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Comparison of Pure and Mixed Integration Schemes in Material Point Method Simulations of Hammer-Driven Conductor Casing Installation in Undrained Cohesive Soils

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Abstract. The installation of conductor casing is a critical step in offshore oil well construction, providing structural support. Numerical simulations are essential for optimizing this process, however, challenges such as large soil deformations require advanced methods, such as the Material Point Method (MPM). While MPM has proven effective for large-deformation geotechnical problems, its conventional formulation faces limitations in stress accuracy. To address these challenges, this study evaluates the driving of conductor casing using two numerical methods: pure MPM and MPM-Mixed Integration (MPM with Gauss integration). The soil is modeled via the Mohr-Coulomb criterion, with geotechnical parameters derived from real data from the Campos Basin in Brazil. Results show that MPM-Mixed Integration provides superior stability under large deformations, reducing numerical problems and improving stress distribution accuracy.

Keywords: Material Point Method, Mixed Integration Scheme, Numerical Simulation, Impact Driving, Conductor Casing.

1 Introduction

The conductor casing is essential for wellbore integrity during offshore oil well construction, its main roles include stabilizing loose shallow formations and providing a structural base for other casings [1]. Installing conductor casing can be done by methods like jetting or impact driving. This technic carries out three-stage process: self-weight penetration, suction penetration, and hammering penetration [2]. Numerical simulations in geomechanical analyses traditionally employ continuum-based methods, such as the Finite Element Method (FEM) or Finite Difference Method (FDM). However, these approaches face limitations when modeling penetrating objects in soil, particularly due to mesh distortion and convergence loss [3]. To address these limitations, particle-based numerical approaches like the Material Point Method (MPM) have emerged as promising alternatives. These methods overcome the challenges by eliminating mesh distortion issues while maintaining the ability to model large deformations effectively [4], [5], [6]. Effective numerical simulation requires careful model calibration through adjustment of critical parameters (e.g., time step size, damping coefficients, or material stiffness parameters etc.), and selection of appropriate integration methods.

The literature contains several studies analyzing these calibration parameters in numerical models of penetrating objects, such as Tenorio et al. [7], who conducted analysis of how time step size and MPM specifications affect numerical convergence. Moreover, analyses of contact parameters have been advanced by researchers such as Moormann et al. [8] developed a 2D axisymmetric model incorporating a penalty function method to evaluate soil-pile contact interactions, using MPM-Convected Particle Domain Interpolation (CPDI). Given this context, the present study evaluates conductor casing installation in undrained cohesive soils, typical

of offshore environments, comparing two numerical approaches: pure Material Point Method, and MPM-Mixed Integration (featuring Gauss integration). The investigation specifically assesses which integration scheme demonstrates superior adaptability for this application.

2 Methodology

2.1 Material Point Method Formulations: Pure MPM and MPM-Mixed Integration Schemes

This study employs ANURA3D, an open-source software based on MPM, for model development. The MPM is a hybrid Eulerian-Lagrangian approach that combines the advantages of both particle-based and mesh-based numerical methods. In this framework, the computational domain is discretized using a background Eulerian mesh; material bodies are represented by Lagrangian particles that carry all constitutive properties (stress, strain, history variables) and move through the mesh. This dual characterization enables MPM to handle large deformations naturally, avoiding the mesh distortion issues inherent to pure Lagrangian methods while maintaining the material history tracking that Eulerian methods typically lack [4], [9]. Figure 1 shows the typical MPM algorithm.

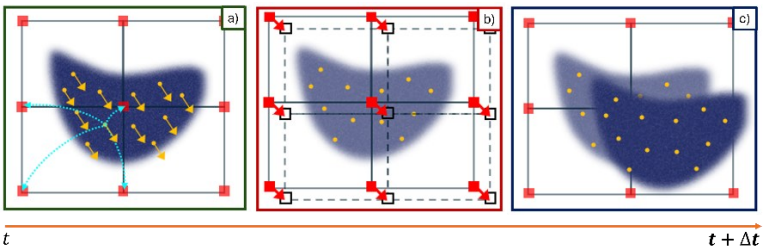


Figure 1. Illustration of the Material Point Method (MPM) Simulation Cycle (Authors, 2025).

As shown in Fig. 1, the method enforces momentum conservation as expressed in eq. (1). The numerical integration proceeds through the stages: (Fig. 1a) calculation of shape functions for all material points across the background mesh, transfer of mass and momentum properties from particles to nodes, (Fig. 1b) solution of the momentum equations on the mesh (eqs. 2 and 3), and (Fig. 1c) projection of updated nodal velocities and positions back to the material points. Where, $\boldsymbol{\sigma}_p$ is the Cauchy stress; $\nabla N_I(\boldsymbol{x}_p)$ is the shape function gradient; V_p and ρ_p are material point volume and density; $\boldsymbol{b}$ is body force; and f_I^{int}, f_I^{ext} are internal/external nodal forces.

$$\rho \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \nabla \cdot \sigma + \rho \boldsymbol{b} \tag{1}$$

$$f_I^{int} = -\sum_p \boldsymbol{\sigma}_p \nabla N_I(\boldsymbol{x}_p) V_p \tag{2}$$

$$f_I^{ext} = \sum_p N_I(x_p) \rho_p \boldsymbol{b} V_p. \tag{3}$$

A rigid body penetrates soil involves large deformations; thus, conventional MPM integration scheme can exhibit cell-crossing instability. Specifically, as a particle transitions from one element to another, discontinuous shape function gradients at the element edges introduce unphysical nodal forces, compromising solution accuracy. An adopted solution for mitigating cell-crossing instability is the mixed-integration scheme implemented in the Anura3D code [10]. First, it calculates a representative constant stress value for each element by averaging the stresses of all material points contained within that element at the current time step (t), as shown in eq. (4). Second, it computes internal forces using a Gauss integration technique analogous to conventional FEM procedures.

$$\sigma_{av} = \frac{\sum_{MP=1}^{no_{MP,el}} \sigma_{MP} \Omega_{MP}}{\sum_{MP=1}^{no_{MP,el}} \Omega_{MP}}. \tag{4}$$

Where σ_{av} is the average stress, $no_{MP,el}$ is the number of material points in the element; σ_{MP} and Ω_{MP} are the stress and the volume of material point, respectively. The Gauss integration scheme for internal forces is applied exclusively to elements within the continuum's interior.

2.2 Numerical model of the conductor casing driving process

The 2D axisymmetric Material Point Method (MPM) model was developed using an unstructured mesh of 4,884 triangular elements (2,539 nodes) and 13,805 material points, with six points per element in the soil region and one point in the conductor region. The 0.91 m diameter, 58.8 m long conductor casing (linear weight: 3,470 lb/ft) was modeled as a rigid body with density of 7860 kg/m³. Figure 2 shows boundary conditions and the adaptive mesh scheme, combining a compressing soil region with an upper moving mesh referenced to the conductor. Soil-structure interaction employed Al-Kafaji (2013) [11] adhesive contact algorithm. Figure 2 also shows the material properties used in the four clayey soil layers.

Properties	Value				Unit
	Layer 1	Layer 2	Layer 3	Layer 4	
Classification	Very soft clay	Very soft clay	Soft clay	Medium clay	
Initial porosity	0.58	0.58	0.47	0.43	-
Density	1,475.40	1,687.91	1,799.32	1,838.82	Kg/m³
Effective Poisson's ratio	0.49	0.49	0.45	0.40	-
K0 value	0.96	0.96	0.82	0.67	-
Effective young modulus	17,896.07	27,823.35	60,440.32	83,865.38	kPa
Effective cohesion	9,27	36.23	93.95	148.92	kPa
Effective friction angle	36.29	37.99	40.35	37.63	degrees
Friction angle (contact)	0.43	0.44	0.47	0.44	rad
Adhesion factor	4.64	18.12	46.97	74.46	kPa
Water properties					
Density	1000				kg/m³
Intrinsic permeability	1.02x10-9				m²/s
Bulk modulus	2.15x104				kPa
Dynamic viscosity	1.57x10-6				kPa.s

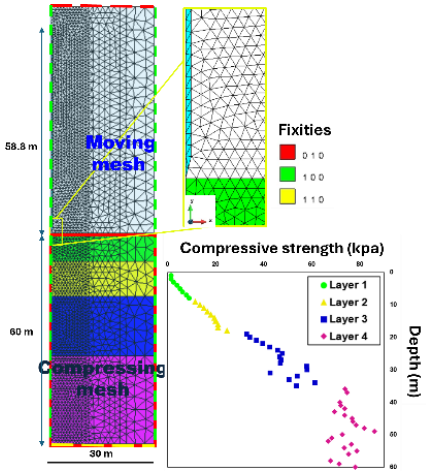


Figure 2 - Numerical model and material properties considered in conductor casing driving simulation (Authors, 2025).

The simulation proceeded in two phases: (1) self-weight equilibrium under gravitational acceleration, followed by (2) hammering impacts corresponding to a 0.46 m/s initial velocity at 1s intervals. The Campos Basin clay was modeled using a Mohr-Coulomb constitutive law. This was possible because under undrained conditions, rapid loading generates substantial excess pore pressure while preventing significant solid-fluid phase movement, making pore pressure dissipation negligible [12].

3 Results

Figure 3 compares the effective stress magnitude distributions obtained using the pure MPM integration scheme (Fig. 3a) and the MPM-Mixed integration approach (Fig. 3b). The stress field comparison reveals significant differences between integration schemes. The pure MPM formulation produces irregular stress distributions with patchy high-stress zones and abrupt fluctuations between adjacent regions, indicative of numerical discontinuities. These artifacts likely stem from cell-crossing instability, particularly evident in sudden color transitions at element boundaries. In contrast, the mixed MPM-Mixed integration yields smoother, more, demonstrating progressive transitions between high/low stress zones and eliminating spurious peaks near rigid boundaries. Figure 3c presents the displacement ratio comparison between the integration schemes. While both methods ultimately achieve the target penetration depth of -18m, the mixed integration approach completes the process approximately 10 seconds faster, representing an 18% improvement in computational efficiency. The depth-time penetration profile demonstrates the superior performance of the MPM-Mixed integration scheme compared to pure MPM.

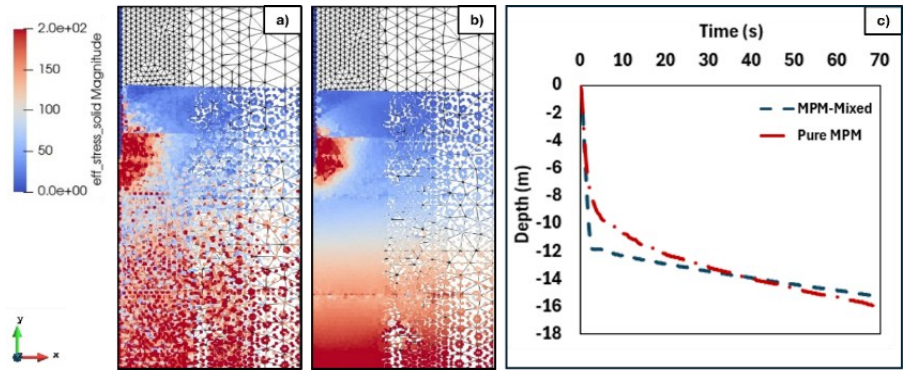


Figure 3. Comparison of effective stress magnitude (kPa) for: (a) Pure MPM; (b) MPM-Mixed integration; (c) Penetration depth versus time (Authors, 2025).

4 Conclusions

This comparative analysis demonstrates that the MPM-Mixed integration scheme outperforms pure MPM in simulating conductor casing penetration through undrained clay providing superior numerical stability by eliminating issues such as spurious stress oscillations and unphysical penetration patterns while maintaining smooth stress gradients. Additionally, the mixed integration approach enhances accuracy through improved resolution of stress concentrations at soil-structure interfaces and energy conservation, preventing underestimation of penetration resistance. For geotechnical problems involving large deformations and complex interfaces, the MPM-Mixed scheme represents a robust alternative that balances physical fidelity with computational practicality.

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References

[1] J. Wang, J. Yang, Q. Xue, M. Ma, and H. Wan, "Influence of Various Factors on the Structural Stability of Conductors During Piling,". *The 34th International Ocean and Polar Engineering Conference*, Rhodes, Greece, Jun. 2024.
[2] Y. Wang, D. Gao, and J. Fang, "Finite element analysis of deepwater conductor bearing capacity to analyze the subsea wellhead stability with consideration of contact interface models between pile and soil". *J Pet Sci Eng*, vol. 126, pp. 48–54, Feb. 2015. doi: 10.1016/j.petrol.2014.11.036.
[3] M. Nazem, et al., "Integração da tensão e refinamento da malha para grandes deformações em geomecânica". *Int J Numer Methods Eng*, 2006.
[4] A. Yerro, V. Girard, F. Ceccato, and M. Martinelli, "Modelling unsaturated soils with the Material Point Method. A discussion of the state-of-the-art". *Geomechanics for Energy and Environment*, 2022.
[5] S. Fu, Z. X. Yang, N. Guo, and R. J. Jardine, "Material point method simulations of displacement pile and CPT penetration in sand considering the effects of grain breakage". *Comput Geotech*, vol. 166, p. 105945, Feb. 2024, doi: 10.1016/j.compgeo.2023.105945.
[6] M. Martinelli and V. Galavi, "Investigation of the Material Point Method in the Simulation of Cone Penetration Test in dry sand". *Comput Geotech*, vol. 130, pp. 266–352, 2021.
[7] M. S. C. Tenório *et al.*, "A Numerical Investigation of Conductor Casing Installation Using Material Point Method". *Ibero-Latin American Congress on Computational Methods in Engineering (CILAMCE)*, Dec. 2024, doi: 10.55592/cilamce.v6i06.8222.
[8] C. Moormann, S. Gowda, S. Giridharan, and, F. Hamad, "Simulation of offshore piling using advanced dynamic material point method (MPM)". *DFI-India Conference*, Chennai, India, Oct. 2017.
[9] V. P. Nguyen, A. de Vaucorbeil, and S. Bordas, *The Material Point Method*. Cham: Springer International Publishing, 2023. doi: 10.1007/978-3-031-24070-6.
[10] F. Ceccato, et al., "Anura 3D MPM Software, Scientific Manual," 2022.
[11] I. K. J. Al-Kafaji, "Formulation of a Dynamic Material Point Method (MPM) for Geomechanical Problems," Tese de Doutorado, Universität Stuttgart, Stuttgart, 2013.
[12] F. Ceccato, A. Yerro, and G. Di Carluccio, "Simulating landslides with the material point method: Best practices, potentialities, and challenges," *Eng Geol*, vol. 338, 2024, doi: 10.1016/j.enggeo.2024.107614.