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A review of applications for which the virtual element method is particularly useful

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Abstract. This work presents an overview of some applications for which the virtual element method is shown to be remarkably useful, and what features of the method contribute to its success. The virtual element method (VEM) originated from a reformulation of the mimetic differences method to fit the framework of finite element method (FEM), and can be concisely described as a polytopal generalization of the FEM. Naturally, the question of when to prefer one method over the other arises, which comprises the motivation for this work. A review of the VEM literature is conducted focusing on works that showcase notable applications of the method. These are then analysed to discern which unique feature of the method has set it apart. An overview of these applications and characteristics is then synthesized to present the findings. The selected applications are: subsurface flow in porous rocks, topology optimization, contact problems, fracture mechanics, polycrystalline materials, granular materials and magnetohydrodynamics.

Keywords: Virtual element method

1 Introduction

Introduced a little over a decade ago in Beirão da Veiga et al. [1], the virtual element method can be described as a generalization of the finite element method for polytopal elements of arbitrary order. The method works by constructing implicitly defined ansatz spaces, tailored to the element geometry, with certain key properties that ensure computability and optimal convergence rates.

The method is designed so that explicit knowledge of the ansatz functions is not required, earning them the "virtual" qualifier. Their projection into a suitable polynomial space is used instead, leading to rank-deficient system. The missing rank is then supplied by a stabilization term.

Ever since its introduction, the method has seen a lot of theoretical development, and has amassed a considerable amount of applicational publications. The present work reviews a selection of the VEM literature, aiming to provide a partial, and far from exhaustive, answer to the question: What are virtual elements good for?

The remainder of the work is structured as follows. The methodology for the literature survey is in section 2. A brief introduction to the VEM in section 3. The analysis of applications in section 4, followed by concluding remarks in section 5.

2 Methodology

The procedure used to direct the literature survey is quite simple. Having a clear seminal paper [1], it has become a custom for subsequent works to cite it when first alluding to the method. Therefore, a good initial sample of VEM publications can be found by querying all the papers citing this original one.

With this sample, the following procedures are performed. First, the works that only mention the method in passing as disregarded, leaving only those that actually concern it. Second, the remaining works are categorized as either developing the method itself, or being application-oriented. Third, the latter group is then clustered by field of application.

A small selection of the first class of works is mentioned in section 3, to aid in a characterization of the methods capabilities. Whereas the application-oriented works are manually clustered and analysed regarding their particularities, then summarized in section 4.

3 Virtual element method

The following is a very brief introduction to the virtual element method. It will focus on the original formulation, as it is sufficient to showcase the general workings of the method.

According to Ciarlet [2], finite elements are defined by triples of a domain, a function space, and a corresponding set of degrees of freedom. For the virtual element, consider as domain a generic polyhedron P with a generic polygonal face F for the 3D formulation; and a generic polygon F for the 2D case, as illustrated in Figure 1(a). Whenever some affirmation applies to both 2D and 3D, E will be used as a wildcard to represent both domains. The boundary of E is denoted ∂E. The function space V_k is defined implicitly (that is, as the space of all functions that satisfy certain requirements) as shown in eq. (1). For this definition k indicates the order of convergence of the method, which is achieved by ensuring $V_k(E) \supseteq P_k(E)$ by construction, as one can verify in the following definitions.

$$\begin{aligned} B_k(\partial F) &:= \left\{ v \in C^0(\partial F) : v|_e \in P_k(e) \ \forall e \in \partial F \right\}; \\ V_k(F) &:= \left\{ v \in H^1(F) \cap \mathcal{B}_k(\partial F) : \Delta v \in P_{k-2}(F) \right\}; \\ V_k(P) &:= \left\{ v \in H^1(P) : v|_F \in V_k(F) \ \forall F \in \partial P,\ \Delta v \in P_{k-2}(P) \right\}. \end{aligned} \tag{1}$$

The set of degrees of freedom for such elements are those presented below.

(i) Values at the vertices;
(ii) Values at the $k-1$ points of the Gauss-Lobatto quadrature for each edge;
(iii) Internal moments with respect to the scaled monomial basis of P_{k-2} on each face;
(iv) Internal moments with respect to the scaled monomial basis of P_{k-2} on the volume (if 3D).

This set of degrees of freedom induces a set of canonical basis functions $\{\phi_i\}$. These degrees of freedom are sufficient for the construction of an orthogonal projection operator $\Pi_k : V_k(E) \to P_k(E)$ (to be rigorous, a small modification in the space definitions is required in the 3D case [3]). Finally, any function in this space can be split into its polynomial projection and the remainder, as illustrated in Figure 1(b). Essentially, the method works by introducing the polynomial projections instead of the ansatz functions in the problem's formulation, leading to rank-deficient system. The missing rank is supplied by an ad-hoc stabilization term.

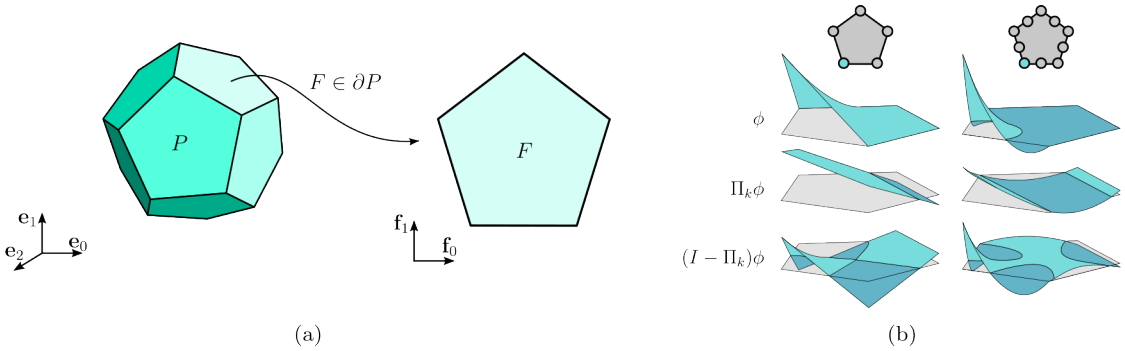


Figure 1. (a) Example polyhedron P with face F. (b) Typical ansatz functions ϕ, for linear and quadratic elements, along with their projection $\Pi_k\phi$ and the remainder $(I - \Pi_k)\phi$

The formulation presented above is the original, from the seminal paper [1]. Since then, several other formulations and modifications have been published. Some, but not all, are: the modification that allowed for an L^2-projector and 3D elements [3]; a serendipity formulation [4], foregoing the internal degrees of freedom (iii) and (iv), and which is shown not to suffer from element distortion as happens with their finite element counterparts; a version of the method with arbitrary regularity [5]; $H(\text{div})$- and $H(\text{curl})$-discretizing versions of the method [6], along with their serendipity formulations [7]; basic principles for mixed VEM [8]; along with many other developments that could not be included in this paper.

4 Notable applications

In this section, a few selected applications are discussed. They were selected because they illustrate how the characteristics of the method can be used in problem solving methodologies.

The selected applications were: subsurface fluid flow in porous rocks, topology optimization, contact mechanics, fracture mechanics, polycrystalline materials, granular materials, and magnetohydrodynamics.

Subsurface fluid flow in porous rock

The flow of fluid (e.g., water, oil) in fractured porous rocks is a phenomenon of interest in the field of oil extraction, but also relevant for other applications such as water management, and pollutant percolation [9]. These problems usually involve a discrete fracture network (DFN), consisting of polygonal fractures distributed over a three dimensional domain. The networks are stochastically generated based on probabilistic data concerning geometry, orientation and hydrological properties of the fractures. The discrete fracture network can be modelled by itself, or embedded in elements representing the porous rock in discrete fracture-matrix (DFM) models.

The number of fractures required for the simulations depends on the scale, goals and specific context of the concrete application, but those can vary from hundreds to millions [10]. Meshing the fractures (and rock matrix) in a way that ensures compatibility across all interections becomes a considerable problem.

With this motivation, Benedetto et al. [9] introduced the idea of using virtual elements to provide mesh compatibility with simpler meshing procedures. This is achieved by a simple clipping procedure, as the VEM allows for polygonal elements and hanging nodes, as well as good behavior with highly distorted shapes that may appear. This approach was further explored in the context of the isolated fracture networks [11, 12], and later extended to the fracture-matrix model [13–15].

Topology optimization

Modern topology optimization follows the density-based approach, where the presence of material is represented as a density field (the design variable). This is treated as a variable along with the displacements for the mechanical problem. Usually, constant ansatz for the density, and low order ansatz for displacements is enough for a good solution. However, this can sometimes lead to numerical instabilities like checkerboard patterns and 1-node hinge connections. Increasing the order of the density ansatz does not always fix the problem, sometimes leading to different instabilities like “layering” and “islanding” (see Rahmatalla and Swan [16]). These instabilities do not happen when using polytopal elements (Gain et al. [17]). Furthermore, it seems that among the polytopal finite element schemes, the virtual element method is computationally cheaper [17], as it does not require explicit computation of the ansatz functions. Neither does it require numerical integration for lower-order ansatz [18]. In Antonietti et al. [19], numerical studies show that VEM with polygonal meshes leads to mesh independence of the optimal solutions, whereas traditional FEM approaches can be suject to mesh-induced suboptimal solutions.

In the 2D setting, noteworthy work has been done in the sense of using VEMs capacity to handle hanging nodes to produce quad-tree based adaptivity. This has been published initially in Zhang et al. [20] for multimaterial optimization with hyperelastic materials under large deformations. It presents an adaptive strategy consisting in refining regions with higher density, and coarsening regions with lower density, to reduce the total number of degrees of freedom of the ensuing algebraic systems. This has been further improved in Zhang and Chi [21], by refining only in regions close to where density changes (local features), coarsening elsewhere.

In Chi et al. [18], the authors present a 3D VEM-based topology optimization framework. Considering that polyhedral elements require more degrees of freedom to discretize the same domain when compared to hexahedra, leading to a much higher rate of mechanical degrees of freedom per design degrees of freedom; the authors decided on a approach that enriches the design variable ansatz, producing a design-wise better solution for the same computational cost. This is achieved by using the usual linear VEM ansatz for the displacement variables, and the lowest order VEM ansatz for magnetostatic problems (Beirão da Veiga et al. [22]) for the design variables; leading to a piecewise continuous design variable approximation. This lead to good performance, avoiding the typical problems of the classical approach, and gaining robustness with respect to distorted elements.

In summary, the offer of computationally cheap low order polygonal elements is one of the main features of VEM for 2D topology optimization. The capacity of handling hanging nodes can be leveraged for very powerful mesh adaptivity procedures, further improving computational efficiency. For the 3D setting, to offset the higher number of degrees of freedom required for polyhedral elements, an enriched ansatz can be used for the design variables. Overall, typical instabilities are avoided, with a tolerable ammount of additional cost.

Contact mechanics

Contact problems require solving the mechanics of all involved bodies, in addition to enforcing non-penetration constraints. Computationally, this requires identifying possible violations of the constraint, and enforcing it when necessary. The constraint introduces a new term to the problem's energy, which depends the constraint enforcement method of choice and the discretization of the contacting surfaces. Many combinations of such methods are available, comprising a spectrum of cost-benefit tradeoff.

Concerning practical applications of computational contact methods, different niches have different needs, requiring different methods. For problems where precision of contact tracions and associated stresses is required (e.g., tyre research, sheet metal forming, etc.), more refined (and expensive) methods are available, such as Mortar methods [23]. For other types of industrial applications, the sheer scale of the problem may require simpler methods to be used. In the automobile industry, the complexity of crash-test simulations in terms of computational expense makes the use of simpler contact methods a necessity [24]. In these cases, precision is not as important as capturing the global behavior of a complex system of interacting parts.

Wriggers et al. [25] have leveraged the geometrical versatility of the VEM to produce a novel computationally inexpensive approach to contact in 2D. It consists in making the simplest contact discretization (Node-to-Node) viable for large displacements and non-matching meshes. This is achieved by inserting and moving nodes as necessary. Subsequent papers have extended this approach to problems with friction and large deformations [26], and for curved edges [27]. Cihan et al. [28] have recently extended the method for the 3D case by means of triangulated surfaces. This VEM-driven contact approach has only been published for linear elements so far.

On the front of higher order methods, it is known that penalty-based Node-to-Segment (2D) discretizations produce significant oscillations in 2nd and higher order elements. Moherdaui et al. [29] shows that the use of VEM helps mitigate the stress oscillations that arise from the biased treatment of the surfaces with 2nd order ansatz. This is achieved by a filtering effect provided by the polynomial projection of the stresses.

Fracture mechanics

Cracks are displacement discontinuities. With the FEM these can be modelled, for example, with mesh modifications (node duplication or interface elements); element function space enrichment (extended methods); phase-field methods, etc. Mesh dependence is not uncommon for the former class of methods, as they either require the crack to follow the mesh, or need significant remeshing. In general, fine meshes are commonly used in regions potentially in the crack path, be that to reduce the mesh influence and provide a better crack-resolution.

Hussein et al. [30] proposed a framework where the crack path is directly represented by splitting elements. This is possible due the robustness to hanging nodes, distortion and concave elements; and can be achieved with significantly less remeshing than FEMs would require. This framework has also been employed for cohesive fracture in Artioli et al. [31], Choi et al. [32].

Hussein et al. [33] introduce a phase-field fracture approach along with localized mesh refinement that follows the crack tip. A moving refined region follows the crack tip, allowing precise phase-field computation of the crack path. The path is then stored as a mesh cut in the underlying coarse mesh, which is once more used when it leaves the area of influence of the tip. This allows an efficient adaptive mesh refinement/coarsening scheme without biasing the crack direction. The relevant information (crack path), being stored as mesh connectivity in a coarse mesh. This allows precise results at significantly lower computational costs.

Polycrystalline materials

These materials present strong enough grain-level anisotropy, and sufficient micro/mesoscale irregularity to render analytical models ineffective. Statistical properties of the polycrystalline structure and material properties are easily obtainable from experimental data. For these types of materials, multiscale approaches are often employed. These consist in have a macroscale model, for which the constitutive relations are numerically computed via simulations on a (variationally linked) microscale representative volume element (RVE).

Marino et al. [34] proposed a one linear virtual element per grain scheme. Grains modelled with FEM can present anisotropy-related locking when minimal meshes are employed, requiring more degrees of freedom for a representative response. Virtual elements are resilient with regards to this locking, even with linear elements. Reducing the computational cost of the micro/mesoscale model produces a much more efficient overall scheme.

This was later extended to multiphysics problems in Böhm et al. [35], where the eletro-magneto-mechanical response of piezoelectric materials is modelled with this scheme showcasing its robustness and cost efficiency. And to highly anisotropic polycrystal plasticity in Böhm et al. [36].

Granular materials

Particle system simulations are employed in many fields, such as industrial processes with bulk materials, soil mechanics, additive manufacturing, etc. Usually contact dominated, these simulations require small timesteps for contact resolution. The sheer amount timesteps can lead to long simulation times.

Depending on the application, particle shape can have significant influence. In those cases, polyhedral particles are preferable to spherical ones. There are established methods for working with rigid polyhedral particles. Another factor that may come into play is particle deformability, making VEM a suitable candidate.

Gay Neto et al. [37] proposed a one virtual element per particle approach, being less computationally expensive than having an internal FEM mesh. This closely approximates deformability at the level of individual particles. Dynamic response has also been shown to be adequate. The work also showcase an example where particle deformation can be the determining factor: In a hopper discharge simulation, rigid particles led to clogging, whereas deformable ones led to a successful discharge.

Magnetohydrodynamics

This application remounts to the methods origins: the mimetic finite differences scheme. Magnetohydrodynamics describes the dynamics of magnetic fields in electrically conducting fluids. Applications are usually associated with physics of plasma (solar wind, fusion reactors, etc.). It couples two very complex physics: Maxwell equations and incompressible Navier-Stokes equations. Both of these require the use of mixed methods, with approximation spaces of different physical quantities satisfying certain relationships to ensure stability. The approximation spaces for the quantities in Maxwell equations have to form an exact subcomplex of the deRham complex, and the same goes for the quantities in the Navier-Stokes equations and the Stokes complex, with the addition of the incompressibility constraint.

The virtual element method has been shown to be particularly helpful in producing spaces that satisfy specified conditions. Beirão da Veiga et al. [38] presents formulations of different virtual element spaces satisfying the stability requirments of Maxwell's equations in 3D. Similarly, Beirão da Veiga et al. [39] shows the virtual element spaces for the 3D Navier-Stokes equations, with the incompressibility constraint satisfied by construction.

These are combined in Beirão da Veiga et al. [40], where a virtual element method for the tridimensional resistive magnetohydrodynamics model is shown, with optimal theorical convergence being obtained in numerical experiments.

5 Final considerations

This work presents an analysis of some of the applications for which the virtual element method's characteristics have been leveraged into new solution schemes. The main properties of the method explored in this sense are: geometrical versatility; robustness with respect to element distortion and concavity; robustness regarding locking; versatility with respect to space definition; and relatively low cost for linear elements.

These are usually leveraged in terms of: mesh compatibilization; mesh modification; mesh adaptivity; attractive cost-benefit coarse schemes; construction of sequences of approximation spaces forming exact subcomplexes.

Many more applications of the method and discussion on the subject can be found on the books published on the topic. For a more mathematically oriented book, see Antonietti et al. [41], and for a more engineering oriented book, see Wriggers et al. [42].

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