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Analysis of Masonry Structures using the Finite-Volume Theory (FVT)

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Abstract. The Finite-Volume Theory (FVT) is an advanced numerical approach for structural analysis in solid mechanics, standing out as a promising alternative to the Finite Element Method (FEM). Based on static equilibrium, FVT discretizes the domain into subvolumes in which average stresses and displacements are evaluated. In this work, three-dimensional FVT is applied to the structural analysis of masonry walls, with the implementation, in MATLAB®, of a computational environment that integrates mesh generation, definition of boundary conditions, numerical simulation, and results visualization. The analysis compares the global behavior of walls modeled with homogenized effective properties, obtained via FVDAM under generalized plane strain conditions, and with the individual properties of blocks and mortar. The results showed high agreement between the two approaches in the elastic regime, with percentage differences in displacements ranging from almost zero for high mortar stiffness to over 16% for low stiffness cases. These findings confirm the validity of using homogenized properties in this context and demonstrate the efficiency of the proposed methodology. Thus, the research contributes both to the advancement of numerical modeling of masonry structures and to the development of computational tools applicable to structural engineering.

Keywords: finite-volume theory, structural analysis, masonry, three-dimensional structure.

1 Introduction

Masonry is widely used in civil construction due to the easy availability of raw materials and its low cost; however, its structural behavior is challenging to model because of the heterogeneity of blocks and mortar [1]. This complexity demands numerical methods capable of accurately representing its constituents, interfaces, and discontinuities.

Its computational modeling can follow three approaches: **micro-modeling**, which represents blocks and mortar individually, ensuring greater accuracy but at a high computational cost; **macro-modeling**, which treats masonry as an equivalent homogeneous material, reducing computational effort but at the expense of detail; and **homogenization**, which estimates effective properties from the microstructure, balancing simplification and realism [1–5].

The Finite Volume Theory (FVT) emerges as a promising alternative to the Finite Element Method (FEM), standing out for enforcing continuity of average tractions and displacements on the faces of subvolumes [6, 7], a key feature for modeling composite materials. As a relatively recent method, FVT has been expanded to various applications, including three-dimensional structures [8]. In this context, this work proposes a 3D FVT formulation and its computational implementation for the structural analysis of masonry, as an alternative to FEM [5, 9].

2 Methodology

The formulation adopted in this work is based on the standard version (or zero-order or Cartesian) of the Finite Volume Theory (FVT), originally proposed for regular rectangular discretizations, as presented by Cavalcante and Pindera [7]. In this approach, the three-dimensional domain of the structure is subdivided into prismatic subvolumes, each with local dimensions $l_{(q)}$, $h_{(q)}$, and $d_{(q)}$ adopted in this work, corresponding to the directions

of the local axes $x_1^{(q)}, x_2^{(q)}$, and $x_3^{(q)}$ respectively, for the subvolume q, as illustrated by Araujo et al. [8] in Figure 1.

2.1 Displacement Field

For the representation of the displacement field within each subvolume, an incomplete second-order polynomial interpolation is adopted, based on Legendre polynomials defined in the local coordinate system. The general expression for this approximation is given by:

$$u_i^{(q)} = U_{i(000)}^{(q)} + x_1^{(q)} U_{i(100)}^{(q)} + x_2^{(q)} U_{i(010)}^{(q)} + x_3^{(q)} U_{i(001)}^{(q)} + \frac{1}{2}\left(3(x_1^{(q)})^2 - \frac{l_q^2}{4}\right) U_{i(200)}^{(q)} + \frac{1}{2}\left(3(x_2^{(q)})^2 - \frac{h_q^2}{4}\right) U_{i(020)}^{(q)} + \\ + \frac{1}{2}\left(3(x_3^{(q)})^2 - \frac{d_q^2}{4}\right) U_{i(002)}^{(q)} \tag{1}$$

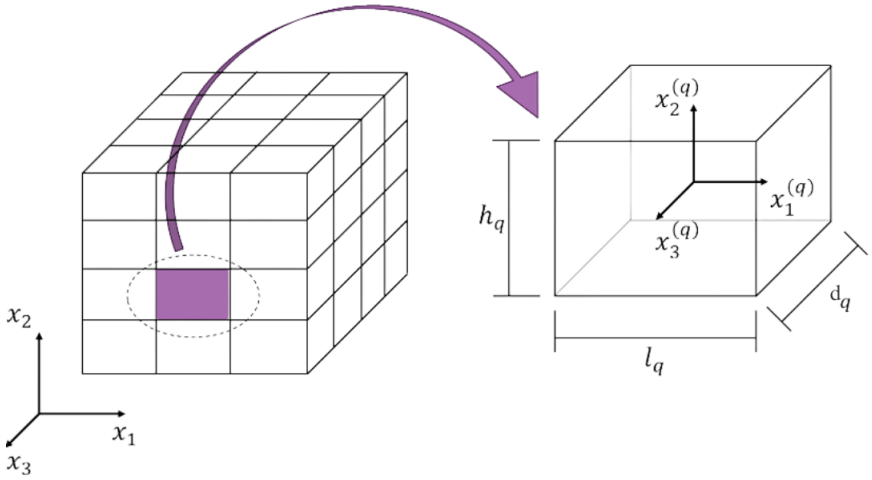


Figure 1. Global and local coordinate systems for a subvolume.

2.2 Kinematic and Static Relations

The kinematic variables are treated as average displacements on the surfaces of the subvolumes, obtained by:

$$\bar{\mathbf{u}}^{(q)} = \mathbf{A}^{(q)} \mathbf{U}^{(q)} + \mathbf{a}^{(q)} \mathbf{U}_{(0)}^{(q)} \tag{2}$$

where: $\mathbf{U}^{(q)} = [\mathbf{U}_1^{(q)}, \mathbf{U}_2^{(q)}, \mathbf{U}_3^{(q)}]^T$ is the vector containing the unknown first- and second-order coefficients of the displacement field, $\mathbf{U}_{(0)}^{(q)} = [U_{1(000)}^{(q)}, U_{2(000)}^{(q)}, U_{3(000)}^{(q)}]^T$ is the vector of unknown zero-order coefficients, responsible for the rigid body displacements of the subvolume, and $\mathbf{A}^{(q)}$ and $\mathbf{a}^{(q)}$ are matrices that depend on the geometric characteristics of the subvolumes [8].

Analogously to the kinematic variables, the static variables are also evaluated as averages, using the same indexing convention adopted for the average displacements on the surfaces. In the standard formulation of the Finite-Volume Theory, the static variables correspond to the average tractions applied on the faces of a generic subvolume. They are related by:

$$\bar{\mathbf{t}}^{(q)} = \mathbf{D}^{(q)} \mathbf{U}^{(q)} \tag{3}$$

where $\mathbf{D}^{(q)} = \mathbf{N}^{(q)}\mathbf{C}^{(q)}\mathbf{E}^{(q)}$ depend on the dimensions and material properties. The matrices composing $\mathbf{D}^{(q)}$ can be found in the work [8].

2.3 Local Stiffness Matrix

Applying the force equilibrium Equations 4 :

$$\sum_{p=1}^{6} \bar{\mathbf{t}}^{(q,p)} S_p^{(q)} = \mathbf{0} \tag{4}$$

where $\bar{\mathbf{t}}^{(q,p)}$ are the average tractions on each face and $S_p^{(q)}$ represents the area of the face of subvolume q and inserting the kinematic Equations 2 and static Equations 3 into this equation, results in a linear system that directly connects displacements and average tractions.
Finally, the local non-symmetric stiffness matrix is obtained and expressed by:

$$\bar{\mathbf{t}}^{(q)} = \mathbf{K}^{(q)}\bar{\mathbf{u}}^{(q)} \tag{5}$$

The resultant force acting on the faces of the subvolume can be evaluated as:

$$\mathbf{R}^{(q,p)} = \mathbf{S}_p^{(q)}\bar{\mathbf{t}}^{(q,p)} \tag{6}$$

Then the system of equations can be rewritten as:

$$\mathbf{R}^{(q)} = \mathbf{S}_p^{(q)}\mathbf{K}^{(q)}\bar{\mathbf{u}}^{(q)} = \bar{\mathbf{K}}^{(q)}\bar{\mathbf{u}}^{(q)} \tag{7}$$

where $\bar{\mathbf{K}}$ is the symmetric local stiffness matrix of the subvolume.

3 Results and Discussion

This work compares the structural response of a masonry wall using homogenized effective properties, obtained via FVDAM for an RVE under generalized plane strain conditions [5], and the same wall, with identical geometry and mesh, modeled with the individual properties of the block and mortar. Based on Anthoine [10], such a comparison is valid in the elastic regime, since the generalized plane strain condition yields a global stiffness close to that of three-dimensional models.

The material properties used in this study (homogenized and for each phase) were obtained from Escarpini Filho and Almeida [5]. For the heterogeneous model, the constituent properties are presented in Table 1. Based on these properties, the three-dimensional structural analysis was carried out using FVT, which constitutes the main focus of this work.

Table 1. Material Properties

Material	Young's Modulus [GPa]	Poisson's ratio [ν]
Block	200	0.15
Mortar	[200, 20, 2, 0.2]	0.15
Rigid Beam	100000	0.3

The model represents a masonry wall composed of 2.5 ceramic blocks per course and five vertical layers, each block measuring 210 mm × 50 mm × 1 mm and separated by 10 mm mortar joints, with rigid beams at the base and top for vertical confinement and global stability (Figure 2).

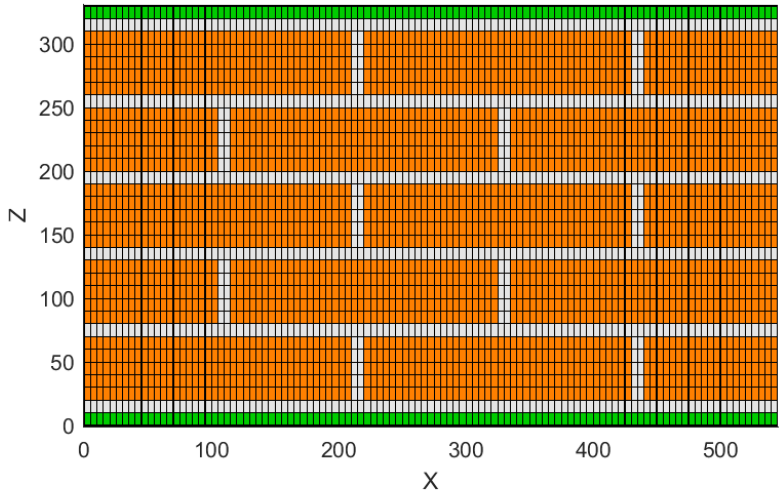


Figure 2. Mesh of the masonry problem.

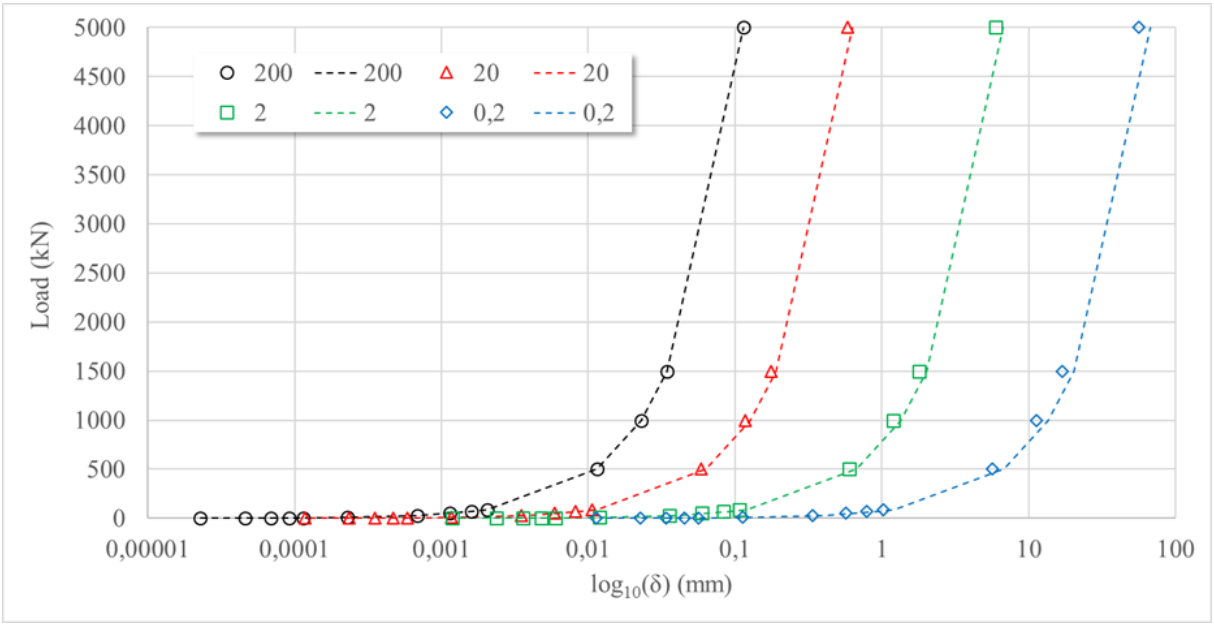


Figure 3. Displacement of the upper rigid beam.

The upper beam was subjected to a load, ranging from 1 kN to 5000 kN, and the displacements (δ) were measured in millimeters at the end opposite to the load application. The objective of this study is to evaluate the behavior of masonry as a function of the mortar properties, thereby identifying their influence on the structural response throughout the elastic phase and under different loading levels.

The results are presented in Figure 3, where the legend indicates the Young's modulus values of the mortar. The dashed lines represent the response of the wall modeled as heterogeneous, while the discrete markers correspond to the homogenized modeling. A high level of agreement between the two approaches is observed throughout the entire elastic range, with only minor differences in displacements for the same load level. This similarity confirms that, for the analyzed regime, the homogenized model can consistently reproduce the global stiffness of the wall, regardless of variations in the mortar's elastic modulus. To further assess this similarity, the percentage differences in predicted displacements between the heterogeneous and homogenized models were cal-

culated for each mortar Young's modulus case, yielding 0.00005%, 7.480%, 11.125%, and 16.247%, respectively. These values indicate that the two approaches are virtually identical for high mortar stiffness, with differences increasing progressively as the mortar becomes more compliant. Small deviations appear only near the loading limit, when the wall begins to exhibit more pronounced deformations, highlighting that the influence of heterogeneity becomes more significant under higher loading levels.

4 Conclusion

This study compared the structural response of a masonry wall modeled with homogenized effective properties, obtained via FVDAM under generalized plane strain conditions, and with the individual properties of the block and mortar, maintaining the same geometry and mesh.

The results indicated a high level of agreement between the two approaches in the elastic regime, regardless of the mortar's elastic modulus, validating the homogenized formulation for representing the global stiffness of masonry. Quantitative comparisons confirmed this behavior, with percentage differences in displacements ranging from nearly zero for high mortar stiffness to over 16% for low stiffness cases. As reported by Anthoine [10], this equivalence is expected within this range of behavior. More noticeable differences only emerged at higher load levels, when heterogeneity began to have a stronger influence on the structural response.

Thus, the use of effective properties proves to be an efficient and reliable alternative for elastic analyses or preliminary studies, reducing computational cost without significant loss of accuracy.

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