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Pressure redistribution due to lateral vibrations in granular columns: insights from a discrete element simulation

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Abstract. The pressure distribution in granular columns is greatly affected by the contact of the grains with the lateral walls of its container. For a granular column statically resting inside a container, the total vertical pressure at the base may saturate, (i. e., it is limited by a maximum threshold value) which may be described by Janssen's model. Herein, we investigate the effect of lateral vibrations on the container's wall, and its effect on the force transmission through the granular material, which results in greater effective pressure at the base of the column. A discrete element simulation is used to model the deposition of granular material within rigid boundaries under a prescribed top load, and then impose horizontal oscillatory motion. The imposed vibration led to a significant increase of the effective pressure at the base, due to the improved force transmission. The impact of amplitude and frequency of the vibration was also investigated in a parametric analysis.

Keywords: granular materials, triaxial shear, fluidization, DEM

1 Introduction

Frictional contact between granular materials and lateral boundaries is a key mechanism of stress redistribution in various engineering systems. Applications such as pile driving, retaining structures, silos, and hoppers exhibit significant load transfer effects due to lateral friction. While the static behavior of some of these systems is relatively well understood, the dynamic response induced by lateral vibrations remains complex and less explored.

In static granular columns, the vertical pressure distribution may be described by Janssen's model [1–3], which accounts for the balance between gravitational loading, applied top pressure, and frictional interactions with the container walls. The lateral friction force depends on the lateral pressure coefficient of the granular material (also known as the earth pressure coefficient or the Janssen coefficient, k) and the wall-grain friction coefficient (μ). Unlike a hydrostatic profile typical of horizontal soil layers, Janssen's model predicts a nonlinear vertical pressure distribution and a saturation of vertical stress at the base. This saturation arises because lateral walls can partially support vertical loads via friction, limiting the transmission of force to the base of the column. As a result, the effective base pressure is significantly reduced, with important implications for the design of silos, in drilling technologies, and other engineering systems with tall columns of granular materials. The impact of lateral vibrations on the Janssen effect, however, has been less studied, and modeling this phenomenon remains a challenge.

The impact of external vibrations on the shear strength of granular materials has been observed experimentally. For example, Xie et al. [4] and Ye et al. [5] have observed shear strength reduction in an experimental triaxial test setup with applied external vibrations. An effective reduction in the friction coefficient between soil and metal surfaces has been observed due to the application of vibrations [6, 7]. Discrete element method (DEM) simulations have been used to understand the behavior of granular materials subjected to vibrations. Numerical studies [8, 9] have observed shear strength reduction in both triaxial tests and in direct shear tests [10], showing the effect of vibration in the internal friction coefficient of the granular material. The interaction between vibrations and granular materials has also been studied in the context of shear flows ([11, 12]) and earthquake fault triggering ([13, 14]).

Although many DEM studies examine the application of loading and vibrations in granular deposition in containers (e.g., [15, 16]), the direct influence of horizontal wall vibrations on pressure redistribution in granular columns has not yet been systematically addressed. While research has explored the limits of the Janssen model for dynamical behavior in different contexts (e.g., [17, 18]), and experimental testing has shown that external

vibrations influence pressure transmission in granular columns [19], there has been little progress in developing analytical or simplified models that specifically describe the effect of horizontal wall vibrations on pressure distribution in columns governed by the Janssen effect. Numerical investigations on this issue remain scarce, despite their importance as a first step toward understanding how vibration modifies the Janssen response. This should also be differentiated from the more well-studied phenomenon of vertical vibration of granular columns causing segregation and convection (such as in the Brazil Nut effect).

The current work investigates the effect of lateral vibrations on the pressure distribution and transmission in a granular column. A discrete element simulation is employed to model the granular column, where the Janssen effect is observed for the effective force at the base of the column, and the effect of lateral vibrations is studied. Different amplitudes and frequencies of vibration are considered in a parametric analysis, and their effect on the effective pressure at the base of the column is evaluated. This is a first step toward a more general understanding of the effect of lateral vibrations on Janssen granular columns, which may have implications for various engineering systems.

2 Methodology

2.1 Janssen classical model

The classical Janssen model for the pressure distribution of a two-dimensional granular column in a rigid container is given by the consideration of the force equilibrium of a slice of the column, leading to the following differential equation:

$$\frac{dP_v(z)}{dz} = \rho g - \frac{2\mu k P_v(z)}{W} \tag{1}$$

where $P_v(z)$ is the vertical pressure at depth z (zero at the top, and H at the base, where H is the height of the column), ρ is the density of the granular material, g is the gravitational acceleration, μ is the frictional coefficient between the grains and the wall, k is the lateral earth pressure coefficient and W is the width of the column. The solution for the vertical pressure distribution is given by:

$$P_v(z) = A \exp\left(-\frac{2\mu k z}{W}\right) + \frac{\rho g W}{2\mu k} \tag{2}$$

For a uniform pressure at the top of the column $P_v(0) = P_0$, the constant A can be calculated as:

$$A = P_0 - \frac{\rho g W}{2\mu k} \tag{3}$$

The classical solution of the Janssen model will be used as a reference for the effective pressure at the base of the column $P_v(H)$, representing the state before vibration where force transmission is limited. The comparison between the effective pressure at the base $P_v(H)$ and the hydrostatic pressure $P_{hydro} = \rho g H + P_0$ is given by the ratio:

$$r_J = \frac{P_v(H)}{P_{hydro}} \tag{4}$$

2.2 Discrete element simulation

The DEM simulation [20] is performed using the open-source software Yade [21]. Spherical particles are used to represent the granular material, and rigid walls are used to represent the container. A base wall is used as a rigid support, and the vertical force applied in this element is monitored throughout the analysis in order to observe the force transmission through the granular material. The analysis is performed in a 2-dimensional domain, in order to reduce the computational cost of the simulation and simplify the modeling process, including the vibration application. This is done by blocking the respective degrees of freedom of the particles in the simulation.

Normal and tangential contact forces are modeled using a spring-dashpot model (Figure 1a). Compression forces F_N are modeled using a linear stiffness K_N and a damping coefficient C_N. No tension can develop between

the particles. The tangential contact forces F_T are modeled using a linear spring with stiffness K_T and a maximum tangential force of $\mu_p F_N$, where μ_p is the particle-particle friction coefficient. The stiffness values are based on the material used (FRICTVISCOMAT material in Yade), and calculated as:

$$K_N = \frac{E_{N1} l_{p1} E_{N2} l_{p2}}{E_{N1} l_{p1} + E_{N2} l_{p2}} \tag{5}$$

where E_{N1} and E_{N2} are the elastic moduli of the material of the particles, and l_{p1} and l_{p2} are representative lengths of the particles, equal to the particle diameter. The material also is characterized by the ratio E_T/E_N, which defines the tangential stiffness E_T. The material also considers a density ρ. The effective normal damping coefficient C_N is based on the normal damping ratio β_N and given by $C_N = \beta_N 2\sqrt{m \cdot K_N}$ where m is the effective mass calculated from ρ and the geometry. Finally, the forces are calculated as:

$$F_N = U_N K_N \tag{6}$$

$$F_T = U_T K_T \tag{7}$$

where U_N and U_T are the normal and tangential displacements, respectively (see figure 1b and 1c). More information is available in the Yade documentation [21] and source code.

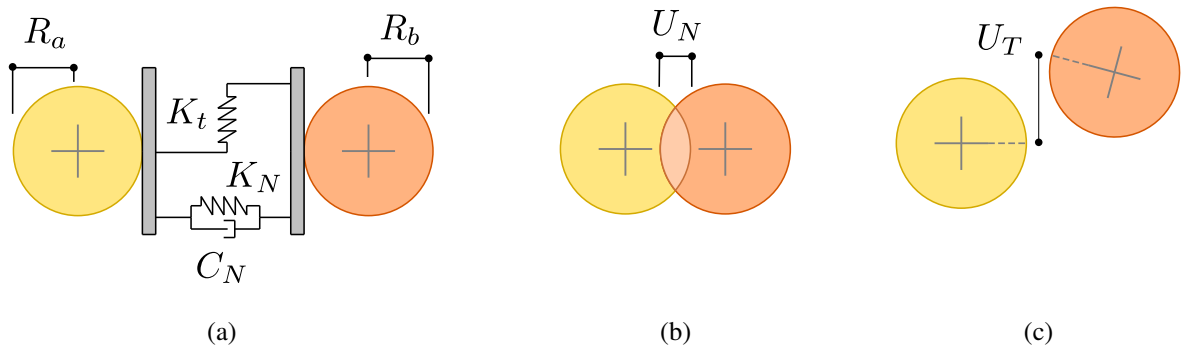


Figure 1. Numerical model: (a) particle-particle contact, (b) normal contact displacement, and (c) tangential contact displacement.

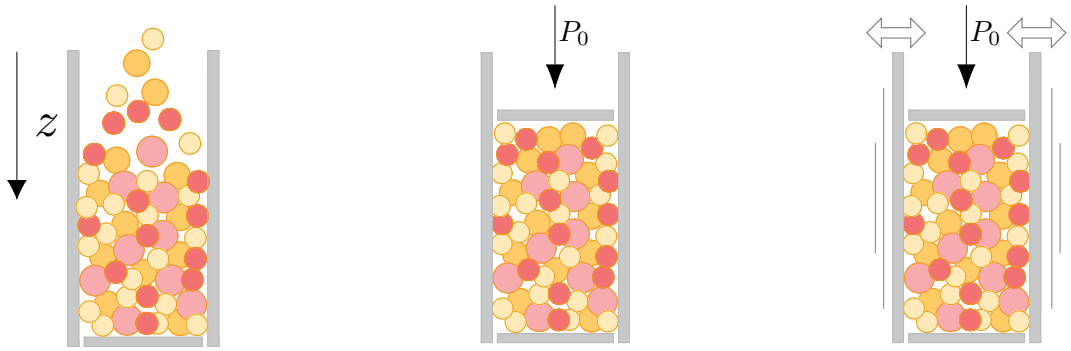
The walls are also assigned to the same material, and the same contact model is used. The time integration is performed by estimating a critical time step periodically, and assigning a safety factor to ensure stability during the simulation. The time step is calculated as:

$$\Delta t_{cr} = \min_i r_i \sqrt{\frac{\rho_i}{E_i}} \tag{8}$$

2.3 Numerical experiment setup

Sample preparation is performed by depositing the granular material by gravity in sequential layers, in order to avoid excessive compaction or impact of the particles (Figure 2a). An advantage of this method is that the lateral friction is naturally mobilized during the deposition process. On the other hand, the sample preparation step may be more time-consuming than other alternatives. After the deposition is finished, a top vertical pressure is applied by a moving wall element created at the top of the column (2b). The vertical pressure is first applied by strain controlled displacements, until reaching a target pressure. It is then controlled by stress measurements, to keep the target stress level (even during the vibration phase). The strain controlled compression is done in a quasi-static manner, in order to avoid excessive disturbance of the granular material. The target top pressure can be adjusted in order to confine the granular material, or apply a very little pressure, equivalent to no pressure at all.

Finally, after the pressure has been applied, the lateral walls are set to vibrate in the horizontal direction (2c). The vibration is applied in a sinusoidal manner in time, controlled by a displacement amplitude A_d and frequency f. The bottom pressure is monitored during the vibration application, and the effective pressure $P_v(H)$ and the ratio r_J are calculated.



(a) (b) (c)

Figure 2. Numerical experiment setup: (a) gravity deposition of the granular material, (b) application of a pressure at the top, and (c) vibration application at the lateral walls.

For the numerical experiment, a column of 1200 particles is used, the diameter follows a uniform distribution with mean 0.002 and limits of 20% the mean value. The grains have density of 2500. The column width is $0.1m$. The lateral wall friction coefficient is set to $\mu = 0.5$. The particle-particle friction coefficient is also set to 0.5. The contact model has parameters $E_N = 10^8 Pa$, $E_T/E_N = 0.5$, $\beta_n = 0.01$. The target stress P_0 is set to $50 \cdot 10^3 Pa$.

Different amplitude A_d and frequency f values are used in the analysis. The amplitude is set to 0.0002 to $0.002m$, and the frequency is set to 150 Hz to 650 Hz. The vibration is applied for approximately 15 vibration periods.

3 Results

The deposition stage of the numerical simulation is first examined and compared with the theoretical Janssen model for static pressure distribution. After deposition, the granular column reaches a height of $0.21\,\mathrm{m}$, corresponding to a height-to-width ratio of approximately 2 (prior to piston loading). The initial void ratio is 0.33, and the total weight of the column is about $1\,\mathrm{N}$.

Due to the mobilization of lateral friction, the Janssen effect develops within the granular column, resulting in a non-uniform pressure distribution along its height. Figure 3 (left) compares the pressure distribution obtained from the numerical simulation with the theoretical prediction of the Janssen model. For this comparison, the average values of the friction coefficient μ, Janssen coefficient k, and material density ρ were extracted from the simulation and used as input parameters in the analytical model. It is possible to notice that the numerical results align well with the theoretical predictions, although differences might be due to local variation of these parameters in the numerical result from the global averaged parameters.

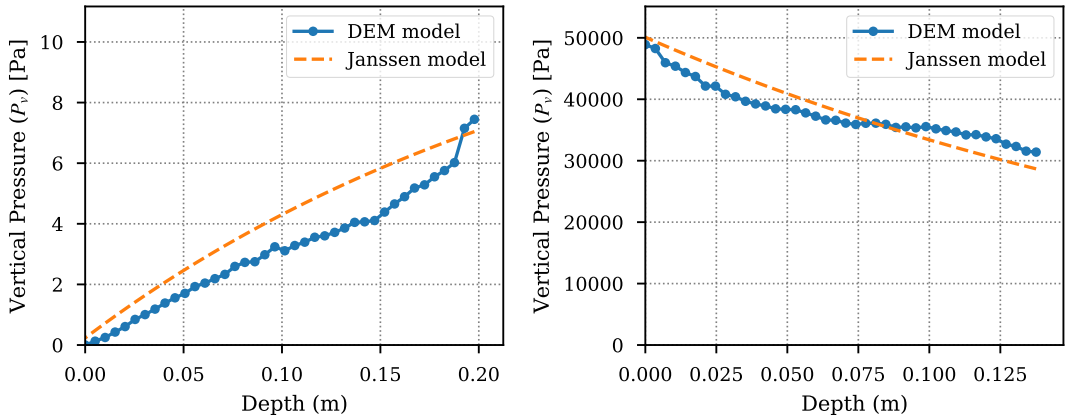


Figure 3. Pressure profile comparison of the numerical model and the Janssen model. Left: after deposition, Right: after top pressure loading

Immediately after deposition, the Janssen ratio r_J is 0.932, which means that approximately 7% of the weight of the material is being supported by the lateral walls. Following the piston loading stage, the pressure distribution is re-evaluated and shown in Figure 3 (right), showing similar values between the numerical and theoretical prediction. As expected, the mobilization of lateral friction limits the transmission of vertical pressure along the column height. After piston application, the Janssen ratio decreases to approximately 0.61 which means that approximately 39% of the weight and top pressure are being supported by the lateral walls.

Subsequently, horizontal vibrations are applied to the lateral walls, in order to observe the effect of vertical pressure redistribution. This excitation induces grain motion, altering the contact conditions at the walls and promoting a global redistribution of stresses within the granular column. Figure 4 shows the evolution of base pressure over time as vibration begins. A sudden increase in pressure is observed at the onset of vibration, followed by oscillation around a mean value. Such behavior indicates that the lateral vibration have a direct impact in the force transmission, and are capable of changing the stress distribution within the column.

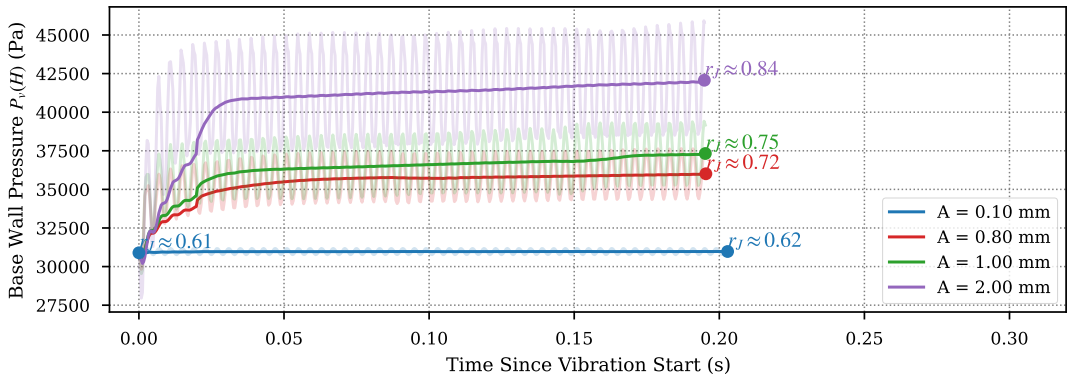


Figure 4. Pressure evolution at the base of the column during vibration application

The impact of the amplitude of the motion was also investigated. Different values of amplitude were considered, while keeping the frequency the same. They are also presented in figure 4. Amplitude values varied from $0.0001m$ to $0.002m$, while keeping the frequency at 250 Hz, leading to a change in the final r_J from 0.62 to 0.84. It is evident the impact that the amplitude of the vibration has on the force transmission through the granular material.

The effect of the frequency of the vibration was also investigated. Frequency values from 150 Hz to 650 Hz were considered, leading to an impact on the final r_J to 0.71 to 0.77, for a constant amplitude of 0.001 m. A spectral behavior was also observed, (see Figure 5, left) indicating that certain frequencies may be more effective at promoting force transmission than others, even if for the range of values investigated, the differences in frequencies have not led to as much difference in r_J as the range of amplitudes. The amplitude effect can be seen in Figure 5 (right), where a clear trend of increasing r_J with increasing amplitude is observed. An almost linear trend is observed, where higher amplitudes lead to higher effective pressure at the base of the column.

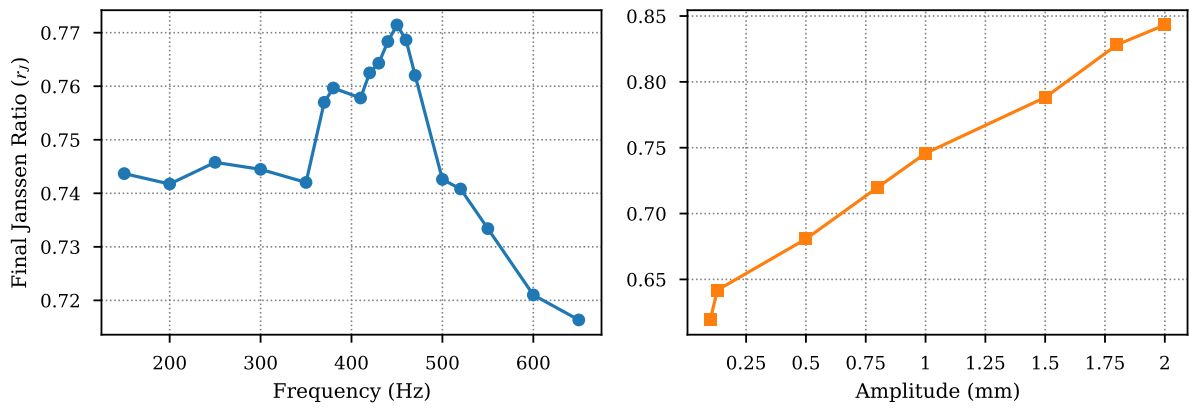


Figure 5. Final Janssen ratio r_J values for vibration frequency variation (left) and amplitude variation (right)

4 Conclusions

The current work has investigated the impact of lateral horizontal vibrations on the force transmission in granular columns using numerical simulations with discrete element modeling. A clear impact of the lateral vibration was observed on force transmission, indicating that both the amplitude and frequency of the vibrations can play a significant role in pressure redistribution in the studied system. The range of amplitude values tested changed the Janssen ratio r_J (the ratio between the effective pressure at the base and the hydrostatic pressure, including a top confinement stress) from 0.62 to 0.84, while the frequency values tested changed r_J from 0.71 to 0.77. Further investigation might involve how the system responds to vibration at different system and excitation scales, which might lead to a better understanding of the effect of lateral vibrations on confined granular columns.

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Authorship statement.

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