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On the dynamics of periodic foundation-structures interacting with frame structures

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Abstract. A lot of research has been performed in the last years concerning dynamics of periodic structures and their ability to create bandgaps at certain frequencies that dramatically reduce the vibration amplitudes of associates systems. The general idea is to create a periodic structure that filters out the vibration amplitude of a given dynamical system. For the case of dynamic systems interacting with the soil the reasoning is more complex. When, a classical structures system is supported by the soil, the behavior is not unilateral. The dynamic behavior of the soil presents geometrical damping, which usually damps out part of the vibration amplitude of the supported dynamical system. But there a price to pay. The vibration amplitude of the system supported by the soil is decreased but it generates outgoing waves that will cause perturbation to the surrounding environment. There is a trade-off, less vibration on the system usually means more waves propagating into the environment. This article will address the idea of incorporating a periodic foundation structure between the soil and the dynamical system. The idea is to investigate if the periodic structure decouples the soil from the dynamic system for given frequencies. The soil is modelled by the boundary elements methods, the periodic foundation structure is described by the finite elements method. The ability of the periodic foundation structure to create a bandgap is distinct directions corresponding to multiple degrees of freedom will also be addressed.

Keywords: Dynamic Soil Structure Interaction, Structure Dynamics, Foundation Dynamics.

1 Introduction

The dynamics of periodic structures have attracted significant attention in recent decades due to their unique ability to create frequency bands—known as bandgaps—where vibrations are substantially attenuated. This property has been widely explored for vibration control in various structural systems, with applications ranging from civil engineering to mechanical and aerospace systems. However, applying periodic structures in foundations, especially in interaction with the soil, presents additional challenges. The dynamic behavior of soil, characterized by geometric damping and wave propagation, adds complexity to the analysis. While the soil can dissipate part of the transmitted vibrations, it also generates waves that propagate into the surrounding environment, creating a trade-off between local vibration reduction and environmental impact.

In recent years, the use of periodic foundations for structural vibration isolation has been extensively investigated. Numerous studies have aimed to validate the feasibility and effectiveness of such systems. Xiang et al. [9] proposed a novel layered periodic foundation and demonstrated its ability to generate frequency bandgaps that effectively reduce vibrations in buildings, acting as a vibration isolator. Xiong et al. [10] examined the performance of periodic foundations under both vertical and longitudinal ground motions, showing that they can significantly attenuate the dynamic response of structures and offer new insights for civil engineering applications. Pu and Shi [7] further demonstrated that a damped layered periodic foundation composed of multiple unit cells considerably enhances the seismic performance of structures across various site conditions. More recently, the works of Jain et al. [6] and Elshazly and Seylabi [4] have stood out by exploring the effectiveness of sandwiched concrete and rubber layers in inducing bandgaps in the response of soil-structure systems, showing that these

foundations can be tuned to target specific excitation frequencies.

This work proposes the incorporation of a periodic foundation between the soil and the superstructure, aiming to investigate whether such a system can act as a filter that decouples the soil from the structure at certain frequencies. Using a hybrid numerical approach—boundary element method for the soil and finite element method for the periodic foundation—the study seeks to understand the formation of multi-directional bandgaps and their effectiveness in vibration mitigation.

2 Problem Statement

The problem consists of linear-elastic structures modeled as one-dimensional bodies of length L. A portion h of the structure's height, referred to in this paper as the "foundation," may be composed of a single material or of alternating layers of two different materials. The different configurations of the structure and its foundation considered in this study are illustrated in Fig. 1: (a) a homogeneous structure supported by a homogeneous foundation, i.e., made of a single material, and (b) a homogeneous structure supported by a sandwich foundation, composed of alternating layers of two different materials. The structure is analyzed under two different support scenarios (Fig. 1). The structure is supported by a rigid base and is subjected to a vertical load (in the z-direction) of unit magnitude applied at the top or the structure is also loaded from the top, but it rests on a flexible soil. The objective is to determine which configuration leads to the greatest reduction in the wall's vibration amplitude.

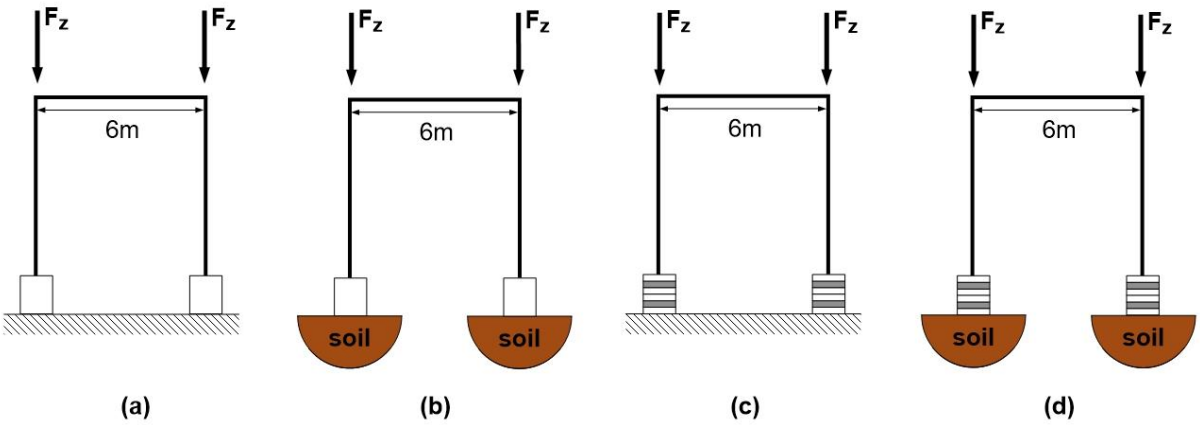


Figure 1. Different configurations of the structure, its foundation and the support mechanism

The sandwich foundation is composed of n identical unit cells stacked vertically (Fig. 2). Each unit cell is divided into two halves, each made of one of two distinct materials, resulting in a foundation consisting of 2n alternating layers of these materials.

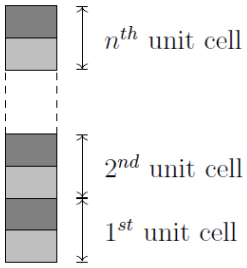


Figure 2. Unit Cell

3 Formulation

Figure 1 illustrates the structure–foundation–soil system analyzed using the proposed methodology. This setup consists of a structure supported by a rigid foundation resting over a soil. The structure is modeled as a frame, and the foundation is modeled as a bar. Both frame and bar models are formulated according to the classical Finite Element Method (FEM), in view of the finite dimensions of the structure and foundation and its linear-elastic behavior. Boundary Element Method (BEM) formulation is used for the soil, so that radiation conditions and wave propagation features of the soil can be represented accurately.

3.1 FEM Model

For the one-dimensional FEM model of the foundation, the foundation is represented as an assembly of two-node, one-dimensional bar elements, each with a single translational degree of freedom per node. The corresponding stiffness and mass matrices are given by Rao [8].

$$\mathbf{K}_{\text{bar}} = \frac{EA}{h_e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \quad \mathbf{M}_{\text{bar}} = \frac{\rho A h_e}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \tag{1}$$

The structure is modeled as a frame. The frame element consists of two nodes, each having three global degrees of freedom: two displacements along the global axes and one rotation. The corresponding stiffness and mass matrices are given by Ferreira and Fantuzzi [5].

$$\mathbf{K}_{\text{frame}} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & \frac{-EA}{L} & & \\ & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} \\ & & \frac{4EI}{L} & 0 & \frac{-6EI}{L^2} & \frac{2EI}{L} \\ & & & \frac{EA}{L} & 0 & 0 \\ & & & & \frac{12EI}{L^3} & \frac{-6EI}{L^2} \\ & sim & & & & \frac{4EI}{L} \end{bmatrix} \tag{2}$$

$$\mathbf{M}_{\text{frame}} = \frac{\rho A h_e}{420}\begin{bmatrix} 140 & 0 & 0 & 70 & 0 & 0 \\ & 156 & 22h_e & 0 & 54 & -13h_e \\ & & 4{h_e}^2 & 0 & 13h_e & -3{h_e}^2 \\ & & & 140 & 0 & 0 \\ & & & & 156 & -22h_e \\ & sim & & & & 4{h_e}^2 \end{bmatrix} \tag{3}$$

3.2 BEM Model

The three-dimensional Direct Boundary Element Method (DBEM) with constant quadrilateral elements is employed to determine the soil response in the frequency domain, ω. The soil model is defined by its density ρ_{Soil}, Young's modulus E_{Soil}, Poisson's ratio ν_{Soil}, and internal damping factor η_{Soil}. In the present analysis, the soil damping is assumed to be frequency-independent and constant, η_{Soil}=const (Beskos [1]). The 3D Boundary Element formulation in the frequency domain is based on the "frequency domain full space fundamental solution" (Dominguez [3]). The BEM implementation described by Carrion [2] enables the evaluation of the dynamic response of 3D soils under applied surface tractions, as well as the interaction between the soil and rigid surface or embedded foundations. The formulation for rigid foundations interacting with a half-space can be derived as a simplification of the case addressed here (Carrion, [2]).

Figure 3 show the real and imaginary parts of the flexibility functions, relating the external vertical excitation, F_z, to the vertical rigid foundation degree of freedom, for the case of a homogeneous half-space.

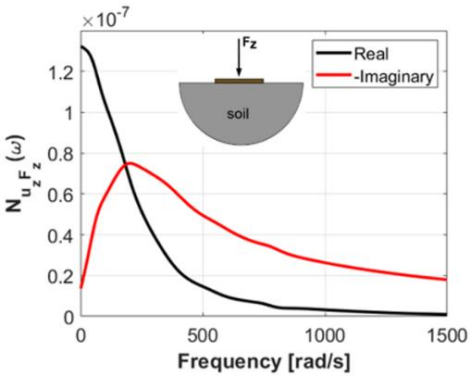


Figure 3. Flexibility for the half-space

After defining the dynamic response of the soil and of the structure the coupling is carried out by the balance of forces and kinematic compatibility at the soil-structure interface.

4 Numerical Results

This section presents and discusses the results obtained from the numerical models described in the previous sections. For clarity in these discussions, the origin of the x–z coordinate system is defined such that z=0 corresponds to the soil's free surface, and x=0 aligns with the left end of the frame. The results are expressed in terms of vertical displacements $u_z(\omega)$ at various points along the system. The displacement $u_{zh}(\omega)$ refers to the vertical displacement at the upper face of the first unit cell located at the bottom of the structure (at a vertical position z=h/2n from the base, where n=5 is the number of unit cells in the sandwich foundation). This point is chosen as representative in case of fixed base because, under this condition, displacements at z=0 are zero due to the rigid base supporting the wall.

Table 1 lists the material properties used in the analysis. These values are selected based on their relevance to engineering practice. Unless otherwise specified, all results consider structure with a height H=6 m and foundations with a height h =1m.

Table 1 – Material Properties

Material	**c_s [m/s]**	**ρ [kg/m^3]**	**E [m/s]**
Concrete	-	2,400	3.00E10
Rubber	-	1,300	1.37E5
Soil	577.35	2.700	8.40E10

All results in this section are computed at z=h/(2n). Figure 4 presents a comparison of the response of the 1D system supported by homogenous foundation or sandwich foundation over a rigid base shown in Fig. 1. While the addition of a homogeneous foundation noticeably influences the system's response, frequency bandgaps in the system's response appear only with the implementation of the sandwich foundation.

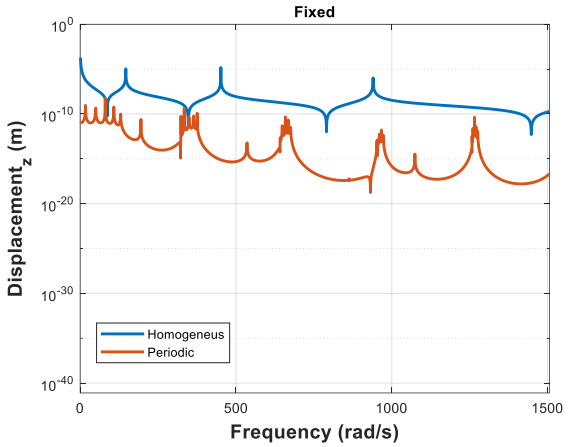


Figure 4. Response of 1D system considering concrete structure with a homogenous or sandwich foundation over fixed base.

Figure 5 presents a comparison of the response of the 1D system supported by homogenous foundation or sandwich foundation resting over a half-space as shown in Fig. 1. Figure 5 illustrates that the presence of soil doesn't change the band gap effect caused by the presence of sandwich foundation.

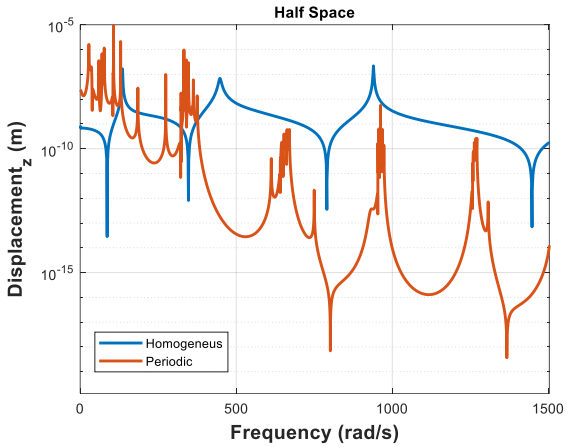


Figure 5. Response of 1D system considering concrete structure with a homogenous or sandwich foundation resting on half space.

Figure 6 shows a comparison of the response of a periodic foundation on a fixed base or on a semi-space. Analyzing the results, it is possible to see that the presence of soil has almost no effect on the dynamic behavior of the structure, which shows us that the periodic structure acts as a filter.

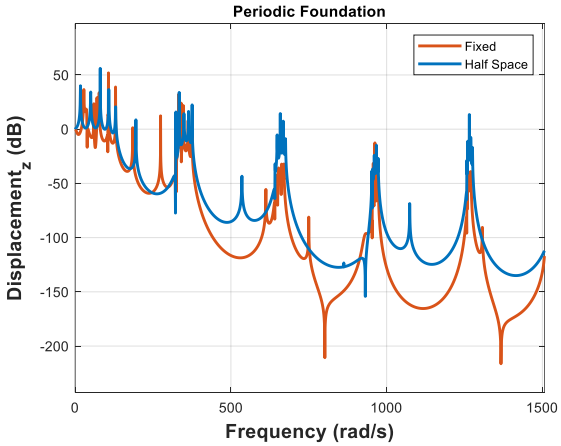


Figure 6. Response of 1D system considering concrete structure with a homogenous or sandwich foundation resting on half space.

Figure 6 shows a comparison of the response of a periodic foundation on a fixed base or on a half-space. Analyzing the results, it is possible to see that the presence of soil has almost no effect on the dynamic behavior of the structure, which shows us that the periodic structure acts as a filter.

To numerically assess the influence of periodic foundations on the analyzed systems (Fig. 1), the modal parameters (natural frequencies, ω_n, and damping factors, ξ_n) are extracted for the four cases considered. For this purpose, the Rational Fraction Polynomial Method (RFPM), as described by Ewins [11], is applied to the structure FRFs presented in Figs. 4 and 5. The resulting modal parameters for the first four modes of each case are shown in Tab. 2. The results show that the periodic foundation configurations (cases c and d) modify the system's natural frequencies compared with those obtained for the homogeneous foundation configurations (cases a and b).

For the homogeneous foundation cases (a and b), soil geometric damping is observed to induce structural damping where none was previously present. In contrast, for the periodic foundation cases (c and d), the transmission of soil influence to the structure is significantly reduced, as evidenced by the near-zero damping factors obtained for case d.

Table 2. Modal parameters extracted for the first 4 DOFs of the analyzed systems

Case a	**Case b**		**Case c**	**Case d**	
Nat. Freq. (rad/s)	Nat. Freq. (rad/s)	Damping F.	Nat. Freq. (rad/s)	Nat. Freq. (rad/s)	Damping F.
$\omega_{n1} = 146.1$	$\omega_{n1} = 135.7$	$\xi_1 = 0.0185$	$\omega_{n1} = 16.7$	$\omega_{n1} = 27.8$	$\xi_1 = 0.00001$
$\omega_{n2} = 451.8$	$\omega_{n2} = 447.8$	$\xi_2 = 0.0172$	$\omega_{n2} = 49.8$	$\omega_{n2} = 59.9$	$\xi_2 = 0.00003$
$\omega_{n3} = 942.8$	$\omega_{n3} = 939.7$	$\xi_3 = 0.0073$	$\omega_{n3} = 80.5$	$\omega_{n3} = 70.2$	$\xi_3 = 0.00002$
$\omega_{n4} = 1740.9$	$\omega_{n4} = 1741.2$	$\xi_4 = 0.0453$	$\omega_{n4} = 108.3$	$\omega_{n4} = 77.1$	$\xi_4 = 0.00001$

5 Conclusions

This study demonstrated the potential of periodic foundations to modulate the dynamic response of soil-supported structures by creating bandgaps in multiple directions and degrees of freedom. The hybrid modeling approach, combining boundary elements for the soil and finite elements for the foundation, enabled accurate capture of the complex interaction between the structure, the periodic foundation, and the soil. Results indicate that while the periodic foundation effectively reduces vibration amplitudes at specific frequencies, there is an inherent trade-off related to the propagation of waves into the surrounding environment. Therefore, this approach paves the way for developing foundations that are not only structurally efficient but also minimize environmental impacts of vibrational transmission. Future work may focus on parametric optimization, experimental validation, and evaluation of different soil types, aiming at practical applications in civil and geotechnical engineering.

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