XLVI CILAMCE
VITÓRIA - ES - BRAZIL - 2025
XLVI IBERO-LATIN AMERICAN CONGRESS ON COMPUTATIONAL METHODS IN ENGENEERING

Nonlinear Analysis of the Von Mises Truss and Its Application in Assistive Rehabilitation Technologies

Felipe Manoel Wandscheer[1], Aref Kalilo Lima Kzam[1]

[1]*Latin American Institute of Technology, Infrastructure and Territory (ILATIT), Federal University of Latin American Integration (UNILA)*
Av. Tancredo Neves, 6731 - Block 6 – Itaipu Parquetec, P.O. Box 2044, Foz do Iguaçu/PR, Brazil
fm.wandscheer.2021@aluno.unila.edu.br, aref.kzam@unila.edu.br

Abstract. This work investigates the nonlinear behavior of the Von Mises truss and its applicability in adaptive biomechanical systems, focusing on the development of assistive devices for motor rehabilitation. The Von Mises truss, composed of two articulated bars subjected to large displacements, is a paradigmatic model for studying structural instabilities such as snap-through, with its response strongly influenced by geometric nonlinearity associated with angular variation of the bars. In this study, the truss will be modeled using the Finite Element Method (FEM) with a geometrically nonlinear positional formulation, adopting the Saint Venant–Kirchhoff constitutive law to describe the material behavior. The modeling will allow for a more accurate assessment of the system's response under different loading conditions, identifying critical transition points and stability regimes. Based on the theoretical and numerical results, the integration of the truss into variable compliance mechanisms in robotic exoskeletons is proposed. Experimental tests confirmed the system's effectiveness in assistive applications, highlighting its ability to store and release elastic energy efficiently. The results demonstrate the potential of the Von Mises truss as a key component in adaptive robotic devices for physical rehabilitation.

Keywords: Von Mises truss, Geometric nonlinearity, Finite element method, Exoskeletons, Assistive technologies

1 Introduction

The study of exoskeletons in the field of human rehabilitation is of undeniable importance, ranging from the recovery of patients affected by trauma [1], to the rehabilitation of individuals with paraplegia [2]. They are also used to mitigate the effects of microgravity—such as bone density loss and muscle atrophy [3]—in astronauts. In the field of military engineering, they are employed with a focus on enhancing the physical performance and endurance of soldiers in combat [4]. Thus, exoskeletons emerge as an innovative solution to contemporary challenges at the forefront of technology.

Although there have been advances in this field, the high energy consumption of the mechanisms and the discomfort experienced by users with the prostheses remain obstacles to be overcome for the adoption of exoskeletons. Furthermore, according to [5], the incorporation of a structural system similar to a Von Mises truss reduces energy expenditure and improves ergonomics. However, as more complex adaptations or new topologies emerge, geometric nonlinearity becomes essential in the analysis of these structures.

Therefore, it is imperative to adopt rigorous numerical methods during the design phase. In the present work, a generalized formulation is presented based on the geometric nonlinear analysis of the Von Mises truss, in parallel with the Positional Finite Element Method [6], which consists of using nodal positions as the degrees of freedom of the discrete system. In this formulation, the Saint Venant–Kirchhoff constitutive law will be used to describe the material behavior. Furthermore, the approach was tested and validated through direct comparison with the analytical solution, demonstrating its consistency and accuracy.

2 Structural Analysis of the Generalized Von Mises Truss

In the field of structural analysis, the Von Mises Truss is widely studied, recognized for its simple configuration, yet of remarkable richness in the investigation of structural stability. Classically, this truss consists of two articulated bars fixed on second-order supports. However, in this work, the approach is extended to a generalized

system that transcends the traditional symmetric configuration, allowing two degrees of freedom at the central node (horizontal and vertical), interpretable as the components of an oblique force (Fig.1). Given the nature of large deformations of the system, the Green-Lagrange formulation was employed to accurately describe the deformation field. Additionally, the constituent bars are modeled as elastic elements, under the crucial assumption that their Eulerian critical loads are high enough that the individual elements never buckle.

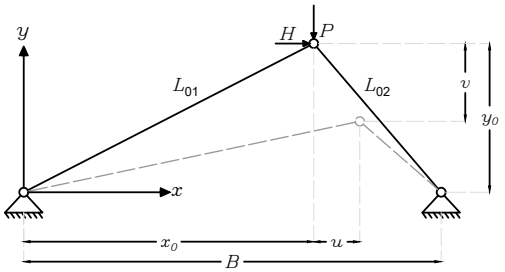


Figure 1. Generalized Von Mises truss in its initial and displaced configuration.

In Figure 1, L_{01} and L_{02} denote the initial lengths of the bars; x_0 and y_0 are the initial coordinates of the central node; u and v represent the horizontal and vertical displacements of the central node, respectively; P and H are the vertical and horizontal components of the force applied at the node; and B is the total span of the truss.

Based on the aforementioned assumptions, the Green-Lagrange strain for bar i can be expressed as:

$$\varepsilon_i = \frac{1}{2}\left(\frac{L_i^2 - L_{0i}^2}{L_{0i}^2}\right), \tag{1}$$

where ε_i is the strain of bar i, and L_i is its length in the deformed configuration. By calculating the strains of each bar using the lengths derived from Fig.1, the total potential energy of deformation of the system can be expressed as:

$$\Pi(u,v) = \frac{1}{8}EA\left[\frac{\left(u^2 + 2x_0u + v^2 - 2y_0v\right)^2}{\left(x_0^2 + y_0^2\right)^{3/2}} + \frac{\left(v^2 - 2y_0v - 2Bu + 2x_0u\right)^2}{\left(y_0^2 + (B - x_0)^2\right)^{3/2}}\right] - Pv - Hu. \tag{2}$$

Here, E denotes the Young's modulus, A the cross-sectional area. According to [6], for the system to be in equilibrium, the gradient of the total potential energy with respect to the degrees of freedom must be null. Therefore, from the nullification of the gradient vector, and considering the vertical force component for the scope of this work, one obtains:

$$P(u,v) = \frac{1}{2}EA\,(v - y_0)\left[\frac{u^2 + 2x_0u + v^2 - 2y_0v}{\left(x_0^2 + y_0^2\right)^{\frac{3}{2}}} + \frac{v^2 - 2y_0v - 2Bu + 2x_0u}{\left(y_0^2 + (B - x_0)^2\right)^{\frac{3}{2}}}\right]. \tag{3}$$

3 Positional Finite Element Method

3.1 Formulation of the geometrically nonlinear positional finite element method for truss structures [7]

For the development of the formulation, the Saint-Venant–Kirchhoff constitutive model was employed, together with the Green–Lagrange strain measures.

Thus, the internal force vector, obtained as the gradient of the strain energy with respect to the nodal coordinates, can be expressed as

$$F_{k\ell}^{\text{int}} = \frac{1}{2}\frac{EA}{L^0}\frac{\partial \Delta}{\partial y_{k\ell}}, \tag{4}$$

where

$$\frac{\partial \Delta}{\partial y_{k\ell}} = 2(-1)^{\ell}\,(y_{k2} - y_{k1})\,,$$

with $k = 1, 2, 3$ and $\ell = 1, 2$. Here, L^0 the initial length of the element, and $y_{k\ell}$ the ℓ-th Cartesian coordinate of node k.

The Hessian matrix, obtained as the gradient of the internal force vector, can then be written as

$$H_{mnk\ell} = \frac{EA}{L^0}\left[\frac{1}{4}\left(\frac{1}{L^0}\right)^2 \frac{\partial \Delta}{\partial y_{mn}}\frac{\partial \Delta}{\partial y_{k\ell}} + \frac{1}{2}\frac{\partial^2 \Delta}{\partial y_{k\ell}\partial y_{mn}}\right], \tag{5}$$

where

$$\frac{\partial \Delta}{\partial y_{mn}} = 2(-1)^{n}\,(y_{m2} - y_{m1})\,,$$

$$\frac{\partial^2 \Delta}{\partial y_{k\ell}\partial y_{mn}} = 2(-1)^{\ell}(-1)^{n}\delta_{km},$$

with $m = 1, 2, 3$, $n = 1, 2$, and δ_{km} being the Kronecker delta.

3.2 Validation of the positional finite element formulation

For the verification of the positional finite element method for truss structures, the percentage errors of the FEM are evaluated with respect to the analytical vertical force obtained in eq. (3). Accordingly, Figure 2 presents the graph comparing the analytical vertical force with the FEM results. In this graph, the red points correspond to displacement-controlled solutions, which are plotted up to $v = 2$ so as to illustrate the capability of generating intermediate points between the local extrema of the P curve, whereas the green points represent the force-controlled solutions. The discontinuity of the green points clearly indicates that the FEM is capable of detecting when the system becomes unstable. Moreover, the graph also displays an illustrative configuration of the system at the local maximum and minimum points of the P force. The maximum percentage error found between the FEM results and the analytical vertical force curve was $1.0053 \times 10^{-6}\%$, thus confirming both the accuracy and precision of the FEM.

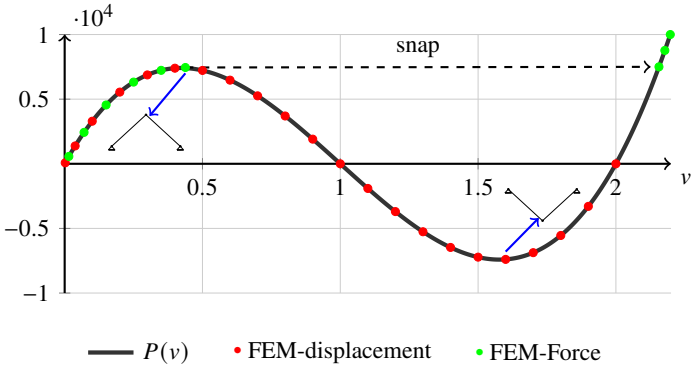


Figure 2. Vertical load $P(v)$ (black), FEM-displacement (red), and FEM-Force (green).

4 The exoskeleton and the Von Mises truss

A generalized Von Mises truss prototype was developed in the laboratory using unplasticized polyvinyl chloride (PVC-U). In order to ensure a balance between lightness and structural stiffness, the members were manufactured from hollow PVC-U bars, each with an internal diameter of 20 mm. Once assembled, the prototype was subjected to an experimental evaluation that focused on analyzing its positions throughout the sit-to-stand

motion, while simultaneously observing the corresponding positions of the lower limb joints. For this purpose, the *Tracker* software was employed, allowing precise monitoring of selected anatomical and structural reference points. More specifically, the points of interest included: Point A, located at the greater trochanter region (upper part of the femur); the knee joint; the central node of the Von Mises truss; and the origin point $(0, 0)$, corresponding to the ankle. The sequential configurations of the prototype, comprising the initial, intermediate, and final frames of the sit-to-stand movement, are presented in Figure 3, thereby providing a visual reference for the positional analysis.

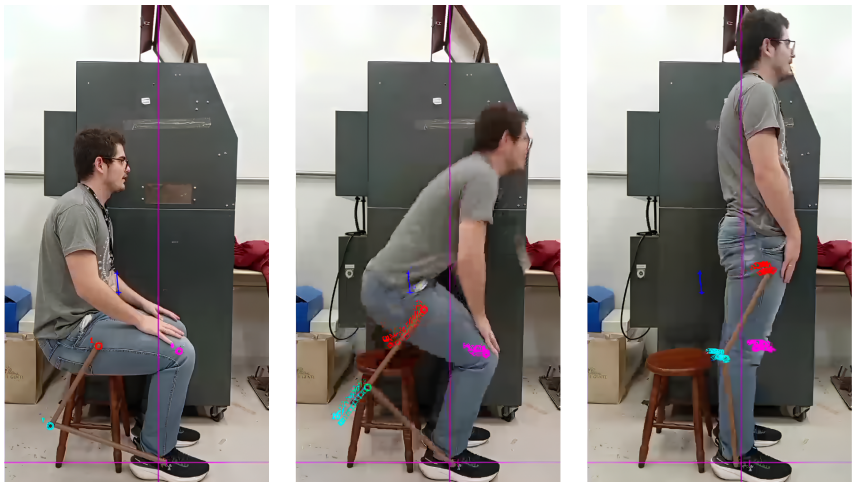


Figure 3. Initial, intermediate, and final positions of the prototype during the sit-to-stand movement (images from Tracker software).

To further investigate the kinematic behavior during the sit-to-stand movement, three experimental attempts were conducted with a participant measuring 1.87 m in height and weighing 95 kg. The trajectories of Point A, corresponding to the greater trochanter region, were tracked across these attempts, capturing the displacement in the x-y plane. The *Tracker* software facilitated precise monitoring of these positions, enabling a detailed analysis of the prototype's performance in replicating lower limb motion. Figure 4 presents the positional data for each attempt, represented by blue dots (Attempt 1), red squares (Attempt 2), and green triangles (Attempt 3). A linear regression was performed from -0.2 to 0, with the resulting slope corresponding to an angle of $51.22\,^\circ$. These results highlight the consistency of the prototype's kinematic response across multiple trials.

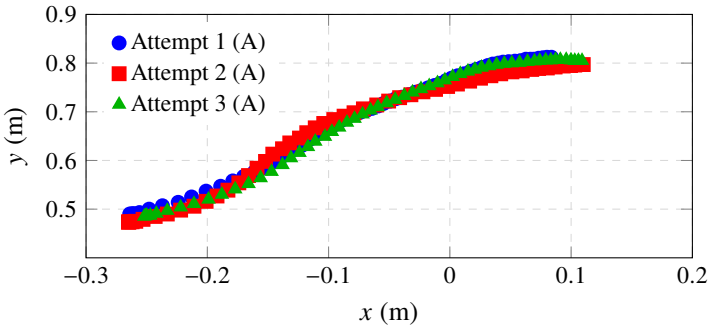


Figure 4. Position y vs. x for Point A (greater trochanter) — Attempts 1 (blue dot), 2 (red square), and 3 (green triangle).

In Fig. 5, the equilibrium path of the central node of the Von Mises truss is presented, considering a cross-sectional area of the bars equal to 94.2 mm^2 and a Young's modulus $E = 3.14$ GPa. The loading conditions consisted of applying horizontal forces of 599.02 N and a vertical force of 745.56 N at the central node. The initial geometry of the generalized Von Mises truss was defined by the coordinates $A(-0.26,\ 0.483)$, representing the femur, $B(-0.498,\ 0.186)$, representing the knee, and $C(0,\ 0)$, representing the ankle. It can be observed in the figure that the displacement of the central node follows the force orientation at an angle of $51.22°$, obtained from the previously mentioned linear regression line. In addition, the deformed configurations of the truss are displayed along the loading process, which was divided into 100 incremental steps to obtain the complete equilibrium path.

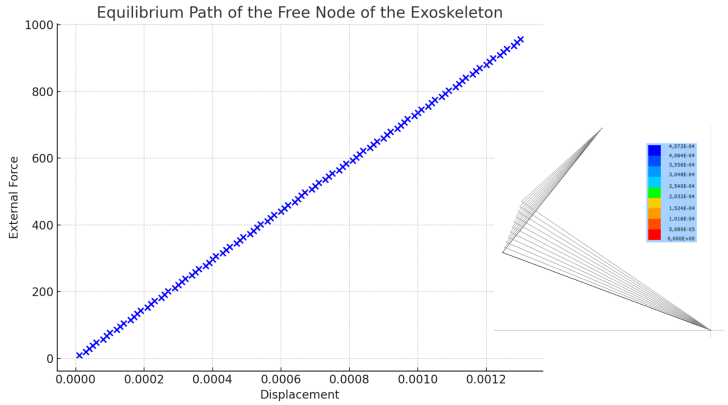


Figure 5. Equilibrium path of the central node of the Von Mises truss under the specified geometric and loading parameters.

5 conclusions

This study demonstrated the effectiveness of the Von Mises truss as a key structural component in assistive rehabilitation technologies. Using a geometrically nonlinear positional formulation within the Finite Element Method, the effects of large displacements were accurately captured and validated against analytical results. Experimental tests with a functional prototype confirmed the truss's ability to efficiently store and release elastic energy during movements such as sit-to-stand transitions. The measured reaction forces — 597.81 N (horizontal) and 746.01 N (vertical) — correspond to the necessary support to sustain the assisted person's body, underscoring the truss's active role in rehabilitation biomechanics. Integrating this structure into exoskeletons offers a promising pathway for more ergonomic and energy-efficient biomechanical devices.

Acknowledgements. The authors gratefully acknowledge the Federal University of Latin American Integration (UNILA) for providing the infrastructure and resources that made this research possible.

Authorship statement. The authors confirm they are solely responsible for the authorship of this work, and that all included material is either their own or properly authorized.

References

[1] Z. Kadivar, C. E. Beck, R. N. Rovekamp, M. K. O'Malley, and C. A. Joyce. On the efficacy of isolating shoulder and elbow movements with a soft, portable, and wearable robotic device. *Wearable Robotics: Challenges and Trends*, vol. 1, n. 1, pp. 89–93, 2016.

[2] A. R. C. Donati, S. Shokur, E. Morya, D. S. Campos, R. C. Moioli, C. M. Gitti, and M. A. L. Nicolelis. Long-term training with a brain-machine interface-based gait protocol induces partial neurological recovery in paraplegic patients. *Scientific Reports*, vol. 6, n. 1, pp. 30383, 2016.

[3] R. Rea, C. Beck, R. Rovekamp, P. Neuhaus, and M. Diftler. X1: A robotic exoskeleton for in-space countermeasures and dynamometry. In *AIAA Space 2013 Conference and Exposition*, pp. 5510, 2013.

[4] C. Applegate. Potential applications of exoskeleton suits in the armed forces, 2022.

[5] D. Dragone, L. Randazzini, A. Capace, F. Nesci, C. Cosentino, F. Amato, and A. Merola. Design, computational modelling and experimental characterization of bistable hybrid soft actuators for a controllable-compliance joint of an exoskeleton rehabilitation robot. *Actuators*, vol. 11, n. 2, pp. 32, 2022.

[6] H. B. Coda. *O método dos elementos finitos posicional: sólidos e estruturas - Não linearidade geométrica e dinâmica*. EESC-USP, São Carlos, São Paulo, Brazil, 2018.

[7] A. K. L. Kzam. Determinação de pontos críticos na trajetória de equilíbrio não linear do elemento finito de treliça posicional. *Revista da Estrutura de Aço–REA*, vol. 9, n. 1, pp. 1–20, 2020.