Reliability-Based Multi-Material 3D Topology Optimization: A Comparative Study Between SIMP and SESO Methods

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Abstract. Topology optimization aims to improve structural design efficiency by determining the optimal material distribution within the solution domain. This paper presents an automated approach for three-dimensional multi-material topology optimization using the Solid Isotropic Material with Penalization (SIMP) and Smoothing-ESO (SESO) methods, implemented in MATLAB. The formulation employs hexahedral finite elements and a power-law interpolation scheme to minimize material discontinuities during the discrete optimization process. A three-dimensional linear elastic analysis is adopted for both methods, integrating commercial materials including steel, aluminum, and concrete. Additionally, the optimization program is coupled with a reliability-based topology optimization analysis module where Young's modulus, force, volume, and compliance are treated as random variables using the First-order Reliability Method (FORM), considering both normal and lognormal distributions with target reliability indices $\beta = 3.0$ and $\beta = 2.5$. Numerical examples, including a Michell-type structure and bridge design, demonstrate the method's effectiveness in achieving material efficiency while ensuring structural reliability. The results show significant reductions in high-performance material usage, confirming the approach's potential for practical engineering applications.

Keywords: Multi-material topology optimization, SESO and SIMP, RBTO.

1 Introduction

Currently, several structural designs are being developed with an increasing focus on optimization, aiming simultaneously to reduce costs and increase the rigidity of structures. Thus, the definition of the appropriate structural topology emerges as one of the most relevant aspects to achieve such objectives. The choice of topology directly influences the overall performance of the structure and constitutes a determining factor in the design and dimensioning process, both in civil engineering and in the aeronautical and mechanical areas.

Given this, several methodologies have been developed to solve topological optimization problems, including the Evolutionary Structural Optimization (ESO) method, proposed by Xie and Steven [1], the Solid Isotropic Material with Penalization (SIMP) method, presented by Bendsøe and Sigmund [2], and the Smoothing Evolutionary Structural Optimization (SESO) method, proposed by Simonetti et *al.* [3].

However, many structures can now be designed more efficiently by using more than one material in their composition. In this context, considering different commercial materials in structural design can result in a significant reduction in costs, while also promoting a substantial gain in structural rigidity.

Several methodologies have been implemented in this scenario, as evidenced in the studies by Li et *al.* [4], Ozcakar et *al.* [5], Zuo and Saitou [6], Simonetti et *al.* [7]. However, the SESO methodology applied to multi-material topology optimization (MMTO) combined with reliability analysis was explored in two-dimensional structures, as presented by Silva et *al.* [8].

Thus, this article aims to propose an innovative approach to apply MMTO to three-dimensional structures, coupled with a reliability analysis using the SIMP and SESO methods, considering Michell-type structures and

bridge structures, using commercial materials such as steel, concrete, and aluminum. Additionally, a comparative analysis is performed between the two methods, considering different values for the reliability index, in order to evaluate their influence on structural performance and, thus, provide more efficient results in future structural design.

2 Topology Optimization with Multi-material

2.1 SIMP method

The SIMP (Solid Isotropic Material with Penalization) method is widely used in topological optimization problems of structures. This method was proposed by Bendsøe and Sigmund in 1999, with some refinements proposed by Rozvany [9], SIMP offers an effective formulation for material redistribution in continuous domains. It is based on the penalization of regions with intermediate densities, guiding the removal of material from a binary solution (0 for the absence of material and 1 for the presence of material). Therefore, the method guides the progressive removal of non-essential material, resulting in optimal configurations with high structural and economic efficiency.

The implementation of MMTO from the SIMP method is shown through the flowchart in Fig. 1.

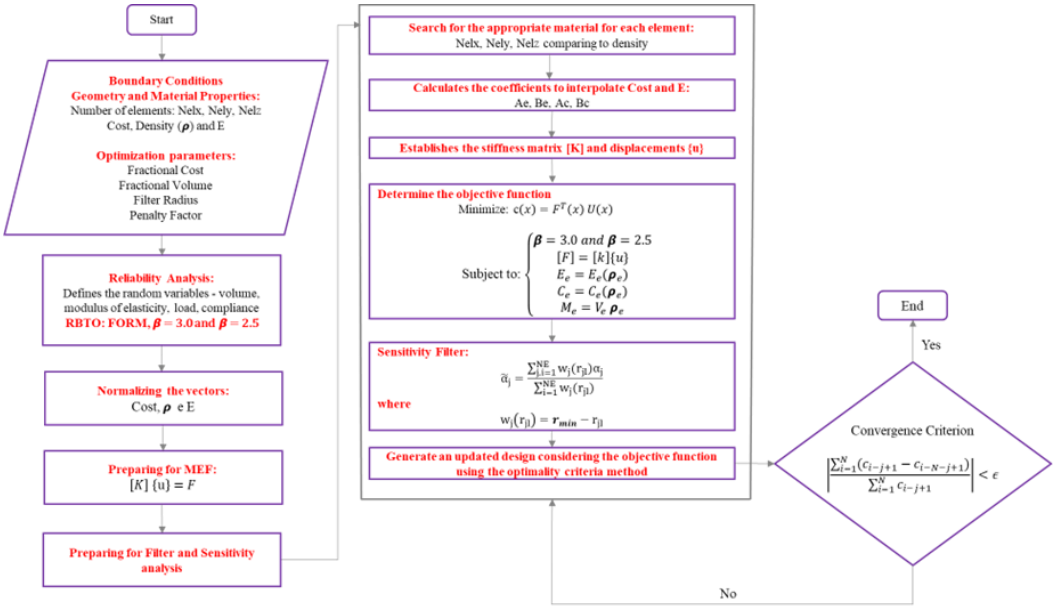


Figure 1. Flowchart of the optimization process using the SIMP method

Initially, the SIMP method was applied to several topology optimization problems using a single material in the design domain. Later, Zuo and Saitou [6] proposed an adaptation of the method to contemplate multi-material problems, introducing a new interpolation function. Equations (1) and (2) present this formulation, applied respectively to two- and three-phase problems.

$$E(\rho) = \rho^P E_1 + (1 - \rho^P) E_2 \tag{1}$$

$$E(\rho_1, \rho_2) = {\rho_1}^P ({\rho_2}^P E_1 + (1 - {\rho_2}^P) E_2) \tag{2}$$

Where: $\boldsymbol{\rho}$ is the artificial density [0,1], P is the penalty coefficient, E_1 and E_2 are the moduli of elasticity of the two phases when $E_1 = 0$ is taken as the empty phase.

2.2 SESO method

The SESO (Smoothing Evolutionary Structural Optimization) method, proposed by Simonetti et al. [3], was developed as an alternative to smooth the element removal heuristic present in the ESO method. This is because the ESO method adopts a unidirectional heuristic approach, which can lead to the hasty removal of structural elements, caused by the attempt to satisfy the maximum compliance constraint, as per Equation 3.

$$c_e < RR.c_{máx} \tag{3}$$

Where: RR is the current rejection ratio, c_e compliance of each element and $c_{máx}$ maximum compliance of the structure.

An alternative to hastily removing elements that could be essential to the structure was to assign a stiffness value to each element, instead of eliminating them based on Eq. 3. Thus, SESO considers that, at each iteration, some elements may be removed from the structure while others may be reinserted, based on the weighting factor given by the ratio $\frac{c_e}{c_{máx}}$. This strategy allows a smoother transition to the optimal solution.

The implementation of MMTO from the SESO method is shown through the flowchart in Fig. 2.

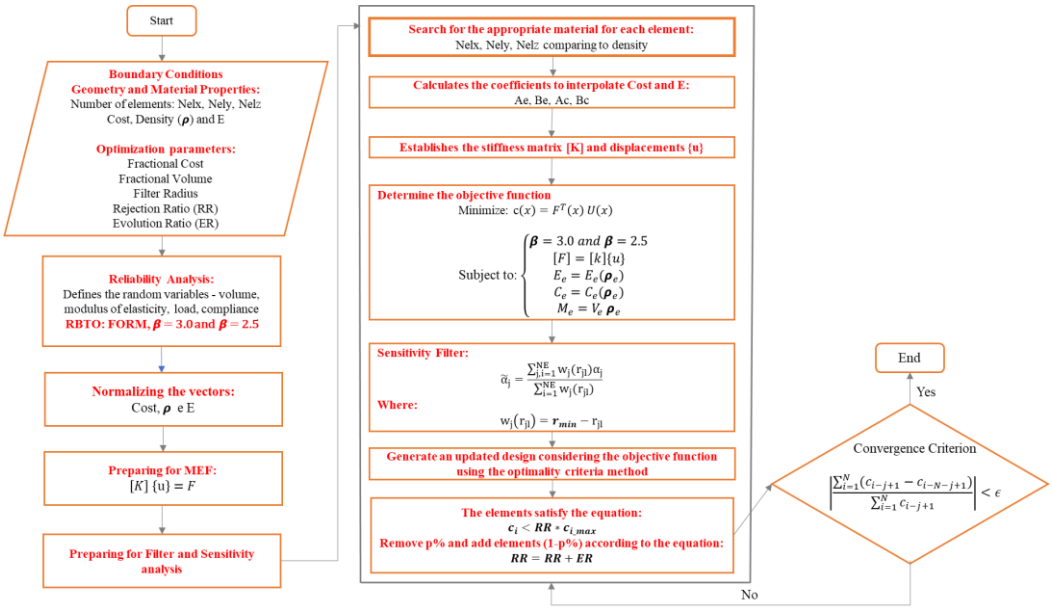


Figure 2. Flowchart of the optimization process using the SESO method

3 Reliability Analysis: First-Order Reliability Method (FORM)

A high-performance structure with a low probability of failure is desired in engineering projects. Thus, reliability analysis emerges as an essential tool for the probabilistic modeling of the uncertainties inherent in the process associated with resistance and stresses, Haldar et. *al.* [10].

Thus, to maximize safety levels in the MMTO approach, several works in the literature (Simonetti et. *al.* [11], Silva et. *al.* [8], Simonetti et. *al.* [12]) have shown satisfactory results from RBTO using the FORM method (First Order Reliability Method).

The FORM method uses Taylor series expansion, approximating the failure function iteratively, and linearizing each point until the result converges to the design point. This point is defined by the shortest distance between the origin and the failure function, known as the reliability index ($\boldsymbol{\beta}$).

In the reliability analysis, the following random variables were used: force, volfrac (fractional volume), E (elasticity modulus), and compliance from a probabilistic distribution function (PDF) of the normal (N) and lognormal (LN) types, with their respective statistical data of the mean ($\boldsymbol{\mu}$) and coefficient of variation (V) as shown in Table 1, adopting a reliability index $\boldsymbol{\beta}$ = 2.5 and $\boldsymbol{\beta}$ = 3.0.

Table 1 - Random variables and their probability distributions

Manufacturing Variable				**Material Variable**				**Variable Actions**			
Variable	**PDF**	$\boldsymbol{\mu}$	V	**Variable**	**PDF**	$\boldsymbol{\mu}$	V	**Variable**	**PDF**	$\boldsymbol{\mu}$	V
Volfrac	LN	1	0.05	E	LN	1.06	0.152	force	N	0	1
				Compliance	LN	1.06	0.152				

The equation used to find the distance from the origin to the failure function d$\boldsymbol{\beta}$, i.e. $\boldsymbol{\beta}$, is given below according to Kharmanda and Olhoff [13]:

$$d_\beta = \sqrt{\sum_{i=1}^{n}\left(\frac{y_i - x_i}{\sigma_i}\right)^2}\ ,\ i = 1, 2, \dots, n \tag{4}$$

Where x_i is the mean of the random variables, Y_i is the random variable and σ_i is the standard deviation equal to 1.0 for normal distributions.

Random variables involved in structural design do not always follow a normal distribution. In many cases, the lognormal distribution provides a more accurate representation of the structure's behavior.

In these cases, a normalized variable, u_i, with lognormal distribution, can be defined as:

$$u_i = ln\left(\frac{y_i\sqrt{1+\left(\frac{\sigma_i}{x_i}\right)^2}}{x_i}\right) \Big/ \sqrt{\ln\left(1+\left(\frac{\sigma_i}{x_i}\right)^2\right)}\ , \qquad i = 1, 2, \dots, n \tag{5}$$

Thus, the reliability index is obtained by normalized random variables.

4 Numerical Examples

The numerical examples presented in this work include a widely used benchmark in the literature, a Michell-type structure, and a bridge based on the study by Li et *al.* [4]. In all cases, the objective function was defined as the minimization of *compliance*, subject to the design constraints related to the elastic modulus and cost, as described in Equation 6.

$$Minimize\ C = \boldsymbol{f^T u} = \sum_{e=1}^{N} E_e \boldsymbol{u_e^T k_e u_e}$$

$$subject\ to: \begin{cases} c_e = c_e\ (\boldsymbol{\rho_e}) \\ E_e = E_e\ (\boldsymbol{\rho_e}) \\ \boldsymbol{Ku = F} \\ M = \sum_{e=1}^{N} V_e \rho_e \\ c = \sum_{e=1}^{N} V_e \rho_e c_e \\ M \le \varepsilon M\ M_0 \\ c \le \varepsilon c\ c_0 \end{cases} \tag{6}$$

Where: C is the objective function, being the compliance; k_e, u_e are the stiffness matrix and the displacement vector of the element, respectively; c_e and E_e are the cost and the modulus of elasticity of the nth element; $\boldsymbol{\rho_e}$ is the normalized density of the nth element; c is the compliance; K, u and F are the global stiffness matrix, displacement vector and force vector, respectively; and V_e is the volume of the nth element; M and C are the mass and cost of the current design domain, respectively; and M_0 and c_0 are the mass and cost of the filled design domain, respectively; εM and εc are the prescribed mass and cost fraction, respectively.

The commercial materials used in the bridge structure were concrete and steel. In the case of the Michell-type structure, in addition to these two materials, aluminum was also used. Table 2 shows the data on the materials used.

Table 2 – Material properties in the optimization of the Michell-type structure

Name	Density (kg/m³)	E (GPa)	Cost (R$/m³)	Color
Empty	0	0	0	White
Concrete	2200	50	300,00	Blue
Aluminum	2700	69	14.845,00	Red
Steel	7860	200	31.440,00	Black

The bridge structure was simulated using the properties indicated in Table 2. To facilitate a comparison of the results with the literature, the color blue was adopted for the concrete elements, but red for the steel elements.

4.1 Bridge Structure

The evaluated bridge has dimensions of 4.0m x 0.6m x 0.55m with a deck thickness of 0.05m. Fig. 3 shows the design domain and its boundary conditions of the structure.

Thus, the structure domain was discretized into 25 parts in each dimension, totaling 84.480 hexahedral finite elements. This structure was subjected to a distributed load of 2000 N/m², applied to the upper surface of the deck. However, the deck was not used in the MMTO domain to validate the results found by comparing them with those of Li et *al.* [4].

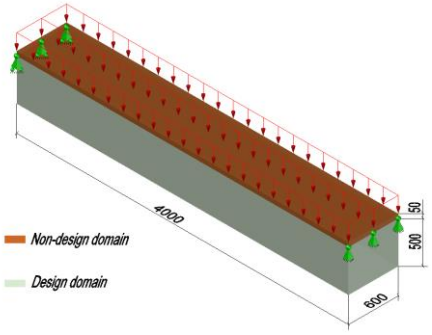
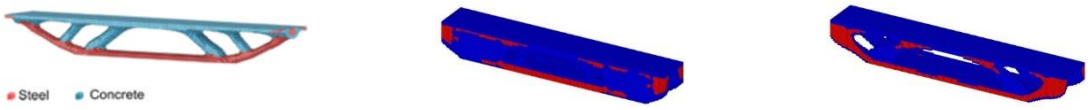


Figure 3. Bridge Structure Design with Boundary Conditions - 3D

The optimal MMTO configuration of the bridge is shown in Fig. 4 in a deterministic manner and compared with the author Li et *al.* [4].



(a) **(b)** **(c)**

Figure 4. Bridge structure, via methods: (a) BESO (Li et *al.* 2024); (b)SIMP; (c) SESO

The evolution of compliance during the optimization process is shown in Fig. 5 from the deterministic SESO and SIMP methods and with reliability using a reliability index equal to $\boldsymbol{\beta} = 2.5$ and $\boldsymbol{\beta} = 3.0$.

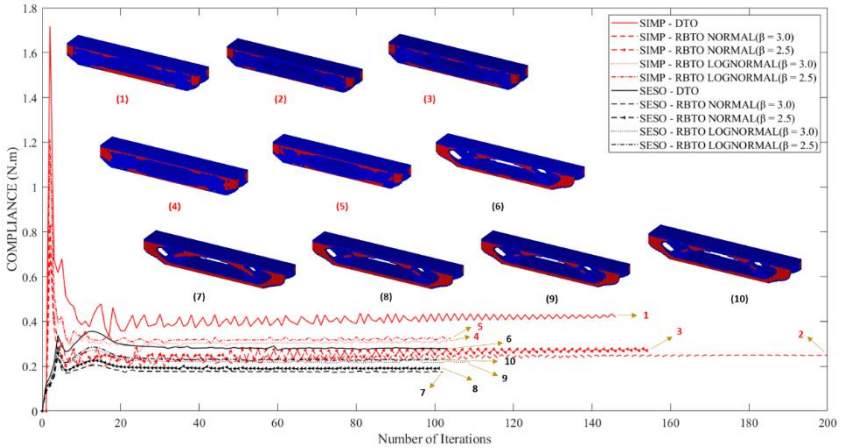


Figure 5. Bridge Structure: Compliance x number of iteration

4.2 Michell-type structure

In addition to concrete and steel, the Michell-type structure also used aluminum to compose the structural materials. This structure was evaluated as a prototype with dimensions of 100mm x 50mm x 2mm.

Fig. 6 shows the design domain and boundary conditions of the structure. The structure domain was discretized with a 1 x 1 x 1mm mesh, totaling 10.000 hexahedral finite elements.

This structured was subjected to a concentrated load of F = 1 kN applied at three different points, located at positions $\frac{n_{elx}}{4}$, $\frac{n_{elx}}{2}$, and $\frac{3\,n_{elx}}{4}$.

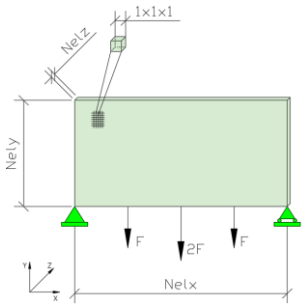
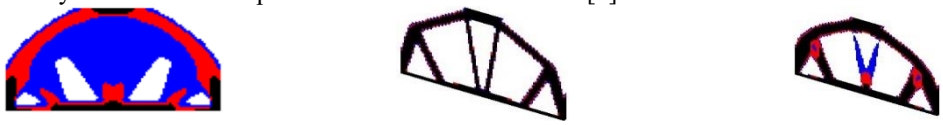


Figure 6. Michell-Type Structure Design with Boundary Conditions - 3D

The optimal configuration of the MMTO of the Michell-type structure is shown in Fig. 7 in a deterministically manner and compared with the author Özcakar [5].



(a) **(b)** **(c)**

Figure 7. Michell-type structure, via methods: (a) BESO (Özcakar, 2021); (b) SIMP; (c) SESO

The evolution of compliance during the optimization process is shown in Fig. 8 from the deterministic SESO and SIMP methods and with reliability using a reliability index equal to $\boldsymbol{\beta}$ = 2.5 and $\boldsymbol{\beta}$ = 3.0.

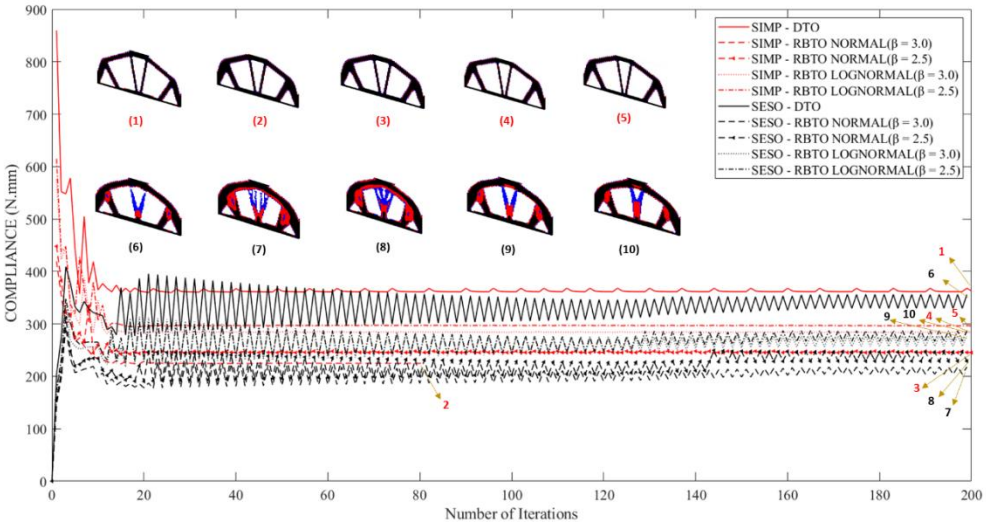


Figure 8. Michell-type structure: Compliance x number of iterations

5 Conclusions

The three-dimensional multi-material topology optimization analysis coupled with reliability, based on Michell and bridge type structures, showed that:

i) The SIMP and SESO methods presented results compatible with those obtained by Özcakar (2021), with relevant distinctions in the use of materials: the SESO method prioritized economic and transitional materials, such as concrete and aluminum, while SIMP focused on the predominant use of steel in the Michell-type structural configuration.

ii) The graphical representations of the bridge show that, by the application of the SESO method, the optimal topology resulted in an arch with a more efficient opening and a more strategic distribution of steel in the structure. This configuration favoured a significant reduction in compliance compared to the MMTO generated by the SIMP method.

iii) In regions with higher loads, MMTO uses steel, which is denser and more resistant, despite the higher cost, while aluminum is used for transitions, and concrete is used in regions with lighter loads, being a more economical alternative.

iv) The reliability analysis integrated with MMTO reduced compliance in both methods when compared to the deterministic results. In addition, a significant reduction in the results found with SESO compared to SIMP was observed, with lower compliance obtained from the Normal distribution compared to the Lognormal.

v) The use of a reliability index $\beta = 3.0$ resulted in lower compliance than that obtained with $\beta = 2.5$, both for Michell-type structures and for the bridge. This behavior was expected since the increase in the reliability index implies a decrease in the allowable force and an increase in volume, which reduces the probability of failure. Consequently, there is a significant gain in terms of structural safety by providing more use of materials in the structures.

vi) The SIMP and SESO methods have shown promise for three-dimensional structures, reducing costs and project compliance. Proving to be a robust and important tool for future structural projects developed by engineers.

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