

The two ground structure configurations used as the initial domain for the analysis, from which the optimized truss towers were generated by the PDS method, are illustrated in Figure 4. Figure 4-a) presents the ground structure with connectivity level 1, and Figure 4-b) displays the configuration with connectivity level 3. In all

visualizations, the gray lines represent the truss elements that are the candidate members to compose the final structure. The black nodes indicate the regular interconnection points within the mesh. The blue spheres, located at the base, denote the restricted nodes that simulate the fixed support points of the tower. Finally, the red spheres, positioned at the top, signal the nodes where the main external loads are applied.

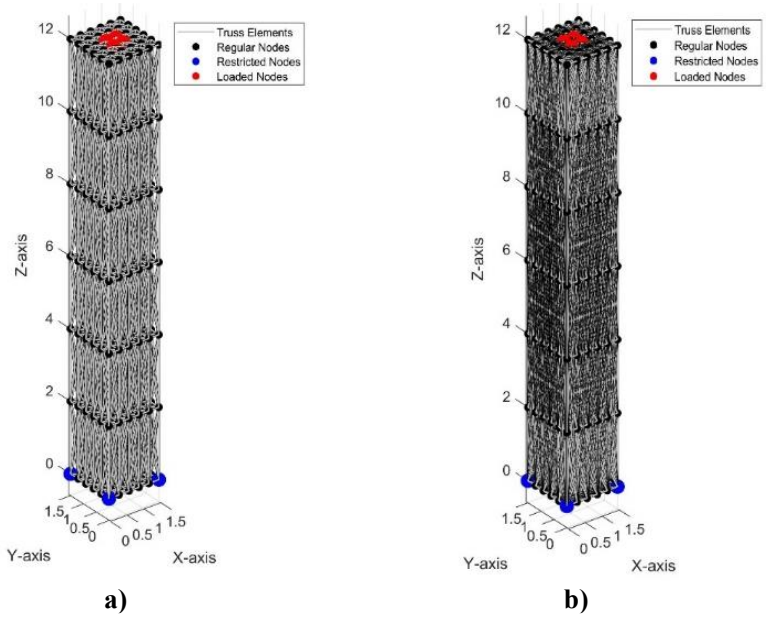


Figure 4. Initial domain (ground structure): a) connectivity level 1 and b) connectivity level 3.

3.2 Three-dimensional truss optimization through the PDS method

The PDS-based topology optimization algorithm is adapted to the three-dimensional ground structure domain. The initial PDS parameters are the same for the two analyses: penalty factor ($PF = 10^{-6}$), selection tolerance ($ST = 10^{-6}$), and convergence criterion ($C_n = 30$). Initially, the convergence criterion is fixed, but topologies that appear with criteria lower than the established one are analyzed for comparison. The absolute stress in each element is chosen as the performance parameter for ranking and choosing the selected elements. Since this is a design domain with discretized elements in truss bars, the number of selected elements is used as one of the prescribed parameters to satisfy at the optimized structure. For the ground structure with connectivity 1, the number of selected elements in the optimized structure was 205. For the ground structure with connectivity 3, the number of selected elements was 160. These values were achieved through an analysis process where the authors varied the number of selected elements until they obtained the truss with fewer elements with at least one topology meeting the ultimate and serviceability limit state criteria.

Several topologies obtained for different convergence criterion values were analyzed. The parameters used to select the best topology for each ground structure connectivity level are allowable displacement at the load application point, maximum buckling utilization index, maximum yield utilization index, and total element length. For the Serviceability Limit State (SLS), the horizontal displacement at the load application point is limited to $H/400$, adopted as a design criterion in accordance with the recommendations of NBR 8800 [11], where H is the tower height. For the Ultimate Limit State (ULS), the structural verification is performed by means of utilization indices. The maximum buckling utilization index is the ratio of the critical compressive load of the most stressed element in the topology to the allowable buckling load, while the maximum yield utilization index is the ratio of the maximum absolute stress of the structure to the allowable yield stress of the material. In both cases, utilization indices must remain less than or equal to 1. The total structure length is the sum of the topology element lengths, a parameter that represents the amount of material used in the structure's construction. The topology chosen at each level of connectivity was the one that satisfied the state limits and had the shortest total length.

3.3 Structural analysis of the chosen optimized topology

Finally, the topologies chosen for each connectivity level are analyzed by comparing their respective displacements of the load application point, maximum absolute stress, total length, maximum buckling utilization index, maximum yield utilization index, deformation of the structure, and stress levels.

The computational environment used for the simulations consists of MATLAB R2023a (64-bit) running on Windows 10. The hardware comprises a 5th Gen Intel(R) Core(TM) i5-10300H processor at 2.50 GHz and 8.0 GB of DDR3L RAM 5.1.

4 Results and Discussions

As mentioned in the previous section, a critical performance indicator for the optimized topologies, the maximum displacement at the load application points, was employed. The admissible displacement for this criterion was established as H/400, adopted as a design guideline in accordance with the recommendations of NBR 8800 [11], which corresponds to 30 mm for the 12 m high tower. Among the topologies meeting the convergence criteria up to C_{30}, two optimized topologies at connectivity level 1, and four at connectivity level 3 registered maximum displacements below the 30 mm limit. The graph representation of these results is provided in Figure 5. These graphs illustrate the maximum displacement at the load application points for each convergence criterion. Green points denote topologies with maximum displacements below the admissible limit, while red points indicate those exceeding it. The admissible limit of 30 mm is demarcated by a horizontal red line. Each graph corresponds to a specific connectivity level: Figure 5-a for level 1, and Figure 5-b for level 3.

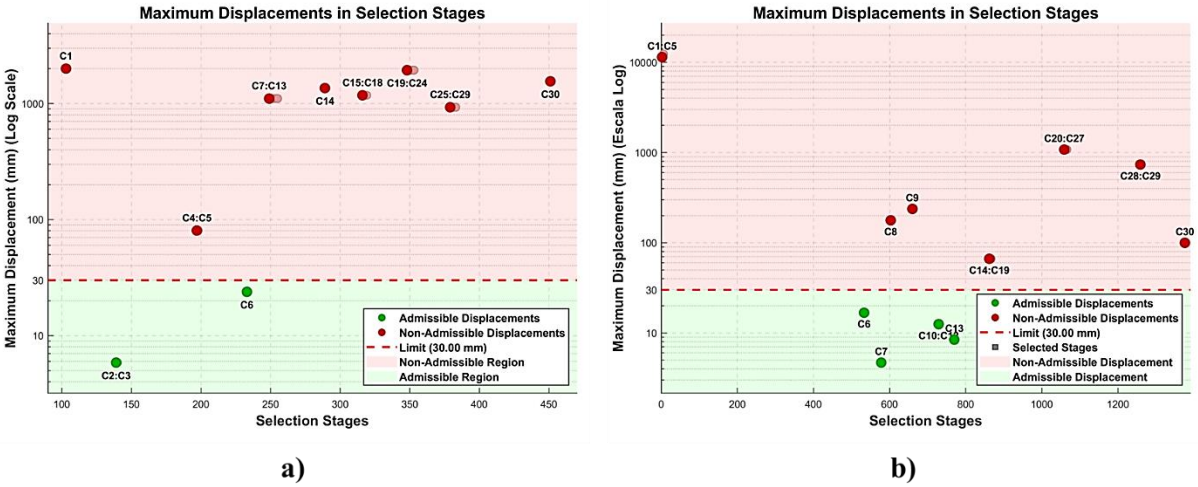


Figure 5. Maximum displacements of optimized topologies: a) connectivity level 1 and b) connectivity level 3.

Figure 6 shows the maximum buckling utilization index for the optimized topologies considering the two adopted connectivity levels (Figure 6-a, level 1, and Figure 6-b, level 3). When the buckling utilization index exceeds unity, it indicates local buckling failure. For connectivity level 1, the two topologies satisfying the displacement limit at the load application point—C_2 (which has the same configuration as criterion C_3) and C_6—do not exhibit buckling failure. Among them, C_6 shows a lower buckling utilization value than C_2, indicating a more stable configuration. For connectivity level 3, the two topologies within the admissible displacement limit—C_7 and C_{13}—also do not present buckling failure. The topology obtained under criterion C_{13} exhibits a lower buckling index compared to C_7, reflecting an improved resistance to local instability.

The same analysis is performed for the maximum yield utilization index, as illustrated in Figure 7. Similar to the buckling case, the two topologies with connectivity level 1 (Figure 7-a) and the four with connectivity level 3 (Figure 7-b) do not exhibit material yielding. In both connectivity levels, the maximum stress values are well below the steel yield stress, confirming adequate strength performance.

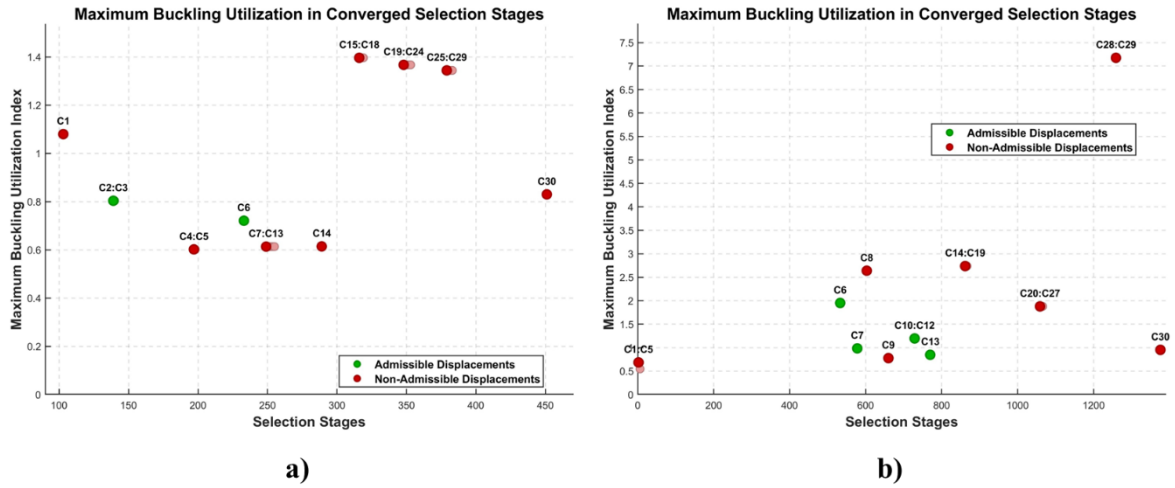


Figure 6. Maximum Buckling Utilization Index for optimized topologies: a) connectivity level 1 and b) connectivity level 3.

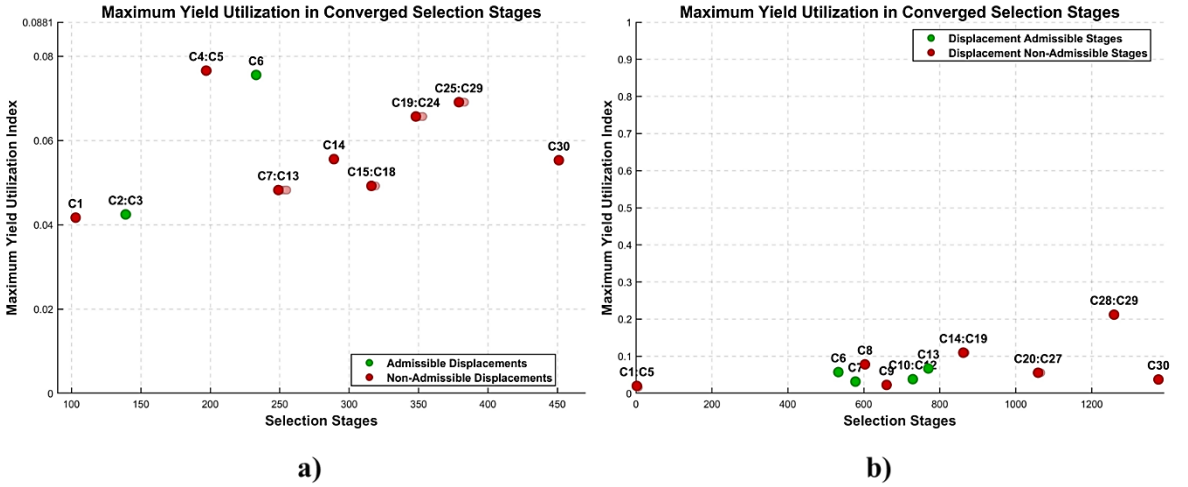


Figure 7. Maximum yield utilization index for optimized topologies: a) connectivity level 1 and b) connectivity level 3.

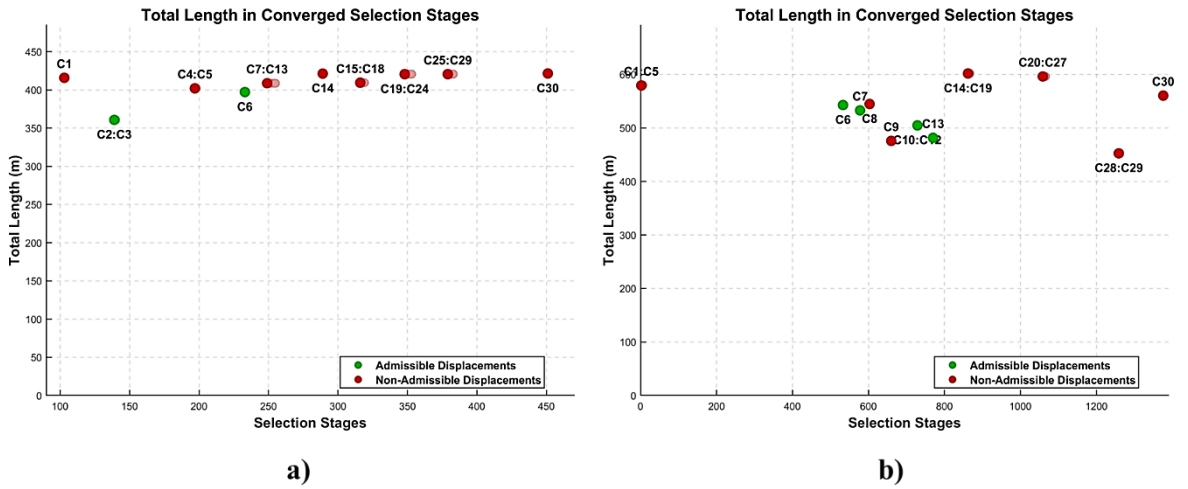
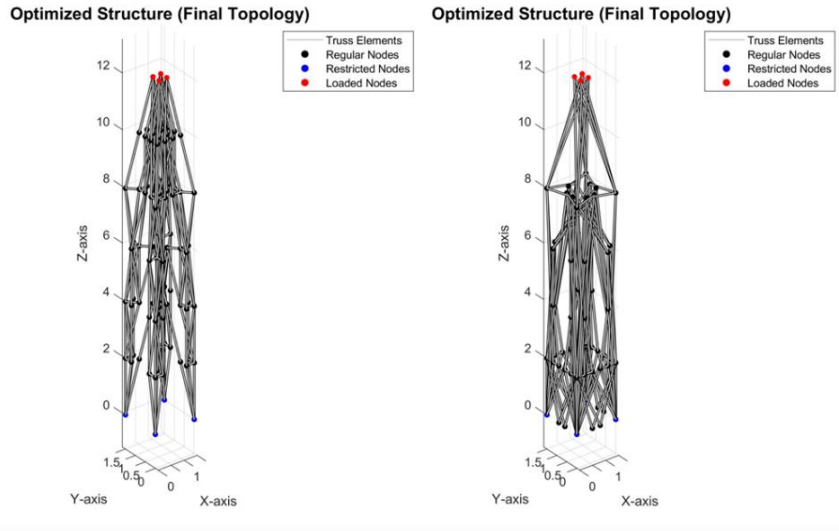


Figure 8. Total lengths of optimized topologies: a) connectivity level 1 and b) connectivity level 3.

Figure 8 presents two graphs illustrating the total lengths of the optimized topologies that met convergence criteria up to C_{30} for connectivity level 1 (Figure 8-a) and for connectivity level 3 (Figure 8-b). For connectivity level 1, topology C_2 presents lower material consumption compared to topology C_6. For connectivity level 3, topology C_{13} presents less material consumption compared to topology C_7.

For connectivity level 1, the topology obtained under criterion C_2 achieved the best overall performance, as it required less material and exhibited a buckling utilization index only slightly higher than that of topology C_6, with both values being quite similar. For connectivity level 3, the C_{13} topology showed superior performance, primarily due to its lower material consumption. In contrast, topology C_7 presented a buckling utilization index close to unity, indicating a high susceptibility to buckling failure. The optimized configurations corresponding to connectivity levels 1 (C_2) and 3 (C_{13}) are illustrated in Figure 9.



a) **b)**

Figure 9. Chosen optimized structures: a) connectivity level 1 – C_2 and b) connectivity level 3 – C_{13}.

Table 1 summarizes the parameter values for the chosen topologies—C_2 for connectivity level 1 and C_{13} for connectivity level 3. The initial truss length of the connectivity level 1 topology is considerably smaller, representing only 7.43% of that of connectivity level 3. Following the same trend, the final truss length of level 1 corresponds to 74.92% of the final length at level 3, clearly indicating that the topology with lower connectivity resulted in substantially reduced material consumption. The connectivity level 1 topology also exhibited smaller displacements at the load application point compared to that of level 3. The buckling utilization indices of both topologies were very similar, while the yield utilization indices were well below the material's yield limit, with the level 1 topology presenting the lowest value. Overall, the topology with connectivity level 1 achieved superior structural performance relative to the topology with connectivity level 3.

Table 1. Parameter values for the chosen topologies

Connectivity Level	Criteria Convergence	Initial Length (m)	Maximum Displacement (mm)	Maximum Absolute Stress (MPa)	Total Length (m)	Maximum Bucking Utilization Index	Maximum Yield Utilization Index
1	C_2	3389	5.90	13.51	360.75	0.8037	0.0425
3	C_{13}	45579	8.40	21.30	481.54	0.8471	0.0700

Figure 10 illustrates the stress distribution in the optimized topology for connectivity level 1 (C_2), while Figure 11 shows the stress distribution of the connectivity level 3 topology (C_{13}) under the applied drag load.

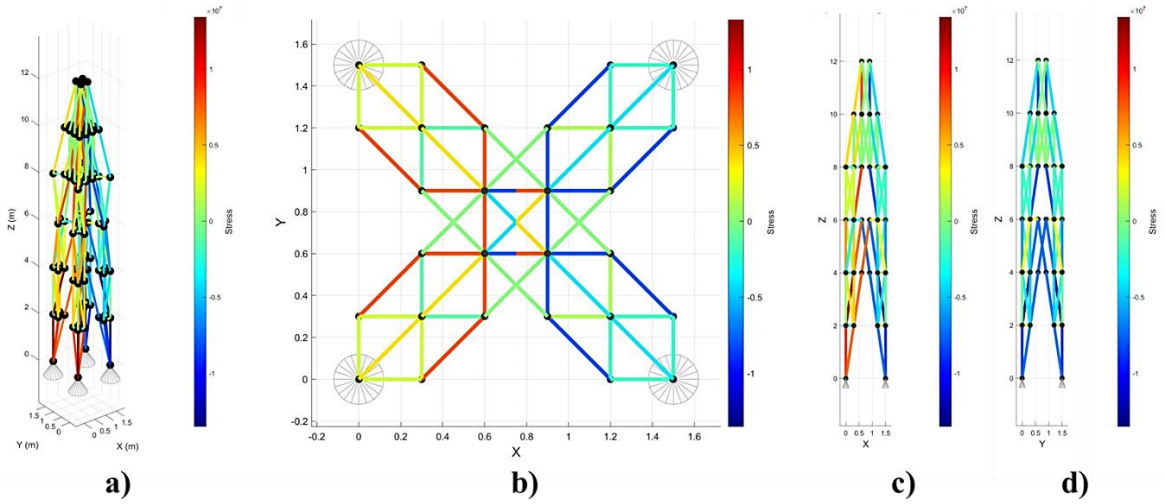


Figure 10. Stress distribution in the optimized topology for connectivity level 1: a) Isometric view; b) *xy* plane; c) *xz* plane and d) *yz* plane.

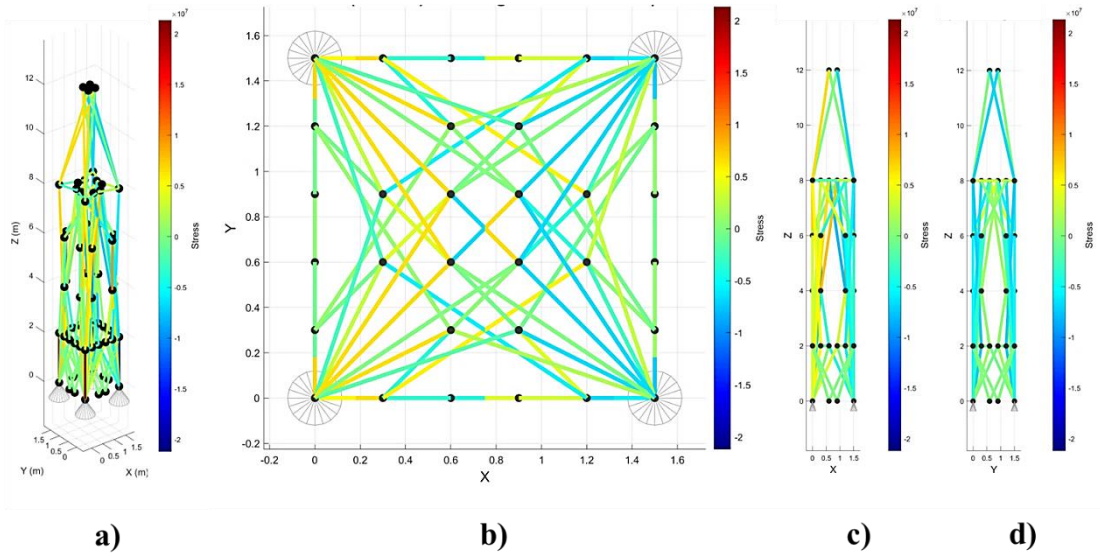


Figure 11. Stress distribution in the optimized topology for connectivity level 3: a) Isometric view; b) *xy* plane; c) *xz* plane and d) *yz* plane.

5 Conclusions

This study successfully extended the Progressive Directional Selection (PDS) method to the topology optimization of three-dimensional truss structures within the ground-structure framework, integrating strength, stability, and serviceability verifications according to Brazilian design standards. The proposed implementation demonstrated that the PDS heuristic can efficiently identify optimized topologies without the need for complex gradient-based formulations, confirming its potential as an intuitive and computationally accessible alternative for structural design.

The results obtained for the two ground-structure configurations—connectivity levels 1 and 3—highlighted the influence of structural connectivity on the optimization outcomes. The topology with connectivity level 1 produced a more compact and materially efficient truss configuration, presenting smaller total element lengths, reduced displacements at the load application point, and lower stress levels when compared to the topology with higher connectivity. Both topologies satisfied the serviceability (SLS) and ultimate (ULS) limit-state criteria established for displacement, buckling, and yielding, but the structure with reduced connectivity achieved superior

overall performance, confirming that a denser nodal interconnection does not necessarily lead to a better structural response. The results for the lattice wind-turbine tower example further reinforced the practical applicability of PDS for real-world engineering problems where both performance and material savings are critical.

References

[1] M. P. Bendsøe and O. Sigmund. *Topology optimization: theory, methods and application*. Springer, 2003.
[2] O. M. Querin, M. Victoria, C. Alonso, R. Ansola and P. Martí. *Topology Design Methods for Structural Optimization*. Academic Press, 2017.
[3] M.P. Bendsøe and N. Kikuchi, (1988). "Generating optimal topologies in structural design using a homogenization method". CMA, v. 71, n.2, pp. 197–224, 1988.
[4] M.P Bendsøe. "Optimal shape design as a material distribution problem". Structural Optimization, v. 1, n. 4, pp. 193–202, 1989.
[5] Y.M. Xie and G.P. Steven. "A simple evolutionary procedure for structural optimization". Computer and Structures, v. 49, n. 5, pp. 885-896, 1993.
[6] X. Huang and M. Xie, Evolutionary Topology Optimization of Continuum Structures:Methods and Applications, John Wiley & Sons, 2010.
[7] L. C. L. Véras and M. A. A Cavalcante, "Topology optimization of continuum elastic structures through progressive directional selection method". In. *Ibero-Latin American Congress On Computational Methods In Engineering (XLI CILAMCE)*, pp. 1-7, 2020.
[8] N. S. N. Cavalcante, L. C. L. Véras and M. A. A. Cavalcante. "Topology optimization of plane trusses using the progressive directional selection method". Cogent Engineering. vol. 12, n.1, pp. 1-18, 2025.
[9] A. P. Gama Júnior. Projeto estrutural otimizado de torres tubulares de aerogeradores em material compósito. Master thesis, Federal University of Alagoas, 2019.
[10] Associação Brasileira de Normas Técnicas, "NBR 6123: Wind loads on buildings", ABNT, 2023.
[11] Associação Brasileira de Normas Técnicas, "NBR 8800: Design of steel and composite structures for buildings", ABNT, 2008.