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Reliability of cold-formed steel frames for serviceability limit states using second order elastic analysis

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Abstract. This paper covers the application of a reliability framework to evaluate the safety presented on the use of second order elastic analysis via finite element method to monitor and design cold formed steel frames in the context of serviceability limit states. The use of improved beam-column elements is discussed for this task, in order to provide a less time-consuming structural analysis under adequate criteria. The uncertainties affecting the frame behavior were studied, and their statistical characteristics were used to derive reliability indices by a first order reliability method. International and Brazilian design procedures and particularities were taken into account. The results were compared to reference to allow discussion regarding the design practice.

Keywords: cold-formed steel frames, reliability analysis, serviceability limit states, excessive displacements, finite element method.

1 Introduction

The finite element method has been widely used in the design practice, due to its versatility and capability to consider the non-linearities in the structural analysis, and it is incorporated in several software commonly used in engineer design routine. Its use, however, must take into account some particularities that affect the load-displacement behavior of structures. In the case of steel construction, local instabilities, non-symmetric sections, semi-rigid connections, perforations, and even material inelasticity are some characteristics that may need to be considered for a trustworthy analysis. For a bearing capacity evaluation, only a shell finite element analysis can deal with all these particularities, but its high computational demand makes its use sometimes prohibitive in the design practice.

The evaluation of service displacements, however, can be carried out using advanced beam-column finite elements, that can deal with elements with non-symmetric sections. Over the past years, several finite element formulations were developed in this context [1], [2], [3], [4]. Design codes such as AISI S100 [5] and NBR 14762 [6] allows the design assisted by finite element analysis, by they do not report a reliability based calibration of a partial safety factor for the structural system. For the ultimate limit state, recent studies assessed the reliability of the design assisted by the finite element analysis, but they rarely cover serviceability limit states [7], [8], [9], [10], [11], [12], [13].

This paper is a continuation of the work developed by Faria, Freitas and Mapa [14], and assess the reliability of a long-span cold-formed steel portal frame for the serviceability limit state of excessive displacements. Two common service displacements will be evaluated: vertical displacement due to gravity loads and lateral drift due to wind loads. Now, normative prescriptions regarding target reliability indices and service load combinations will be considered, in the context of designs based on by the American Load and Resistance Factor Design (LRFD) and the Brazilian Limit States Method. More random variables, including the semi-rigid nature of the joints has now been incorporated into the reliability analysis. The finite element developed by Liu, Gao and Ziemian [4] and further implemented in MASTAN2 was used to calculate the structural displacements, needed to the evaluation of the limit state function. Outcomes of the improvement of this reliability framework were discussed.

2 Reliability analysis procedure

2.1 Reliability analysis for serviceability limit states

The reliability index β is a parameter used to evaluate the safety presented by structures. It is directly related to the probability of failure P_f as $P_f = \Phi(-\beta)$, and for ultimate limit states, P_f represents the probability of the load effect S overcome the structural resistance R. For serviceability limit states, specifically for the case of excessive displacements, P_f represents a probability of exceedance, with exceedance representing cases where the displacement Δ of an arbitrary point of the structure is greater than the corresponding normative displacement Δ_{lim}. The limit state function governing the problem is given by $g(\mathbf{X}) = \Delta_{lim} - \Delta(\mathbf{X})$, with, Δ being a function of the vector $\mathbf{X} = \{X_1, X_2, \cdots, X_n\}$, composed by random variables X_i representing the uncertainties affecting the behavior of the structure. The direct derivation of P_f by eq. (1), considering a general g as function of $\mathbf{X}$ is practically impossible, even for simple problems, and for the specific case of this paper, $\Delta(\mathbf{X})$ is calculated by a finite element analysis, and therefore, $g(.)$, doesn't have an explicit form to be worked on. First-order reliability methods (FORM) are usually employed in reliability analyses of structural systems [7], [8], [10], [11], and had presented good results when compared to those found by simulation techniques [15], with significantly less computational effort. This way, FORM was used in this paper, coupled with a finite element formulation to evaluate the limit state function, as described in section 2.2.

$$P_f = \int \cdots \int_{g(\mathbf{X}) \leq 0} f_{\mathbf{X}}(x_1, x_2, \cdots, x_N) \mathrm{d}x_1 \mathrm{d}x_2 \cdots \mathrm{d}x_N \tag{1}$$

2.2 Reliability analysis procedure

A first-order reliability method (FORM) was built at MATLAB environment [16] to evaluate both probability of exceedance P_f and the related reliability index β. β and P_f are mutual related by the expression $P_f = \Phi(-\beta)$, that uses the cumulative distribution function Φ of the standard normal random variable. For n considered random variables, the limit state equation reported in section 2.1 was updated to $g(\mathbf{X}) = \Delta_{lim} - \theta\Delta(X_2, X_3, \cdots, X_N)$, with $X_1 = \theta$ representing the model uncertainty, which accounts for the assumptions and simplifications inherent in the finite element analysis used to derive Δ. The FORM method is based on Hasofer and Lind [17] Advanced-First-Order-Second-Moment (AFOSM) method adapted for non-normal random variables [18]. A flowchart with the implemented algorithm is shown at Figure 1. More details regarding this reliability analysis procedure can be found at some cited references [17], [18], [19].

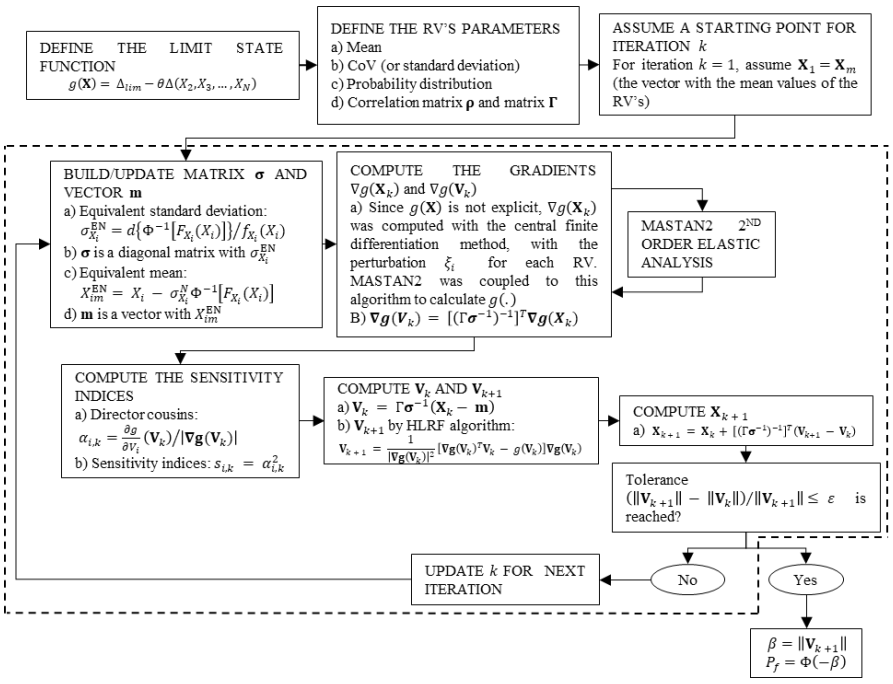


Figure 1. Reliability index derivation

As reported in Figure 1, numerical differentiation process was needed to calculate the partial derivatives needed to build the gradient vector $\boldsymbol{\nabla g}(\mathbf{X})$. The finite element analysis related to $\Delta(X_2, X_3, \cdots, X_N)$ was carried out using MASTAN2 [20] second-order elastic analysis. The previous conference paper [14] compared some results of this analyses with full-scale experimental tests on cold-formed steel frames, and indicated good accuracy, with safe displacement predictions inside the elastic regime. For θ, the parameters $\theta_m = 1.00$ and $V_\theta = 0.05$ for mean and standard deviation, and lognormal distribution were adopted [12], [21].

3 Application: warehouse portal frame

A common application of cold-formed steel members is in the construction of warehouses, with elements on the main structural frame or in secondary parts, such as panels, purling, girts and sheets. A central warehouse portal frame with a channel cross-section was selected for the analysis (Figure 2). It was considered designed just at its serviceability limit states, such that $\Delta(\mathbf{X}_n) = \Delta_{lim}$; $\mathbf{X}_n$ is the vector with the nominal values of the random variables. Four values of nominal dead load D_n were selected, and nominal values of the variable loads were applied, until the frame reached its displacement limit. To guarantee that no local instabilities or yielding occur under the service loads, critical local and distortional buckling loads, and the compliance with eq. (2) were verified, when the demanding load effects $M_{r,x}$, $M_{r,y}$ and P_r from factored actions were compared to their respective factored resistances $M_{d,x}$, $M_{d,y}$ and P_d. The considered uncertainties related to the frame stiffness were: Young's modulus, moment of inertia and joint stiffness (bases, eaves, and apex joints). Their statistics are described in Table 1.

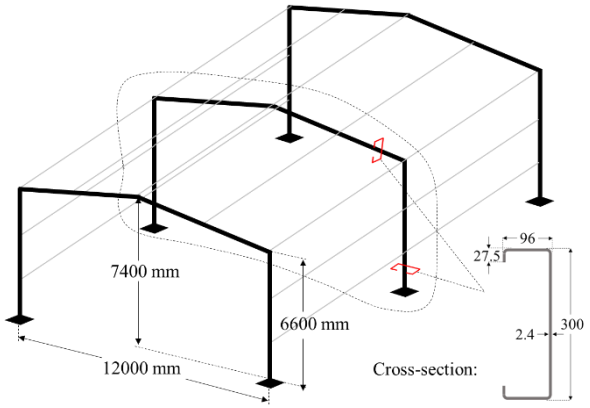


Figure 2. Portal frame geometry

$$\frac{P_r}{P_d} + \frac{M_{r,x}}{M_{d,x}} + \frac{M_{r,y}}{M_{d,y}} < 1 \tag{2}$$

Table 1. Random variables related to the frame stiffness

Random variable	Mean	Coefficient of variation	Probability distribution	Reference
Young's modulus, E	$1.00E_n$	0.06	Lognormal	[22]
Moment of inertia, I	$1.00I_n$	0.05	Normal	[23]
Joint stiffness, K	$1.00K_n$	0.30	Lognormal	[24]

The target reliability indices specified for serviceability limit states are usually lower than those specified for ultimate limit states [21], the consequences of a non-attendance does not relates to the safety of the structure or its users. This paper addresses reversible serviceability limit states. Based on the serviceability criteria defined by Galambos and Ellingwood [13] for this case, the checking load was chosen to ensure that the deflection limit is not exceeded annually. ASCE/SEI 7 [25] commentaries indicates a maximum annual probability of exceedance of 5% for serviceability limit states, correspondent to an equivalent target reliability index of $\beta_0 = 1.64$. No target values were found for the Brazilian codes regarding serviceability limit states. Then, the target associated to the

above cited code was considered as reference.

3.1 Case 1: dead and imposed loads

The first load case analyzed was the portal frame under gravity loads only. For this, only dead (permanent) and live (imposed) loads were considered (Figure 3), and the vertical displacement at the apex joint was the evaluated degree-of-freedom for serviceability. For serviceability checks, ASCE/SEI 7 [25] specifies the following unfactored load combination: $\sum S_d = D_n + L_n$, while NBR 8681 [26] and NBR 14762 [6] indicates $\sum S_d = D_n + L_n$ for irreversible, or $\sum S_d = D_n + \psi_1 L_n$ for reversible serviceability limit states. ψ_1 is a reduction factor that converts L_n to a frequent value $\psi_1 L_n$ – what results in a load combination that repeats many times along the design life of the structure. In the case of this paper, irreversible serviceability limit states correspond to damage in non-structural members, since the analysis performed is elastic, and eq. (2) was checked. By NBR 14762 [6], $\psi_1 = 0.7$ for imposed loads on roofs. Span divided by 240 and by 250 was the normative limits selected for ASCE/SEI 7 [25] and NBR 14762 [6] respectively.

The dead load was modelled as a normally distributed variable, with mean $D_m = 1.05 D_n$ and CoV $V_D = 0.10$ [27]. The reference period is 50 years, but the properties of D are considered invariant for any reference period. The imposed imposed load on the roof was modelled as Gumbel-type random variable, with mean $L_m = 0.45 L_n$ and CoV $V_L = 0.40$ [12], for a reference period of 1 year.

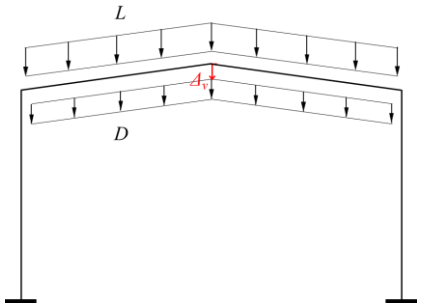


Figure 3. Case 1: dead and imposed loads

3.2 Case 2: dead and wind loads

The lateral displacement of the right eave joint was evaluated in the second load case. For comparison, the load pattern used by Arrayago and Rasmussen [12] was selected, considering the frame under dead (permanent) and wind loads (Figure 4). Here, the lateral displacement of the right eave joint was monitored. ASCE/SEI 7 [25] also indicates an unfactored load combination when the wind load is considered. However, the serviceability wind speed, correspondent to a 10-year mean recurrence should be used. Then, the combination $\sum S_d = D_n + \psi_a W_n$ was used, with a factor $\psi_a = 0.7$ to convert the nominal 50-year wind load to the specified 10-year wind load [12], [28]. ASCE/SEI 7 current version [25] specifies maximum wind velocities for a 700-year return period for category II buildings. Then, based on the eq. (3) conversion factor defined by Peterka and Shahid [29], [30] (3), $\psi_a = 0.44$ can be used to convert the nominal 700-year wind load to the specified serviceability load. For NBR 8681 [26] and NBR 14762 [6], the combination for reversible serviceability limit states $\sum S_d = D_n + \psi_1 W_n$ was used, with $\psi_1 = 0.3$. The displacement limit considered both for ASCE/SEI 7 [25] and NBR 14762 [6] is the height divided by 400.

$$\frac{W_{n,700}}{W_{n,50}} = [0.36 + 0.10 \ln(12T)]^2 = [0.36 + 0.10 \ln(12 \times 700)]^2 = 1.60 \tag{3}$$

The same stochastic parameters considered for D in section 3.2 were used for Case 2. The eq. (3) factor was used to make the annual wind load statistics presented by Galambos and Ellingwood [13] to be related to the nominal 700-year wind load. Then, the following statistics were used to perform the reliability analysis for the ASCE/SEI 7 [25] criteria: $W_m = 0.21 W_n$ and $V_W = 0.60$. Beck and Souza Jr. [31] calculated annual wind statistics for average central-south Brazilian regions, obtaining $W_m = 0.33 W_n$ and $V_W = 0.47$. W_n in Brazilian NBR 6123 [32] corresponds to the wind load calculated from the wind velocity with 50-year return period. W was considered

to follow the Gumbel distribution in both cases.

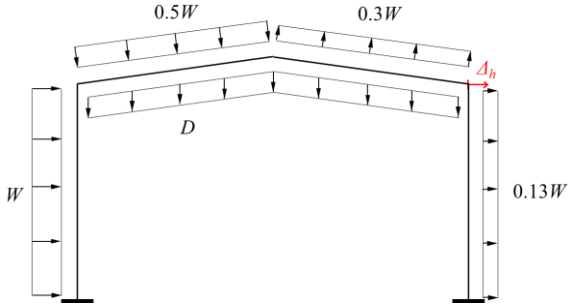


Figure 4. Case 1: dead and wind loads

4 Reliability results

Figure 5 shows the annual reliability indices β_1 found for cases 1 and 2, following ASCE/SEI 7/AISI S100 [5], [25] and NBR 8681/NBR 14762 [6], [26] design prescriptions. For Case 1, β_1 increases as the ratio L_n/D_n grows. In Case 2, this growth is almost imperceptible. The target $\beta_0 = 1.64$ was met for ASCE/SEI 7/AISI S100 [5], [25] design, in Case 1, but failure to reach the target in case 2, as was the case for NBR 8681/NBR 14762 [6], [26] design in both cases. For the design at the serviceability limit, the obtained indices for the Brazilian codes correspond to the respective average values of $P_e = 16\%$ and $P_e = 46\%$ for annual probability of exceedance for cases 1 and 2. Significant difference was seen between the results from US and Brazilian designs, that can be attributed to their different load statistical properties, and the use of the ψ factors. The lower values found for by β_1 when compared Faria, Freitas and Mapa [14]'s $\beta_1 = 2.38$ and $\beta_1 = 2.02$ for a similar structure can be attributed to the consideration of more uncertainties in this paper. For the same load case, Faria, Freitas and Mapa [14] considered only E, D and L as random variables. The values in Figure 5 are consistent to those found by Arrayago and Rasmussen [12] (values ranging between 2.0 and 2.5), with differences attributed to the reliability method employed and, differences between the materials and the geometry of the cross-section ands a different serviceability criteria for the reference period.

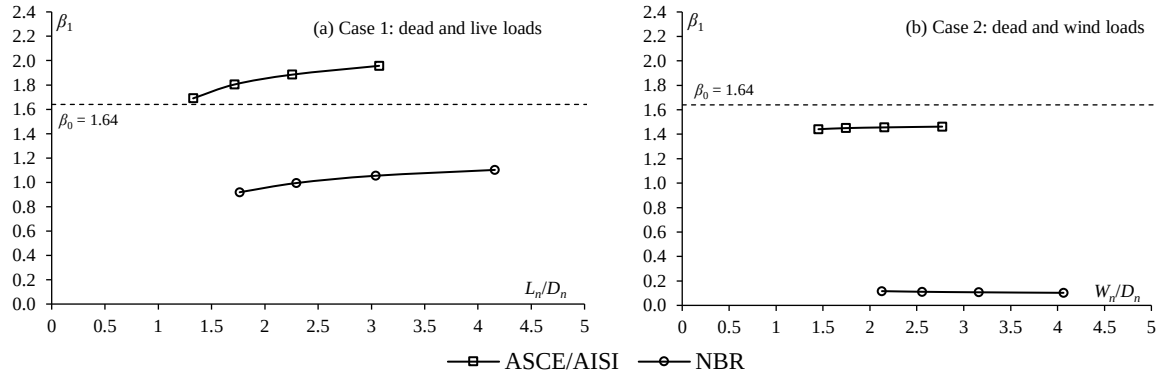


Figure 5. Annual reliability indices

The sensitivity indices s_i described in Figure 1 for the last iteration of the reliability derivation procedure can be used as a measure of the influence of each random variable in the reliability. These results are presented in Figure 6. The influence of the imposed load increased as L_n/D_n got higher, but it was not seen in Case 2 for W_n and W_n/D_n. Live and wind loads had the higher s_i values, due to their mechanical significance in the problem and their high coefficients of variation adopted in section 3. The dead load had minor influence in both cases. The model uncertainty had more influence in Case 1. Regarding the frame stiffness, the joint stiffnesses were the most influential variables; eave and apex joint stiffnesses in Case 1 and base and eave joint stiffnesses in Case 2. It is obviously related to the direction of the load applications and the monitored degree of freedom.

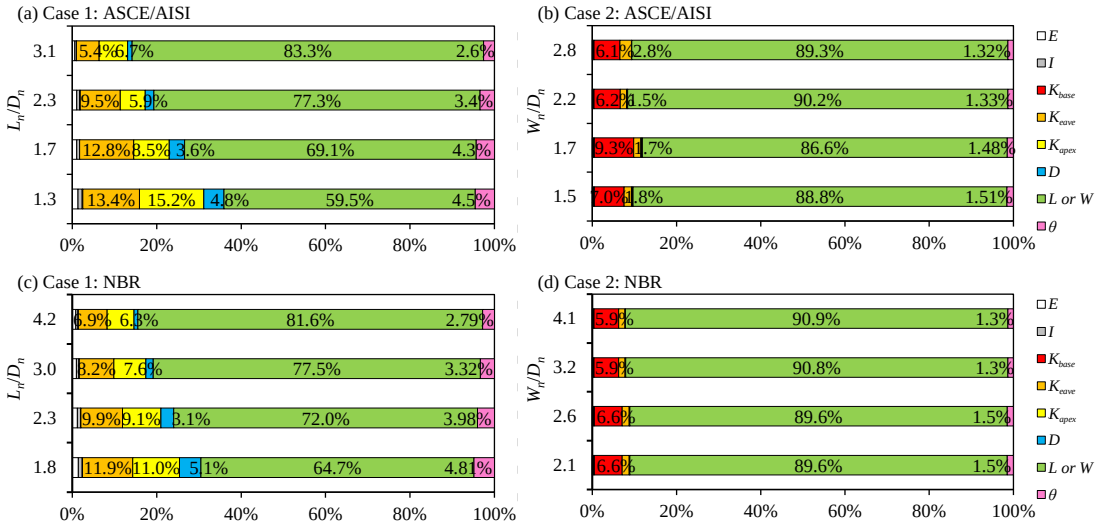


Figure 6. Sensitivity indices

5 Conclusions

In this paper, advances in the study presented by Faria, Freitas and Mapa [14] were made. A new range of variable-to-dead load values were examined, and the previous reliability analysis method for serviceability limit states was improved, considering more uncertainty sources, the design guided by the Brazilian design codes and the specific consideration of a reversible servicieability limit states criteria. A target value of $\beta_0 = 1.64$ was taken as the reliability index related to the maximum probability of exceedance of the displacement serviceability limit states, and the design guided by NBR 8681/NBR 14762 [6], [26] did not meet this target. Then, a serviceability criterion must be better defined for these codes. The target was reached in only gravity load case analyzed for the design by ASCE/SEI 7/AISI S100 [5], [25] codes. The values for the wind load case were close, but slightly under the limit value. Among the new random variables considered in this paper, the joint stiffnesses had significant influences on the results, being responsible for lower values of β_1 than those from previous paper [14]. The irreversible serviceability limit state is under study for this reliability framework.

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