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Study of the Effect of Pressurization on the Vibration Frequencies of "Aircraft Hoop Frames"

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Abstract. In this article, structures referred to as "hoop frames" are modeled. Their physical and behavioral characteristics are akin to closed planar frames. These structures are generally composed of high-strength metal alloy elements. Such systems are part of the structural framework responsible for supporting the fuselage of commercial aircraft, where pressurization loads act directly. These loads induce high axial tensile stresses in the structural elements, altering the geometric stiffness of the system and, consequently, its vibration frequencies. By employing a program based on the Finite Element Method, numerical models composed of bar elements are developed and subjected to various levels of loading due to pressurization. The objective of the study is to evaluate the influence of geometric stiffness on the vibration frequencies of these structures. For a more accurate representation, the structures will also be modeled without external constraints, so as to capture their dynamic behavior while adhering to standard conditions.

Keywords: geometric stiffness, finite element method, vibration frequency, dynamics analysis.

1 Introduction

At present, commercial aircraft play a vital role in global mobility, allowing vast distances to be traversed in relatively short periods of time. Although flight may appear to be a straightforward matter from a physical standpoint — requiring merely the application of acceleration to a body such that its resultant velocity is sufficient to "overcome" the force of gravity — in practice, a host of additional factors, including structural ones, influence and enable the phenomenon. Among these, the article will focus particularly on those related to the structural composition of the aircraft, more specifically the elements responsible for supporting the fuselage, known as the hoop frames.

In general, aerial vehicles are subjected to various mechanical forces during flight, which act directly upon their components. Among them, the wings, for instance, play a fundamental role in lift, stability, and control of the aircraft, while the fuselage — the main body of the airplane — is responsible for housing both passengers and crew. These, along with other components, together form an aerodynamic body capable of sustaining itself in the air.

Structurally, the fuselage is composed of thin-shell elements, which form the outer skin and are capable of absorbing bending moments and normal stresses when subjected to loading. These shell components are supported by internal structures similar to flat frames, formed by prismatic bar elements capable of withstanding axial, bending, and shear forces, commonly referred to as hoop frames.

The fuselage of an aircraft is constantly subjected to forces arising from external — atmospheric and aerodynamic — and internal pressures. Atmospheric and aerodynamic pressures, basically, are generated by the

surrounding atmosphere and the action of airflow against the aircraft's exterior. Internal pressures, in turn, stem from the aircraft's pressurization system. These pressures also play a vital role in the aircraft's structural equilibrium, as they are crucial for achieving a balance between external and internal pressures, thereby increasing the overall stiffness of the airframe and enabling it to maintain its shape while in motion.

Internal pressurization also grants significant benefits to the structural system, as the tensile forces and stresses it induces can positively influence the stiffness of structural elements, enhancing both their stability and resistance. This indeed represents a valuable gain in terms of structural efficiency, considering that the phenomenon of flight is directly linked to dynamic theory, which is closely related to the stiffness and mass of structural components.

Research studies, such as those conducted by Htet [1] and Pagani et al. [2], explore the optimization of structure stiffness in order to improve the dynamic response under critical conditions. Therefore, stiffness analysis is crucial to preventing structural failures, mitigating undesirable vibrations, and extending the operational lifespan of the aircraft. Understanding how these factors influence structural response is indispensable in the design of efficient aerospace vehicles. Within this context, scientific investigation provides valuable insight into the phenomena involved and must be taken into account.

2 Finite Element Method - FEM

The models developed in this study were created based on the Finite Element Method (FEM), one of the most widely used techniques for solving engineering problems, and the foundation for most commercial structural analysis software. FEM can be understood as a technique that transforms elastic bodies into structures composed of smaller elements, connected by nodes. This process is known as discretization and is applicable to two-, three-, and one-dimensional structures.

By considering the boundary conditions and characteristics of each element, it is possible to formulate equations of motion, that yield a system of differential equations capable of describing the behavior of the structure. The solution to this problem, typically obtained numerically, results in vibration modes, natural frequencies, and dynamic responses, important data used in the analysis, assessment, and design of structures.

The elements used in the model are one-dimensional bars, each with two nodes and three unconstrained degrees of freedom per node — two translational and one rotational, amounting to a total of six. Since this is a flat 2D analysis, the other degrees of freedom, out of the plane, are constrained. In this configuration, the bars are capable of exhibiting forces in terms of axial — tensile and compressive stresses — shear, and bending moments, according to the degrees of freedom as shown in Figure 1.

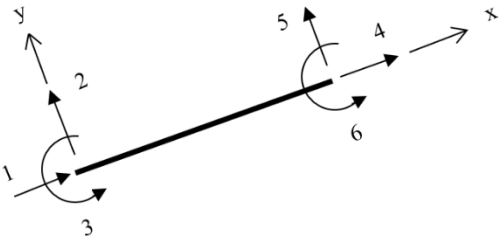


Figure 1 - Flat frame bar

Another important factor in FEM is the type of material model to be used, as it will not only determine the behavior of the structure but also the type of analysis to be employed. For the case study in this work, it is assumed that the materials will behave within the linear elastic region, represented by the inclined line, as shown in the model of Figure 2,

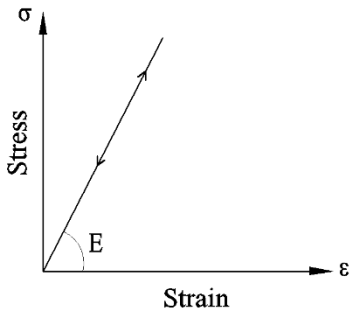


Figure 2 – Constitutive model

where σ represents stresses, ε strain, and E is the modulus of elasticity of the material. Therefore, the analysis to be employed for the problem in question is nonlinear geometric elastic.

With regard to the dynamic evaluation, a modal analysis is of particular interest, as it is responsible for providing the vibration modes of the structure and its natural frequencies. Unlike the linear geometric case, where the modes are obtained without considering applied loads, the models developed here take into account pre-existing loads, arising from the internal pressurization effect of aircraft. This process is characterized by a non-linear geometric analysis, as the effects of the applied load alter the stiffness of the elements, causing the relationship between load and displacement to become non-linear. In this context, the effect of the applied load on the vibration modes and natural frequencies resulting from the geometric non-linearity is particularly noteworthy (Wahrhaftig et al. [3-4]).

Natural frequencies and vibration modes are intrinsic properties of structures, and can be either damped or undamped. For the present case study, particular interest lies in the undamped free vibrations, whose equation of motion is given by,

$$[M]\{\ddot{u}\} + [K]\{u\} = 0 \tag{1}$$

where $[M]$ is the mass matrix, $\{\ddot{u}\}$ the matrix containing the acceleration vectors, $[K]$ the stiffness matrix of the structure obtained from the stiffness of each element, and $\{u\}$ is the matrix containing the displacement vectors.

Considering the effect of the applied load on the structure, the matrix $[K]$, which in the linear case depends only on the physical and mechanical properties of the elements, now becomes composed of two other matrices, such that $[K] = [K_0 + K_G]$. Rewriting equation (1), we obtain

$$[M]\{\ddot{u}\} + [K_0 + K_G]\{u\} = 0, \tag{2}$$

where $[K_0]$ is the constitutive stiffness matrix, dependent on the physical and mechanical characteristics of the elements, and $[K_G]$ is the geometric stiffness matrix, which varies according to the level of the applied load.

Equation (1) is an ODE (Ordinary Differential Equation), whose solution is the following trigonometric equation,

$$\boldsymbol{u} = \hat{\boldsymbol{u}}\cos(\omega t - \theta). \tag{3}$$

where $\hat{\boldsymbol{u}}$ is the vibration amplitude, and the solutions represent forms known as undamped vibration modes, where all coordinates of the system vary harmonically over time, at the same frequency — referred to as the undamped free vibration frequencies — and all in phase.

By differentiating (3) twice with respect to time, substituting into the equation of motion (2), and canceling the harmonic function, we obtain the following system of homogeneous algebraic equations,

$$([K_0 + K_G] - \omega^2[M])\hat{u} = 0. \tag{4}$$

For non-trivial solutions to exist, the determinant of the term in parentheses must be zero, leading to

$$det([K_0 + K_G] - \omega^2[M]) = 0, \tag{5}$$

which results in a polynomial equation of degree n in the variable ω^2, known as the frequency equation. The n solutions for ω_i, in this case, are real and positive, representing the natural frequencies of the system, expressed in *rad/s*. It is important to note that it is from $[K_G]$ that the effect of geometric non-linearity is incorporated into the system.

3 Case Study

With the aim of providing a practical application of the formulations presented, a study was developed to assess the behavior of flat frames — hoop frames — similar to those forming the fuselage of aircraft. The numerical models are composed of bar elements, connected by rigid nodes, as previously described, developed using SAP2000 software. Analyses were conducted to evaluate the natural vibration modes considering the effects of geometric non-linearity, resulting from the application of an external load to the structure. Figure 3 illustrates the dimensions of the modeled frame, as well as the longitudinal spacing between each one, from the perspective of the aircraft body.

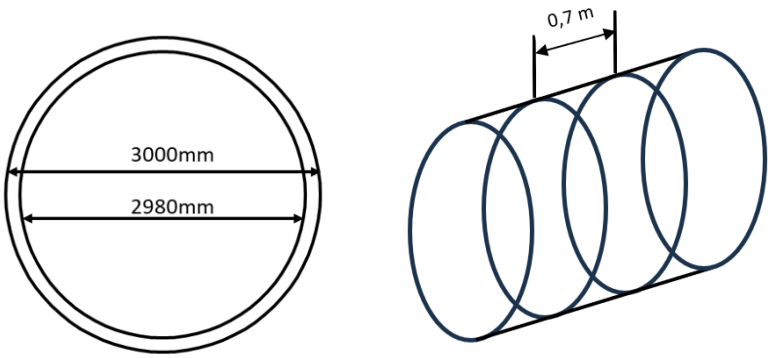


Figure 3 - Dimensions and characteristics of the structure

The material used has a longitudinal modulus of elasticity E = 70 GPa, density ρ = 2.710 kg/m^3, and cross-sectional area A = 16,6 cm^2.

The models comprise 24 bar elements and 24 nodes, arranged in a radial division, as shown in Figure 4.

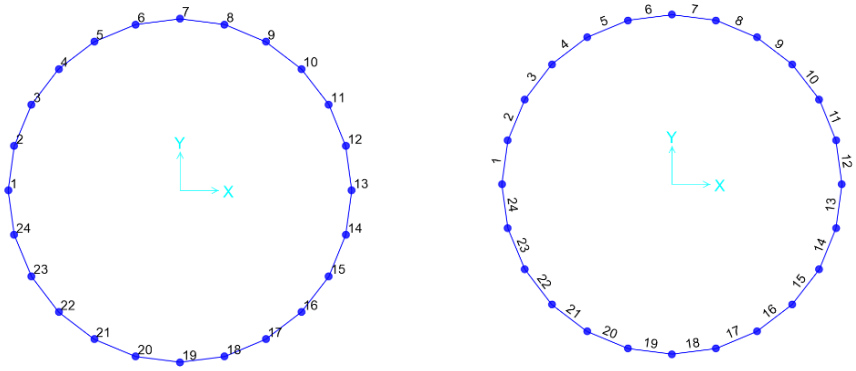


Figure 4 – Element numbering

To evaluate the influence of geometric stiffness on the vibration modes of the structure, the loads corresponding to internal pressure will be applied progressively, starting from an initial condition with no load, and increasing to pressures of 50 kPa, 100 kPa e 150 kPa. Given the longitudinal spacing of 0,7 m between frames, the following equivalent distributed loads corresponding to pressure are obtained: 35 kN/m, 70 kN/m e 105 kN/m. Figures 5, 6, and 7 illustrate these loading cases.

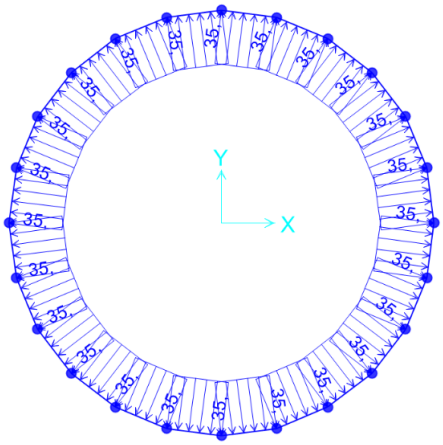


Figure 5 – load corresponding to 50 kPa

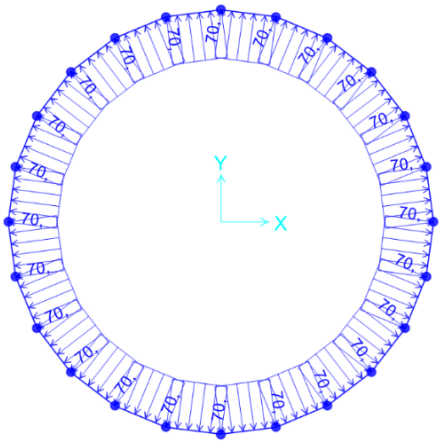


Figure 6 – load corresponding to 100 kPa

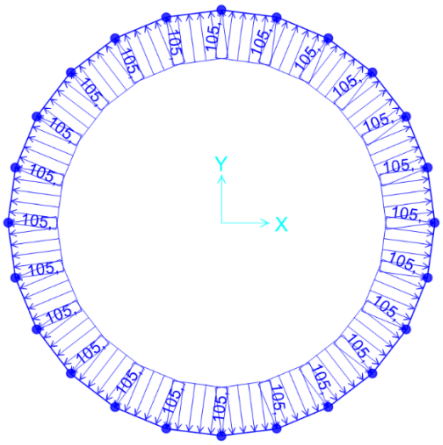


Figure 7 – load corresponding to 150 kPa

The frames studied were also modeled without considering external restraints, representing a condition similar to flotation, as if suspended in air during flight. This allows the boundary conditions to reflect the characteristics of the problem, lending consistency to the data and the results obtained.

4 Results and Discussion

Following the processing of the models, two aspects are of principal interest in this study: the frequencies, expressed in *rad/s*, obtained from solving the eigenvalue problem, and their corresponding vibration modes. Since the structure lacks external restraints, the first three modes obtained correspond to rigid body motion, two translational and one rotational: heave, sway, and roll, pendulum around the noticed point, as illustrated in Figure 8 and corresponding respectively to modes 1, 2, and 3 in the planar 2D analysis.

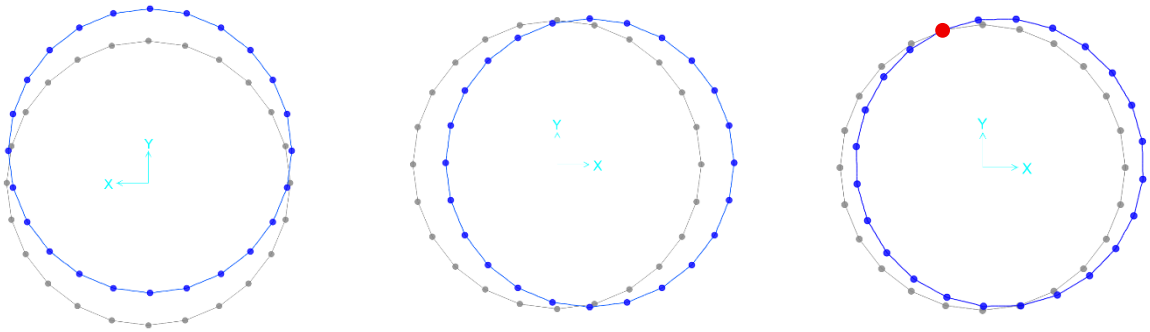


Figure 8 – Rigid body vibration modes

In the figures, the nodes and bars shown in gray represent the initial, not displaced position of the structure. It is worth noting that these three rigid body modes exhibit zero natural frequency. For numerical reasons, values considerably smaller are generally obtained, but not exactly zero.

The first three rigid body modes obtained were not influenced by geometric stiffness, in the case of a structure without loading. However, it was observed that the third mode, in the presence of external loading, had its physical behavior altered, transitioning from a roll type of motion to a rotational movement accompanied by an expansion, called a "breathing" effect, as can be seen in Figure 9.

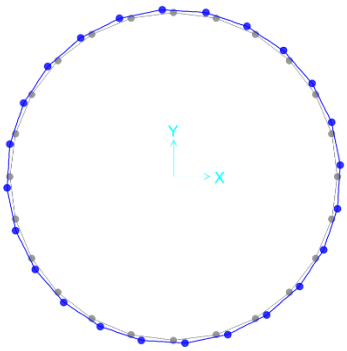
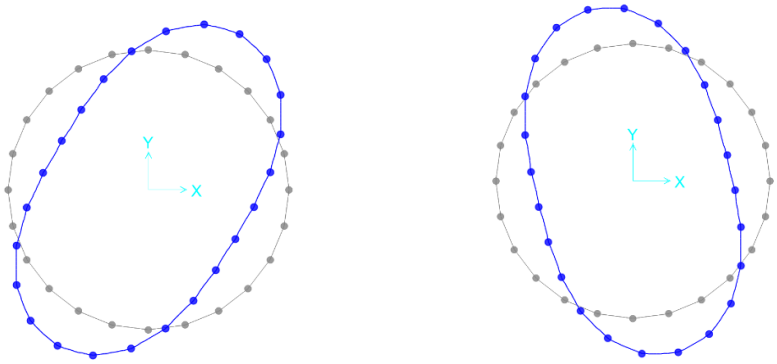


Figure 9 – Mode 3 considering external load applied

It was also observed that only from the fourth vibration mode onwards the effects of geometric stiffness begin to influence the structure. This is due to the fact that only vibration modes that produce distortion and deformation in the structural sections can be affected by the applied load, and thus by stiffness. These higher-order modes, also referred to as elastic modes, are typically obtained in symmetrical pairs, as illustrated in Figure 9 for modes 4 and 5.



Mode 4 Mode 5

Figure 10 – Elastic vibration modes

Therefore, for the study and analysis of the frame's behavior under the lens of non-linear geometric elastic analysis, the frequency values corresponding to elastic modes 4 and 5 were extracted from the software for each level of internal pressure considered, as shown in the following table. The column labeled "Variation" refers to the percentage change with respect to the immediately preceding frequency, progressively.

Condition	Frequency (rad/s)	Variation (%)
Without internal pressure	72,094	-
With 50 kPa internal pressure	120,884	67,68
With 100 kPa internal pressure	154,990	28,21
With 150 kPa internal pressure	182,826	17,96

In order to quantitatively study the effect of pressure on the structure, starting from the unloaded condition, the percentage variation relative to the unloaded case was also calculated for each applied pressure.

Condition	Variation (%)
Without internal pressure	0,00
With 50 kPa internal pressure	66,50
With 100 kPa internal pressure	114,98
With 150 kPa internal pressure	153,59

Although numerical data alone are sufficient for many types of analysis, graphical outputs also contribute significantly to clarifying the physical behavior of the system.

5 Conclusion

Based on the analyses carried out, a significant impact of internal stresses on the obtained frequencies was observed. The tensile stresses generated by internal pressure tend to increase the geometric stiffness of structures overall, and in proportion to their intensity. Thanks to computational programs based on FEM, it is possible to compute and assess these effects in structural analyses with both precision and adherence to analytical models.

The increasing trend in frequency values represents a theoretically expected result. Given the relationship $\omega^2 = K/M$, one may expect that, for a structure of constant mass, the increase in frequency is proportional to stiffness K, as was indeed observed. This point further leads us to reflect upon two fundamental aspects of aerospace structures: lightness and stiffenss.

Taking into account the physical behavior, it was found that the presence of external loads can alter not only the circular frequencies, ω, but also the manner in which the structure tends to vibrate. This effect further brings relevance to nonlinear dynamic analyses, as it allows for an understanding not only of how structures behave under such influences, but also brings the possibility of inducing favorable behaviors, with the aim of shifting the natural vibration modes away from resonance with external dynamic loads, thereby avoiding resonance-related problems.

The study recorded an increase of 153,9% in vibration frequency for an internal pressure of 150 kPa, when compared to the unloaded case. Given that commercial aircraft typically maintain internal pressures on the order of 80 kPa, the analyses corroborate the importance of internal pressurization in aviation and highlight the significance of employing non-linear models when evaluating the dynamic behavior of framed structures.

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References

[1] T. L. Htet. "Structural analysis and topology design optimization of load bearing elements of aircraft fuselage structure." IOP Conference Series: *Materials Science and Engineering*. Vol. 709, n. 4, pp. 1-10, 2020.
[2] A. Pagani, R. Azzara and E. Carrera, "Geometrically nonlinear analysis and vibration of in-plane-loaded variable angle tow composite plates and shells". *Acta Mechanica*, p. 1-24, 2022.
[3] A. M. Wahrhaftig; K. M. M. Magalhães; R.M.L.R.F. Brasil and K. Murawski. "Evaluation of Mathematical Solutions for the Determination of Buckling of Columns Under Self-weight". *Journal of Vibration Engineering & Technologies*. vol. 9, pp. 733–749, 2021.
[4] A. Wahrhaftig; R. Brasil and L. Nascimento, "Analytical and mathematical analysis of the vibration of structural systems considering geometric stiffness and viscoelasticity". In: Numerical Simulations in Engineering and Science. Intechopen, 2018.