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Three-dimensional topology optimization considering geometric nonlinearity: A comparative study between SESO and BESO

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Abstract. The study of Topology Optimization (TO) in three-dimensional structures under geometrically nonlinear behavior remains limited in the literature. This paper aims to apply three-dimensional TO elasticity problems considering geometric nonlinearity, employing the total Lagrangian formulation. Two optimization methods are compared: Geometrically Nonlinear Smoothing Evolutionary Structural Optimization (SESO-GNL) and Bi-directional Evolutionary Structural Optimization (BESO-GNL), both of which use bidirectional heuristics that allow the iterative removal and addition of inefficient elements. Finite element models were developed in both MATLAB and Fortran, using 8-node linear hexahedral elements, with the objective of minimizing structural compliance. The results demonstrate that both SESO-GNL and BESO-GNL are robust and efficient in solving benchmark problems from the literature, and the present implementations appear to be accurate and reliable.

Keywords: Topological Optimization, BESO, SESO, Geometric Nonlinearity.

1 Introduction

Topology Optimization (TO) is a well-established technique for the design of efficient structures, aiming to determine the optimal material distribution within a given domain while satisfying mechanical performance criteria constraints. Traditionally, TO has been applied within the context of linear elastic analysis, where displacements are small and the relationship between load and displacement remains within the linear regime.

However, for certain structural systems such as slender structures or those subjected to high loading conditions the effects of Geometric Nonlinearity (GNL) become significant, greatly influencing the structural response. In such cases, the conventional linear approach may lead to suboptimal or even unstable solutions, requiring the adoption of TO considering Geometric Nonlinearity (TOGNL).

The work of Buhl et al. [1] presents a TO method for structures with large deformations, using a total Lagrangian FEM, adjoint sensitivities, and the Method of Moving Asymptotes with filtering and continuation schemes. It shows that compliance-based designs degenerate under single loads, but robustness can be achieved through multiple load cases or complementary elastic work, with nonlinear effects becoming critical in buckling or snap-through scenarios. Subsequently, Gea and Luo [2] expanded the field by proposing a microstructure-based approach associated with a nonlinear formulation, consolidating the research in TOGNL. In later studies, Gea and Luo also applied the Sequential Convex Approximation method, highlighting that the inclusion of nonlinear effects can lead to stiffer topologies. Lee and Park [3,4] explored nonlinear dynamic loading, introducing the concept of equivalent static loads to approximate the dynamic response at each time step. To address mesh distortions in low-density elements, they reformulated the objective function, prioritizing the minimization of flexibility in critical

regions.

Several researchers have advanced in this topic, including the contributions by Chen et al. [5], Zhu et al. [6], Han et al. [7], Werner et al. [8], and Zhao et al. [9]. One of the main challenges in TOGNL lies in the numerical degradation induced by regions with near-zero material density, which leads to ill-conditioning of the tangent stiffness matrices and hampers convergence of the iterative process. To address this issue, the literature proposes various strategies, such as: (i) element removal and reinsertion strategies, Bruns and Tortorelli [10]; (ii) the use of nearly-incompressible hyperelastic materials with low stiffness, Luo et al. [11]; (iii) connectivity parametrization, Moon and Yoon [12]; (iv) deformation scaling, Van Dijk et al. [13]; (v) polyconvex constitutive model in conjunction with a relaxation function, Lahuerta et al. [14]; (vi) strain energy-based interpolation schemes, Wang et al. [15]; and (vii) condensation techniques using super-elements, Hou et al. [16].

In this context, the present study also proposes the application of the Smooth Evolutionary Structural Optimization (SESO) method, as introduced by Simonetti et al. [17], within the TOGNL framework, implemented in MATLAB. For validation and comparison purposes, the results obtained using SESO are contrasted with those generated by the BESO method (adapted from the program in [8]) and with the HYPLAS (HYperelastic PLastic Analysis System) system, developed by Souza Neto et al. [18], which supports both linear and nonlinear structural analyses. In this work, methodologies for incorporating TO into HYPLAS were implemented. The results indicate that both SESO and BESO methods are robust and effective in generating optimized topologies, even under large-displacement regimes.

2 Topological Optimization

2.1 TO formulation

Considering the classical topology optimization problem aimed at maximizing structural stiffness or equivalently, minimizing the average compliance C - subject to a volume constraint, the TOGNL for the BESO and SESO methods can be expressed as follows:

$$Min\ C = \frac{1}{2}(f_{ext})^T u$$
$$subject\ to \begin{cases} R = f_{int} - f_{ext} = 0 \\ V(\boldsymbol{x}) = \sum_{i=1}^{n} x_i V_i - V^* \leq 0 \\ X = \{x_1 x_2\, x_3\, \dots\, x_n\},\ x_i = 1\ or\ x_i = x_{min} \\ E_i = x_i^p E_0 \quad and \quad \rho_i = x_i^p \rho_0 \qquad i = 1,..,n \end{cases} \tag{1}$$

with C being the structural compliance, V_i is the volume of each element i, V* is the prescribed target volume; $\boldsymbol{x}$ represents the design variables, where $x_i = 1$ and $x_{min} \rightarrow 0$ correspond to the presence and near absence of material, respectively. The total number of finite elements is n, u is the displacement vector, and, p is the penalization factor. E_0 and ρ_0 denote the Young's modulus and the mass density of the base material, respectively. R is defined as the residual force vector, where f_{int} is the internal force vector and f_{ext} is the external force vector.

2.2 Density filter and sensibility number

In the BESO and SESO methods, which rely on a bidirectional heuristic, the removal of a structural element during the iterative process results in a reduction of the system's global stiffness. This, in turn, leads to an increase in the stored strain energy. Consequently, in TO procedures aimed at compliance minimization, the ideal strategy is to remove those elements whose impact on the objective function is minimal. This is achieved by excluding the elements with the lowest sensitivity values, thereby ensuring the least possible deterioration in the mechanical performance of the structure.

Conversely, the addition of elements in the BESO/SESO methods to regions where the strain energy is high ensures a balanced removal-addition process during optimization. This prevents the algorithm from getting trapped in local minima and promotes efficiency and robustness in achieving the optimal solution.

To mitigate abrupt variations in the density field during the evolutionary process which may compromise numerical stability and lead to topological irregularities a mesh-independent spatial filter is applied. This filter

aims to smooth out numerical instabilities such as checkerboarding, mesh dependency, and local minima. Mathematically, spatial filters are implemented as additional constraints in the optimization problem formulation.

As the filter radius increases, the design variable gradients are restricted to smaller values, resulting in smoother transitions between solid and void regions. This leads to a greater number of elements with intermediate sensitivity values. The procedure's sensitivity to the filter radius becomes especially evident when the radius is excessively large, as it restricts the design variable gradients to such a low rate of change that the optimization problem may have no feasible solution for the proposed initial condition. One of the most widely adopted approaches is the use of density filters, as proposed by Bruns and Tortorelli [20], expressed by:

$$\bar{\alpha}_j = \frac{\sum_{j=1}^{N} H_{ij} v_j \alpha_j}{\sum_{j=1}^{N} H_{ij} v_j} \tag{2}$$

where H_{ij} is the weighting factor defined as $H_{ij} = R - d_{ij}$, with R being the prescribed filter radius that defines the influence domain of the filter, and d_{ij} the distance between the centroids of elements (i) e (j) and v_j denotes the volume of element. To control the rate of variation in the density field throughout the optimization procedure which may impair numerical convergence and introduce discontinuities in the topology during intermediate iterations a sensitivity smoothing technique is adopted. This technique is applied to the sensitivities used to identify inefficient elements for removal. The process modifies the sensitivity value from eq. (2) by computing the arithmetic mean of the values from the two previous iterations, thereby promoting a more gradual evolution of the topology. Accordingly, the smoothed sensitivity, $\hat{\alpha}_j$ is obtained by:

$$\hat{\alpha}_j = \frac{\left(\bar{\alpha}_j\right)^k + \left(\bar{\alpha}_j\right)^{k-1}}{2} \tag{3}$$

2.3 Removing isolated regions

In topology optimization, the connectivity among active elements within the structural domain is essential to ensure structural feasibility and to avoid isolated regions ("islands") that do not contribute mechanically to the system. To mitigate this issue, a routine was implemented to identify and remove isolated elements. This is achieved using a connectivity matrix H (H_{ij} indicates the connection between elements i and j). Isolated elements are those with no connections to any neighboring elements, i.e., when $H_s(e_1) = 0$, which can lead to numerical instabilities in nonlinear simulations involving large displacements.

The removal of such elements may cause numerical instabilities because elements with near-zero density exhibit near-zero stiffness, compromising the positive-definiteness of the tangent stiffness matrix—an essential condition for the convergence of the Newton–Raphson method. If unaddressed, this effect may result in a poorly conditioned or even singular system, thus jeopardizing the solution of the nonlinear equilibrium equation, which represents the equality constraint in eq. (1).

According to [1], an effective solution is the conservative removal of the degrees of freedom associated with nodes that are exclusively connected to penalized (low-density) elements, as given by:

$$H_s(e) = \sum_{j=1}^{n} H_{ej} \tag{4}$$

here, e denotes the index of the current element. An element is considered isolated when $H_s(e) = 0$.

2.4 Geometrical nonlinearity integrated with TO

The approach considers finite deformations in structures subjected to large displacements, a common condition in slender components or under critical loading. The formulation is integrated into a research-oriented code environment, aiming to enhance the accuracy and robustness of the resulting topological designs. The inclusion of geometric nonlinearity is achieved through the Total Lagrangian formulation, which uses the initial

configuration of the structure as the reference frame. The equilibrium equations are solved using an incremental-iterative scheme based on the Newton–Raphson method, in which the material and geometric tangent stiffness matrices are updated at each iteration. The core of the analysis involves the evaluation of the residual vector, defined as the difference between the internal and external force vectors, that is:

$$R(u) = f_{int} - f_{ext} = 0 \tag{5}$$

where the convergence is achieved when the norm of the residual vector falls below a predefined tolerance, indicating that the structural equilibrium condition has been satisfied at each load step.

The internal forces are computed by integrating second-order stress measures (such as Piola–Kirchhoff stresses) over the element's reference volume, reflecting the current deformation state of the material. The external forces are derived from the imposed boundary conditions, such as nodal loads, surface pressures, or distributed forces. The difference between these quantities provides the correction vector used to update the nodal displacements. This residual evaluation process is fundamental to ensuring the robustness of the nonlinear analysis, particularly within the TO framework, where the mesh and material distribution are iteratively updated.

Thus, considering the second-order tensors E_{kl} and S_{ij}, which represent the Lagrangian deformation E and the Piola–Kirchhoff stress tensor S, respectively, and the fourth-order tensor D_{ijkl}, which represents the elastic constitutive matrix D:

$$S_{ij} = D_{ijkl} E_{kl} \tag{6}$$

where the internal forces are obtained by integrating the second Piola–Kirchhoff stress tensor S over the element's reference volume, weighted by the strain–displacement matrix B. Thus, the internal force is calculated according to the eq. (6)

$$f_{int} = \iiint_{\Omega_0} B^T S d\Omega \tag{7}$$

In the topology optimization stage, the BESO or SESO methods are employed to evolve the structural geometry based on the sensitivity criterion associated with the total strain energy, which now reflects the nonlinear behavior of the structure. Material update is performed via adaptive parameters for element addition and removal - removal ratio, RR and addition ratio AR, as indicated by [7] - while the equilibrium analysis is re-evaluated at each optimization cycle considering the new structural state. This integration allows the derivation of optimized topologies that inherently incorporate the nonlinear structural response under realistic loading and deformation states. The flowchart of all the process is shown in figure 1.

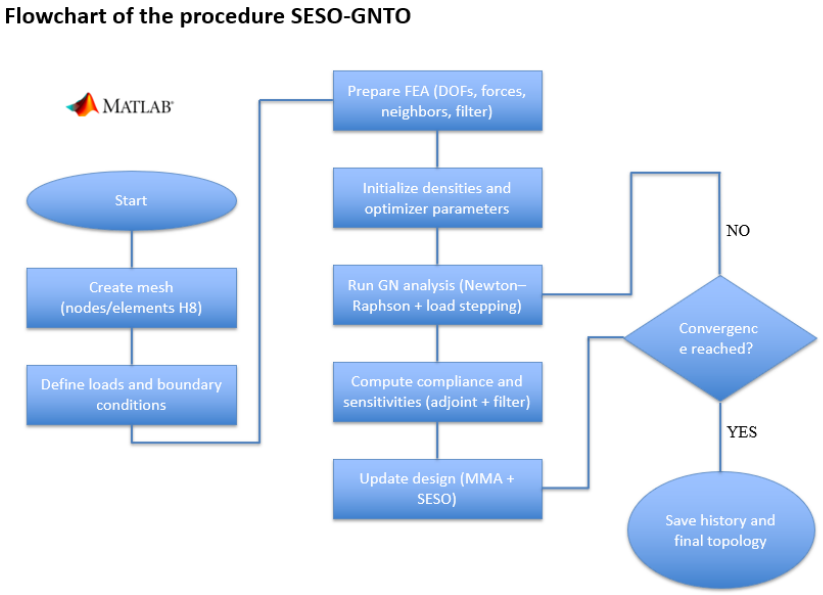


Figure 1. Flowchart of the SESO-GNTO procedure for geometric nonlinear topology optimization.

3 Numerical examples

3.1 Doubled clamped beam

The beam is a classical benchmark proposed by Huang and Xie [19], where the design domain is shown in Fig. 2a, with a total applied load o $P = 30N$. The beam dimensions are L_x = 160 cm, height L_z = 20 cm and thickness L_y = 1 cm. The material is linear elastic, with a Young's modulus of 30 MPa and Poisson's ratio of 0.3. The target material volume is set to 20% of the initial design domain. The initial mesh consists of 3.200 linear hexahedral elements with 8 nodes each (160 x 1 x 20). The developed codes are identified as follows: (c1) SESO-MATLAB; (c2) BESO-MATLAB; (c3) BESO-HYPLAS. The parameters adopted for the optimization methods include a removal ratio (RR) of 1%, an addition ratio (AR) of 2%, and a filter radius of 2.5 cm for the MATLAB-based codes and 1.75 cm for the HYPLAS implementation. The analyses were conducted using 10 uniform load increments (each accounting for 10% of the total load) and a residual convergence tolerance of 10^{-2}.

The optimal configurations obtained by the three methods are shown in Fig. 3 and are consistent with the optimal topologies presented in [19], which used 51,200 quadrilateral elements. The final objective function values, i.e., the strain energy at the final configuration and the normalized processing times (t) are as follows: 5 J and t for (c1), 4.6J and $1.16t$ for (c2) and 1.3 J and $0.48t$ for (c3), respectively. It is noteworthy that, during the sensitivity analysis, the strain energy was evaluated according to eq. (3), with t = 1,506 seconds.

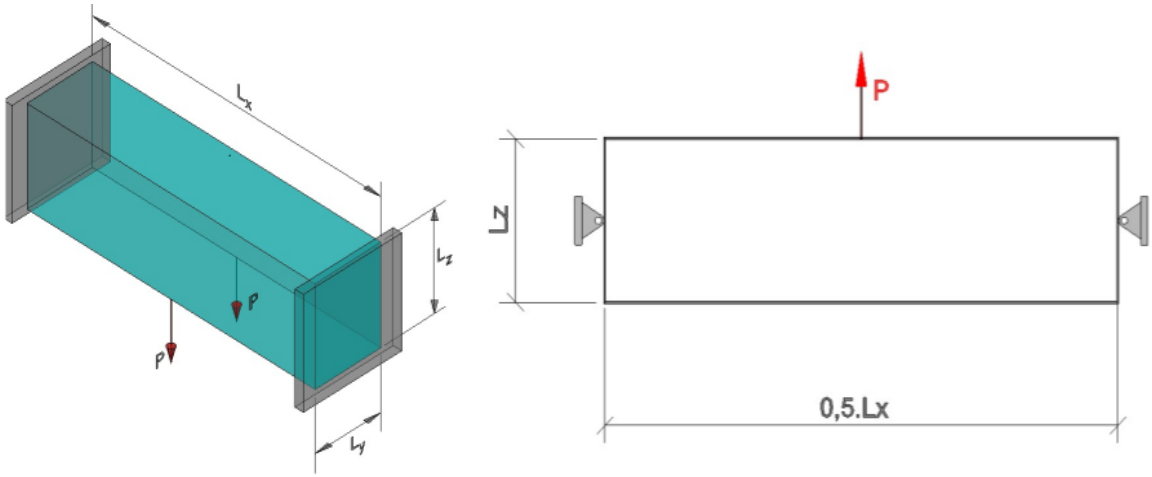


Figure 2. Design domain for both examples

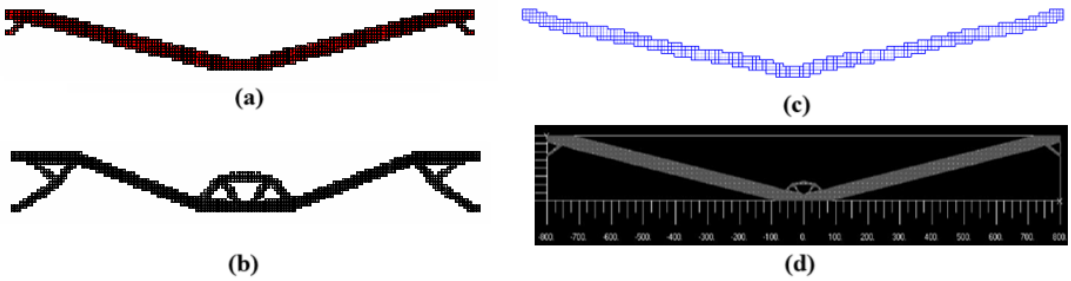


Figure 3. Optimal topology for the present codes: (a) c1; (b) c2 (c) c3 (d) Huang and Xie [19]

3.2 Double-supported beam

Figure 4d shows the design domain, as proposed by [2] and [20]. The beam dimensions are L_x = 80 cm, height L_z = 20 cm, and thickness L_y = 1 cm. A total of 3,200 eight-node hexahedral finite elements and a total load of $P = 200\ N$ is applied upward at the center of the top edge. The material is linear elastic, with a Young's modulus of E = 1 GPa, Poisson's ratio of 0.3, and a final material volume of 20% of the initial design domain. The results

are presented in Fig. 4. Mean compliance and total runtime *t* (final designs) were: c1 - 345 N·cm, t = 3,320 s; c2 - 617 N·cm, t = 3,335 s; c3 - 232 N·cm, t = 230 s. Reference [2] reports a mean compliance of 4.21 × 10^2 "N·m", which is unit-inconsistent with their own setup, given loads in Newtons (N) and lengths in centimeters (cm), the compliance unit should be N·cm. Interpreting the reported value as 421 N·cm aligns it with our range. Reference [20] presents the same problem using SPLP and reports 325 (no unit); under the same reasoning, this should be 325 N·cm.

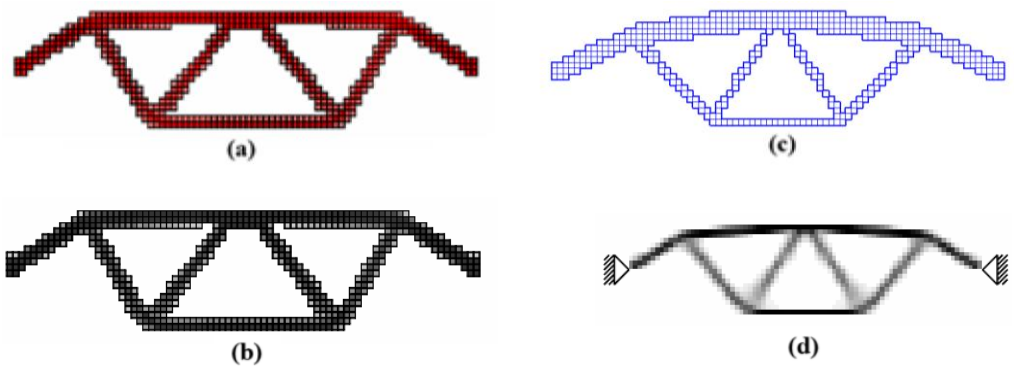


Figure 4. Optimal topology for the present codes (a) c1; (b) c2; (c) c3 (d) [2]

4 Conclusions

This study synthesized the process of integrating geometric nonlinearity (GNL) into 3D topology optimization (TO) within an elastic, Total-Lagrangian framework. We emphasized two pillars—numerical stability and method robustness—and implemented both BESO and SESO in a less-conventional yet promising TO-GNL workflow. Validation on classical benchmarks indicates that both formulations were correctly developed. Under a controlled comparison, SESO exhibited good shape fidelity on a coarse mesh, reproducing canonical layouts with 3,200 linear hexahedra where the literature used ~51,200 quadrilaterals. The smoothing step (Eq. (3)) produced clean, well-connected members, suppressing islands/checkerboards and reducing CAD/fabrication post-processing. The SESO layouts also recovered the expected arching/compression and tension-tie patterns under the same 20% volume fraction and material model.

Regarding robustness and convergence, SESO remained stable under large displacements. In practical terms, it achieved compliance competitive with BESO while relying on a simple, repeatable parameter set (RR = 1%, fixed filter radius). This low-tuning burden is attractive in engineering workflows where robustness and repeatability outweigh marginal objective gains. We note additionally that MATLAB-based formulations, while generally slower in wall-clock time than compiled implementations, are easier to implement, audit, and share, and are widely accepted in current research workflows—making them a pragmatic choice for prototyping and dissemination despite the runtime trade-off.

Importantly, the comparative aim is preserved: both SESO and BESO, as implemented here, proved effective for solving 3D GNTO problems. While SESO offered practical conveniences—mesh efficiency, smooth/connected designs, and stable convergence with minimal tuning—BESO remained a competitive baseline and, in some cases, matched or slightly improved individual metrics. Future work will use the present SESO/BESO methodology as the core optimization engine to incorporate reliability-based multi-material TO and dynamic TO driven by CFD-derived loads, enabling the evaluation of real engineering problems.

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