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Reliability assessment of built-up steel beams subjected to bending

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Abstract. Although built-up cold-formed steel members are commonly used under bending loads, the structural design of such sections requires further investigation. This study proposes an assessment of the reliability of built-up cold-formed steel members subjected to bending and designed using the Direct Strength Method (DSM). A comprehensive database was assembled, comprising 87 specimens with open cross-sections and 61 with closed cross-sections, formed from singly and doubly stiffened C-sections, all exhibiting double symmetry. Based on this database, a statistical analysis of the experimental results was carried out to identify the probability distribution that best fits the model error variable, along with its corresponding parameters. In addition to model error, the statistical parameters of material properties, cross-sectional geometry, loading type, and load ratio were considered in the reliability study, and a sensitivity analysis was performed to evaluate the relative importance of these variables. Subsequently, the reliability indices of members designed according to load combinations from the North American specification (AISI S100, 2016) in the LRFD (Load and Resistance Factor Design) and LSD (Limit States Design) formats, as well as from the Brazilian standard (NBR 14762, 2010), were compared. The results showed that the LRFD load combination yielded reliability indices closer to the target value, while the LSD format presented lower indices, and the Brazilian load combination yielded intermediate performance.

Keywords: Built-up beams; DSM; FORM; Reliability; Cold-formed steel.

1 Introduction

The use of cold-formed steel profiles in structural engineering has been increasing due to their high structural efficiency, manufacturing flexibility, and speed of installation [1]. To increase the load-carrying capacity, it is common to combine individual profiles, creating built-up members. The Direct Strength Method (DSM) is the most effective approach for designing cold-formed steel members, both simple and built-up, as it considers local and distortional buckling modes [2]. Despite this, design codes such as NBR 14762 [3], which is still under public consultation and scheduled for publication in 2025, and AISI S100 [4] provide few specific guidelines for the design of built-up sections. The practice of summing the capacities of individual profiles results in an excessively conservative approach. Because of this, engineers often adopt conservative assumptions based on their experience and judgment when designing built-up members [5]. Therefore, many studies aim to adapt the DSM to account for these sections.

[3] and [4] codes utilize limit states methods, with coefficients for both resistance factor and load factors. In the USA and Mexico, load combinations follow the Load and Resistance Factor Design (LRFD), while in Canada, the Limit States Design (LSD) is used. The calibration of these coefficients is based on the reliability index (β), a fundamental safety parameter [6]. To calculate the reliability index β, methods such as First-Order Second-Moment Method (FOSM), First-Order Reliability Method (FORM), and Monte Carlo Simulation (MCS) are employed. Therefore, this study aims to verify the applicability of the Direct Strength Methodto built-up sections under bending, evaluating their structural reliability. The goal is to contribute to the development of more accurate and safe analysis and design methods for these built-up sections.

2 Theoretical nominal strength

The available flexural strength (Mn), calculated by the Direct Strength Method, should be the smallest value among the nominal flexural strength for global buckling (Mne), the nominal flexural strength for local buckling (Mnl), and the nominal flexural strength for distortional buckling (Mnd), according to Toledo et al. [7]. The [4] standard establishes the mathematical expressions for calculating these resisting moments, which take into account the different instability modes that can occur in cold-formed members subjected to bending.

3 Structural reliability

Finding the right balance between safety and economy is a major challenge in structural design. For this reason, a strong relationship between the resistance of structural elements and the loads applied to them must be established. However, structures are subject to various uncertainties throughout their service life, from the design and construction phases to their use and maintenance. According to Holický [8], many of these uncertainties are inherent to the structural system and cannot be completely eliminated, so they must be explicitly considered during the design and verification stages.

Structural safety can be assessed using the reliability index (β), which indirectly represents the probability that the structure will perform its function satisfactorily without reaching an undesired limit state. A higher value of β indicates a lower probability of failure and, consequently, greater structural reliability. The reliability index β is a unified measure widely used in international design codes and recommendations based on reliability.

The exact calculation of β can be complex in real-world scenarios due to the large number of variables involved and the non-linear nature of the problem. For this reason, approximate methods are usually employed for its determination. Methods such as First-Order Second-Moment Method (FOSM), First-Order Reliability Method (FORM), and Monte Carlo Simulation (MCS) are employed. These are fundamental tools for the probabilistic analysis of structural safety, allowing for the quantification of reliability in a rational and consistent manner.

4 Methodology

In this research, the database from Pereira [9] was used, containing information on the experimental resisting moment and the geometric and mechanical characteristics of the prototypes. The author's selection criteria prioritized studies with variability in the specimen measurements and that addressed built-up sections made from unlipped channel, lipped channel, and doubly symmetric profiles. In addition, the data had to be sufficient to model the sections in the CUFSM v5.01 [10] software and calculate the theoretical resisting moment. Following this selection, the final database was composed of 9 works, totaling 148 specimens of beams under bending. Of these, 87 were open sections and 61 were closed sections.

The Model error (P) was treated as a random variable and analyzed in six groups: all sections (All), open sections (Open), open sections that failed by local mode (O-Local), by distortional mode (O-Dist) and by global mode (O-Global), in addition to the closed section specimen that failed by local buckling mode (C-Local). Using the software MINITAB 19 [11], the most appropriate probability distribution for the data was selected using the Anderson-Darling test, considering five probability density functions (Normal, Lognormal, Gumbel, Weibull and Gamma). Using Microsoft Excel [12], FOSM, FORM and MCS methods were implemented to calculate the reliability indices β and the probability of failure P_f. In the FORM method, the convergence criterion based on consecutive reliability indices was used [13]. With the applied failure function, the method generally converges in 5 iterations, reaching a tolerance of 10^-5. MCS performed 500,000 simulations and its input data is described in Tables 1 and 2.

Table 1: Input data for the load combinations

Standard	ϕ (γ)	γ_D	γ_L	ρ	β_0
AISI S100 (LRFD)	0.90 (1.11)	1.20	1.60	5.0	2.5
AISI S100 (LSD)	0.90 (1.11)	1.25	1.50	3.0	3.0
NBR 14762	0.91 (1.10)	1.35	1.50	5.0	2.5

Table 2. Statistical data for the variables

Variable	M_{LD}	M_G	F	D_{50}	L_{50}
μ	1.10	1.00	1.00	$1.05D_n$	$1.00L_n$
σ	0.11	0.06	0.05	$0.10D_n$	$0.25L_n$
δ	0.10	0.06	0.05	0.10	0.25
PDF	Lognormal	Lognormal	Lognormal	Normal	Gumbel

Table 1 presents the resistance factor ϕ, load factor γ, and the ratio $\rho = L_n/D_n$ — where L_n and D_n are the nominal values of live and dead loads, respectively — as well as the target reliability indices β_0 for each load combination, according to the specification in [4] for LRFD and LSD, and the updated Brazilian code [3]. Table 2 presents the mean (μ); standard deviation (σ); coefficient of variation (δ) and probability density function (PDF) to the uncertainties of the material M, called the "material factor", and geometric property variability F, called the "fabrication factor", both suggested by Ellingwood *et al*. [14] for the situation studied. For the loads, D and L represent the dead and 50-year maximum live load variables, respectively, as proposed by Galambos *et al*. [15].

Equation 1 shows the design equation provided by the specification, where R_n represents the nominal strength. Assuming that, at the limit, the resistance is equal to the design load in equation 1, the nominal loads can be rewritten as a function of the nominal resistance and the defined ρ ratio, according to Equations 2 and 3. Equation 4 shows the proposed failure function based on the usual limit state safety conditions, considering only dead and live load variables and leaving the strength as a function of the coefficients P, M and F.

$$\phi R_n \geq \gamma_D D_n + \gamma_L L_n. \tag{1}$$

$$D_n = \frac{\phi R_n}{(\gamma_D + \rho\gamma_L)}. \tag{2}$$

$$L_n = \frac{\phi R_n}{\left(\frac{\gamma_D}{\rho} + \gamma_L\right)}. \tag{3}$$

$$G(.) = R_n \mathrm{PMF} - (\mathrm{D} + \mathrm{L}). \tag{4}$$

5 Results

5.1 Model error

Table 3 shows a statistical summary, with quantity (Nr); mean (μ); standard deviation (σ) and coefficient of variation (δ) of the P values found and the PDF adjusted for each grouping in the database. In general, the average P values are close to 1.0 with a maximum variation of ±0.04. The exception is the "O -Global" group and the other groups have low δ values, which indicates low dispersion in the results. This may contribute to lower probability of failure values, depending on the PDF, and indicates that DSM is a good fit for this database.

Table 3: Statistics of the model error for the data groups

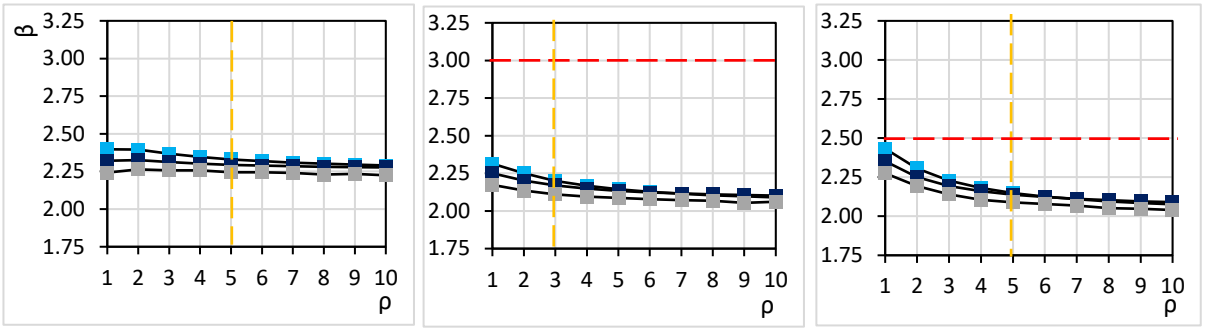
Data Group	Nr	μ	σ	δ	PDF
All	148	1.018	0.133	0.131	Normal
Open	87	1.036	0.153	0.148	Weibull
O-Local	23	1.034	0.143	0.138	Weibull
O-Dist.	55	1.023	0.115	0.113	Weibull
O-Global	9	1.204	0.293	0.244	Normal
C-Local	54	0.983	0.092	0.093	Lognormal

5.2 Reliability analysis

Figures 1 to 6 graphically depict the behavior of the reliability index (β) as a function of the variation in the load ratio (ρ) for each data group studied. The analysis covers the LRFD, LSD, and NBR load combinations and uses FOSM (light blue markers), FORM (dark blue markers), and MCS (gray markers) reliability methods. The

yellow dashed lines in the graphs represent the load ratio (ρ) used for the calibration of each load combination, while the red dashed lines indicate the target reliability indices (β_o) for each design philosophy.

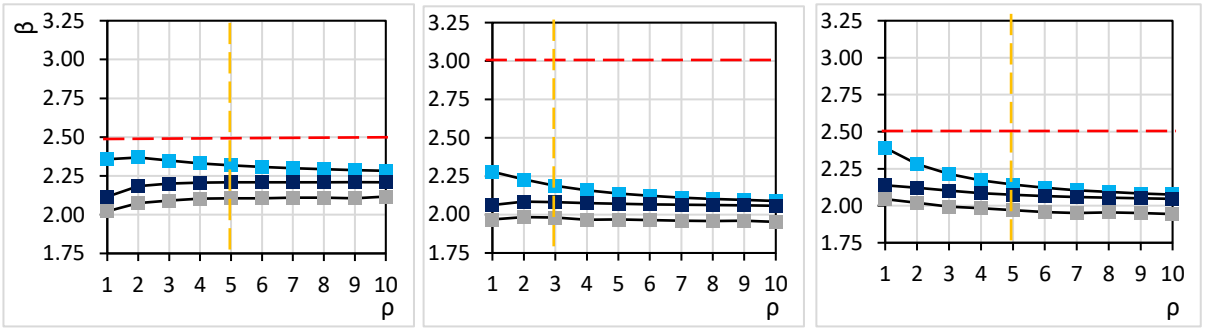
Regarding Figure 1, for the "All" data group, the β values decrease as ρ increases, with this behavior being more pronounced in the updated NBR combination. None of the load combinations achieved their target reliability index (β_o), although the LRFD combination is the closest to the target at the adopted ρ ratio.



a) LRFD combination b) LSD combination c) NBR combination

Figure 1 - β as a function of ρ for the "All" group”

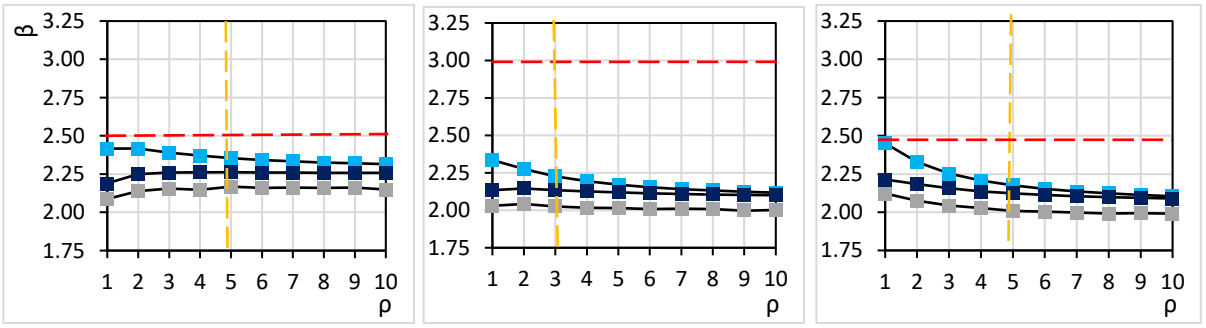
In Figure 2, FOSM method shows a different behavior from FORM and MCS methods, which highlights the precision of these last two methods in compare to FOSM. Also, for the "Open" group, none of the load combinations reached their β_o, even though the LRFD and the updated NBR had values closer to 2.5 in the initial load ratios.



a) LRFD combination b) LSD combination c) NBR combination

Figure 2 - β as a function of ρ for the "Open" group

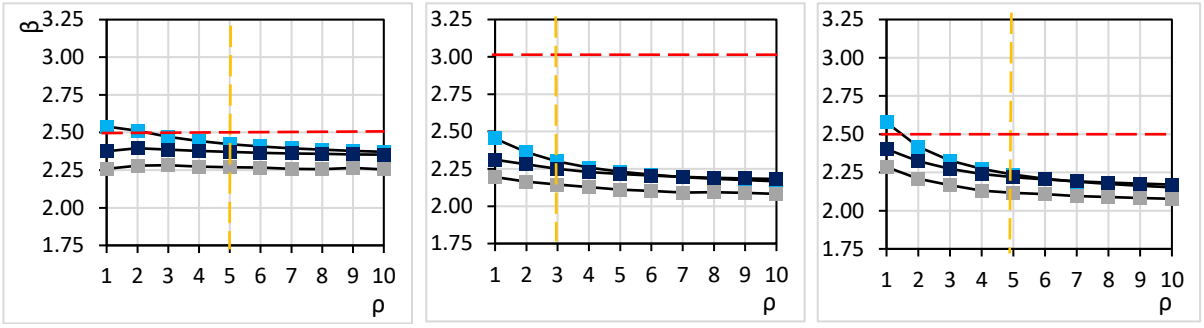
In Figure 3, the "O-Local" group shows that the LRFD and the updated NBR load combinations achieved β values very close to their β_o using the FOSM method in the initial load ratios. However, at the adopted load ratio, the LRFD combination stood out among the three reliability methods, outperforming the others.



a) LRFD combination b) LSD combination c) NBR combination

Figure 3 - β as a function of ρ for the "O-Local" group

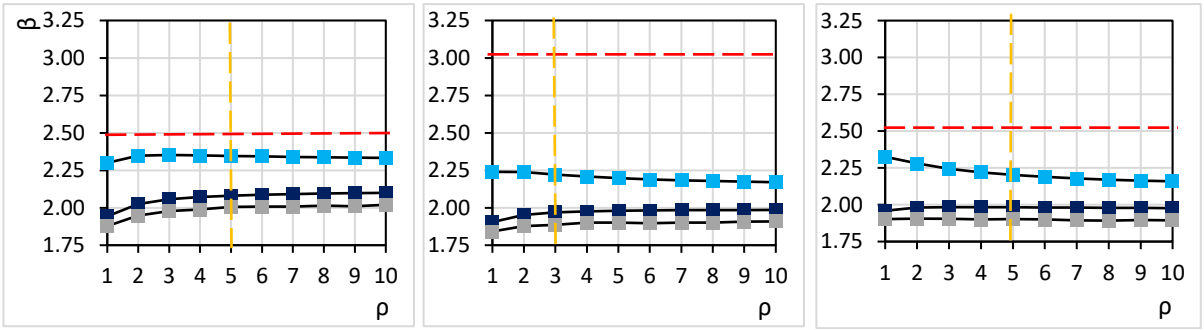
For the "O-Dist" group, Figure 4 shows that, using FOSM method, the LRFD load combination exceeded the β_o at ρ=1 and ρ=2, while the updated NBR exceeded it at ρ=1. This suggests that, for smaller load ratios, these design philosophies could be considered adequate. However, FORM and MCS methods, considered more conservative, did not reach the β_o, indicating that the combination may not be safe for this design configuration. Despite this, the β values at the adopted ρ for this group are very close to the target.



a) LRFD combination b) LSD combination c) NBR combination

Figure 4 - β as a function of ρ for the "O-Dist" group

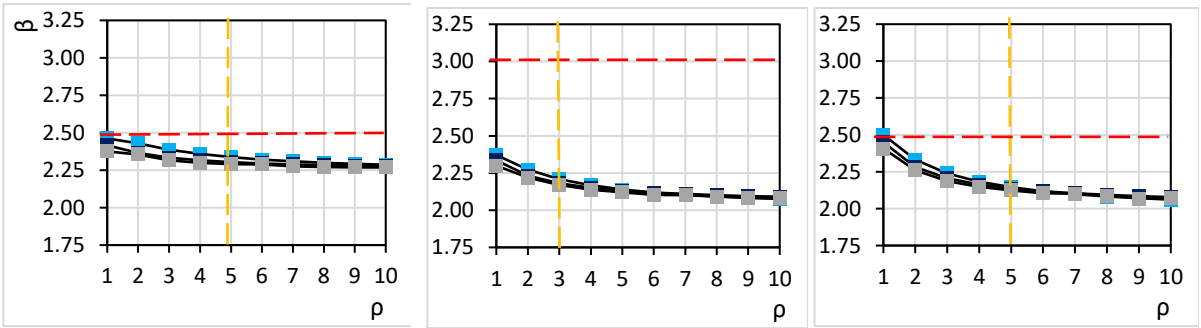
In the "O-Global" data group, the β values of FOSM method differed considerably from the other two, as shown in Figure 5. Furthermore, even with higher β values, FOSM method results did not reach their target in any design philosophy, with FORM and MCS methods generally showing the lowest β values of all data groups. This can be explained by the low number of experimental data for this group, which reduces the precision of the results.



a) LRFD combination b) LSD combination c) NBR combination

Figure 5 - β as a function of ρ for the "O-Global" group

Finally, in Figure 6, for the "C-Local" group, the reliability methodologies presented β values closer to one another in the variation of the load ratio. Only the β calculated by FOSM method for the first ρ ratio of the updated NBR reached its β_o; however, this load ratio is unfeasible for structures with built-up cold-formed steel members.



a) LRFD combination b) LSD combination c) NBR combination

Figure 6 - β as a function of ρ for the "C-Local" group

5.3 Importance Factor

Figure 7 illustrates the importance factors (α) as a function of the load ratio ρ for each of the 5 variables involved in the reliability problem for the six data groups studied. For this analysis, only the load combination that presented the best β results was considered, e.g., LRFD.

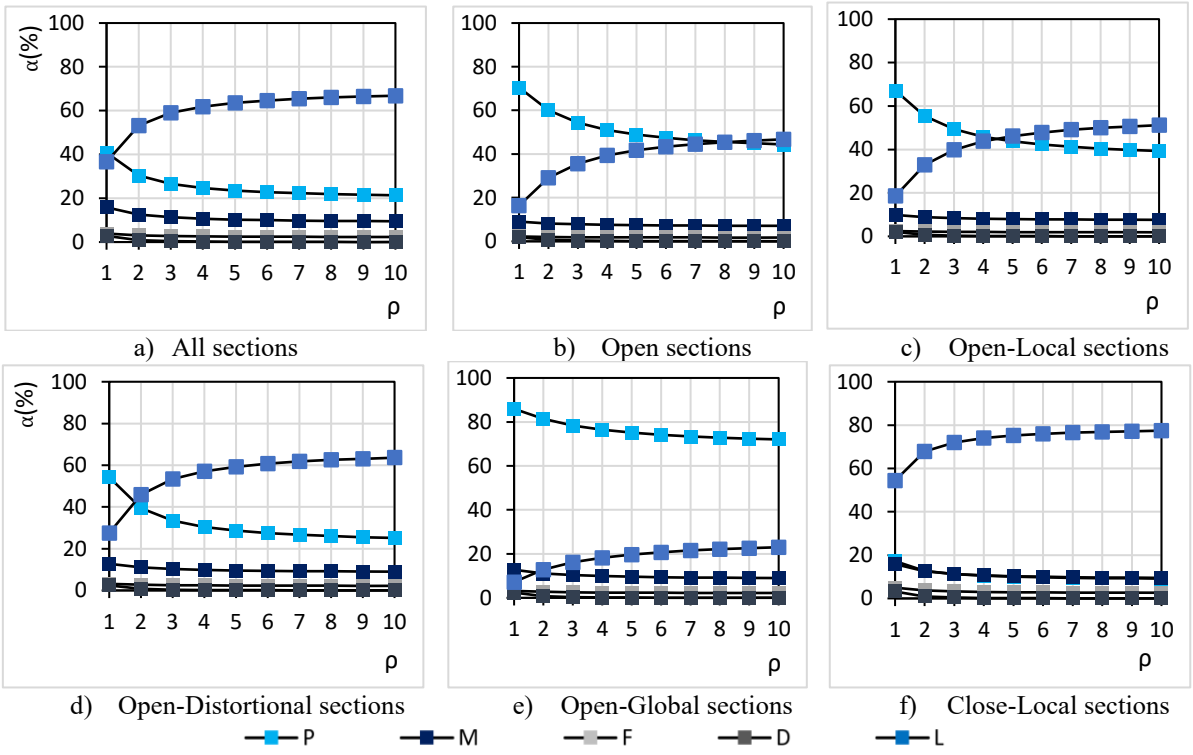


Figure 7 – Importance factor for all groups considering the LRFD combination

The results of the sensitivity analysis, shown in Figure 7, reveal that variables P and L are the two main factors affecting the probability of failure in most cases. The graphical behavior of the first 4 composite groups, mainly composed of open sections, was similar throughout the load ratios. At the initial values of ρ, variable P predominates, then finds a balance with variable L, and only then does the latter surpass the influence of P. In the "O-Global" and "C-Local" groups, variables P and L were the most influential individually, respectively. Considering the sensitivity of the mean value of P to the reliability indices, the variability of P should be a priority in structural design.

Furthermore, it is possible to notice the influence of the δ values on the α results of variable P at the load ratio adopted by the LRFD load combination. In the "C-Local" group, δ was the smallest of all groups, and therefore, the influence of this variable was small, around 10%. In the "All" and "O-Dist." groups, the influence of P was around 25%, where the δ values are slightly larger. Ultimately, in the "O-Local" and "Open" groups, where the δ values are larger, the influence of P is also greater, with a percentage around 50%.

6 Conclusions

This study investigated the structural behavior and reliability of built-up cold-formed steel beams. The main focus was to evaluate the Direct Strength Method (DSM) and the performance of different load combinations. For this, the database from Pereira (2025) was used to compare the nominal moments with the experimental moments through the variable P. From there, the reliability index β values were calculated using FOSM, FORM, and MCS methods. Thus, the main conclusions were: the DSM model fit this design configuration well; the FORM method was more precise than the FOSM, approaching the results of the MCS; the "A-Global" group presented β values further from the target, possibly due to the limited number of data. The database for the global mode of open sections should, therefore, be expanded for a more precise evaluation; none of the load combinations reached the

β_o using the FORM and MCS methods, however, the LRFD combination was the closest to the target for built-up sections, while the LSD was the furthest, which corroborates results found in the literature.

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