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Calculation Models for Combined Bending and Axial Compression in Concrete-Filled Circular Steel Tube Columns

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Abstract. Steel-concrete composite columns are load-bearing elements that combine steel and concrete, with or without reinforcement, working together to resist axial compression and flexural-compressive forces. This solution stands out when compared to steel columns, as the integration with concrete enhances protection against corrosion and fire. Similarly, when compared to reinforced concrete columns, it enables the use of smaller cross-sections while maintaining the same strength. In this context, ABNT NBR 8800:2024, which sets the design standards for composite columns, proposes three calculation models for verifying the resistance to combined flexural and axial compression. The aim of this study is to evaluate the calculation methods outlined in the latest revision of the Brazilian standard, as well as to develop a computational tool using Mathematica Software to determine the flexural-compressive strength of concrete-filled circular sections. Additionally, numerical applications based on examples from the literature are used to compare the results obtained for each calculation method. Thus, the research not only contributes to a deeper understanding of the new regulatory criteria but also aims to assess and compare the calculation models for concrete-filled circular steel tubular columns.

Keywords: Concrete-filled circular steel tube columns; Non-compact sections; Combined bending and axial compression; Computational tool; Normative calculation models.

1 Introduction

According to Griffis [1], the earliest composite constructions in the United States employed steel sections encased in concrete, primarily as a means of providing protection against corrosion and fire. Since then, with the increasing use of this structural system, research has expanded to investigate the advantages of combining both materials in various structural elements, such as slabs, beams, and columns.

At present, it is well established that this solution offers superior performance compared to steel structures alone, particularly under critical conditions such as fire and corrosion, and delays the onset of local buckling in the steel section. Furthermore, when compared to reinforced concrete members, it enables a smaller section-to-strength ratio, meaning that columns of reduced cross-sectional dimensions can achieve the same load-carrying capacity. Additionally, when filled sections are employed, there is a reduction in formwork requirements and greater efficiency during the construction process [2].

In the Brazilian standard ABNT NBR 8800:2024 [3], which governs the design of composite columns, four types of steel–concrete composite columns are defined: partially and fully filled sections, as well as filled rectangular and circular sections. In the present study, the focus is on the circular concrete-filled steel tube (CFST) column, illustrated in Figure 2, where D and t denote, respectively, the external diameter and the wall thickness of the steel tube.

For design purposes, composite columns are also classified according to the slenderness of their cross-section, considering both compression and bending. In the case of circular CFST members subjected to bending, the Brazilian standard allows only two possible classifications for the tube wall slenderness parameter: compact and semi-compact sections.

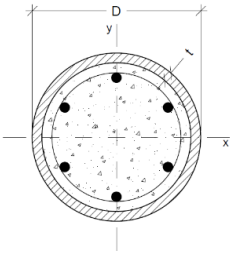


Figure 1: Concrete-filled circular section according to NBR 8800.

However, such slenderness restrictions are not enforced in many parts of the world, including South Korea, several European countries, the United States, and Australia, whose design standards — such as AISC 360-22 [4], Eurocode 4 [5], the Japanese Code [6], and the Australian Code [7] — allow the use of sections with greater slenderness. In this context, several researchers have made notable contributions to the subject. Lai and Varma [8] compared theoretical, code-based, and experimental results regarding the flexural–compressive performance of circular concrete-filled steel tube columns with semi-compact and slender sections. Lin and Zhou [9] developed a methodology to determine the axial compressive strength of short composite columns with circular cross-sections. Moon et al. [10] investigated the flexural behavior of CFST members through finite element modeling, varying the diameter-to-thickness (D/t) ratio and material properties. These studies demonstrated the efficiency of using non-compact CFST sections. Furthermore, other researchers, such as Wang et al. [11] and Varma [12], examined the load-carrying capacity of thin-walled CFST sections under seismic loading, obtaining favorable results for such applications.

Annex M of ABNT NBR 8800:2024 presents three analytical models for columns subjected to combined bending and compression: Model I is the most simplified and can be applied to any section; Model II is valid only for compact sections; and Model III applies exclusively to semi-compact or slender filled sections. Consequently, the main challenges of the present study are twofold: (i) to account for the criteria and formulations established by the relevant Brazilian design standard, and (ii) to determine the flexural strength of semi-compact circular CFST sections, given the absence of practical expressions for this purpose in the recently published NBR 8800.

Accordingly, the present research aims to automate the calculations and analyses of the methods proposed in ABNT NBR 8800:2024 for the flexural–compressive design of semi-compact circular concrete-filled steel tube columns, by employing programming tools, symbolic mathematics software, and calculation spreadsheets, thereby providing the theoretical foundation for the correct application of the Brazilian standard.

2 Methodology

Initially, a literature review was conducted on the design of steel – concrete composite columns subjected to combined bending and compression, focusing on the Brazilian standards — particularly ABNT NBR 8800:2024 — as well as the main international design codes and recent scientific articles on the subject. Based on these studies, calculation routines in the form of flowcharts were developed to sequence the design steps and guide the implementation of the computational tool.

For the study of semi-compact composite columns, whose main challenge lies in determining their geometric properties due to the circular cross-section, the Mathematica software (Wolfram Research, 2024) was employed to solve symbolic equations and numerical problems.

Since NBR 8800 [3] does not provide analytical expressions for calculating the bending moment, $M_{e,Rd}$, corresponding to the onset of compressive yielding in semi-compact circular composite sections — limiting itself to prescribing the stress distribution to be adopted (Figure 2) — it was first necessary to implement functions for calculating the volumes associated with the stress distributions illustrated in Figure 3. This figure depicts the stresses acting on both the steel tube (Figure 3a) and the concrete core (Figure 3b), with these volumes determined based on classical equations for wedges and spherical segments available in the literature. The sum of the volumes of each component thus makes it possible to compute the load-carrying capacity contributed by each material in the composite section.

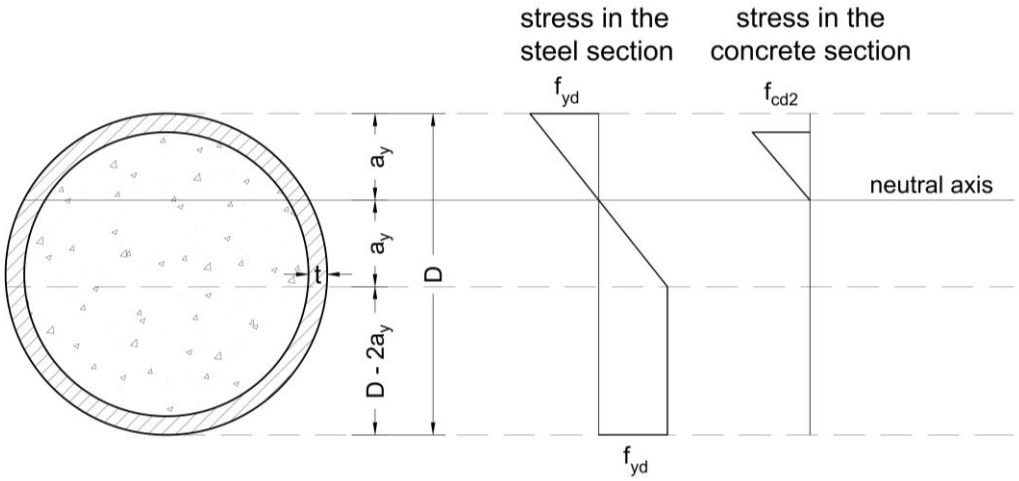
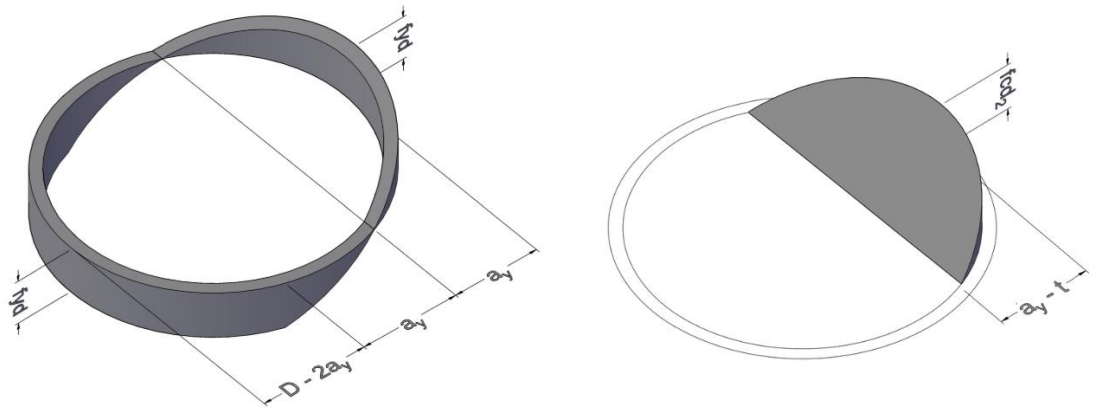


Figure 2: Stress distribution in a CFST section for the calculation of $M_{e,Rd}$.



(a) Stress distribution in the steel section of the composite member.

(b) Stress distribution in the concrete of the composite member.

Figure 3: Calculation of the resultant forces from the stress distribution in steel and concrete.

From this stage, a function was developed to sum the forces of all section components, aiming to determine the position of the neutral axis, a_y. Due to the complexity of the resulting function, the secant method was employed to estimate the value of a_y. This method is a numerical technique used to find roots of nonlinear equations by approximating the derivative through two successive points of the function. Once the neutral axis position was determined, equations from the literature were used to calculate the centroids of the analyzed volumes. Based on these values, it was then possible to perform the calculation of the bending moment corresponding to the onset of section yielding, $M_{e,Rd}$.

To verify the reliability of the developed tool, an auxiliary spreadsheet was created in Excel. This sheet, by discretizing the areas of the circular section, as shown in Figure 4, into 500 strips, approximately calculated the resultant force and the bending moment resistance, $M_{e,Rd}$. This approach allowed for a comparison between the results obtained and, consequently, the validation of the application developed in Mathematica.

Finally, after the determination and verification of the $M_{e,Rd}$, the remaining steps described by the standard for the design of CFTS columns were implemented in the Excel spreadsheet.

For validation, a circular filled section example was adopted, with a non-compact wall and without reinforcement, outer diameter of 152 mm and a wall thickness of 1.7 mm. The section was subjected to the following design loads: a normal force of 200 kN and a bending moment of 10 kNm, in addition to f_{ck}, f_{yk}, E_c and E_y equal to 92 MPa, 328 MPa, 45,400 MPa and 200,000 MPa, respectively.

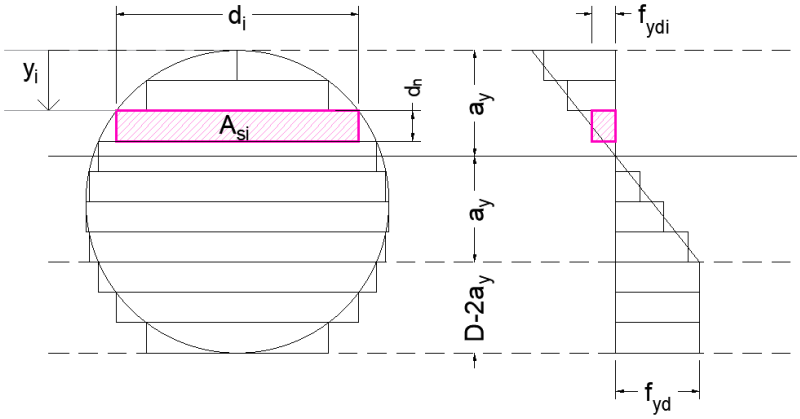


Figura 4: Discretization of the areas of the circular section and stresses used in the verification by Excel.

3 Results and discussions

For the validation of the computational program, the example cited in the previous section was solved using both the spreadsheet developed in this work and the Mathematica software. The results for the calculation of a_y and $M_{e,Rd}$ are presented in Table 1, considering the values obtained from the program developed in Mathematica as well as those calculated by the verification spreadsheet elaborated in Excel.

Table 1. Results obtained for a_y and $M_{e,Rd}$ using the Mathematica tool and the Excel verification tool

Tool	a_y (cm)	$M_{e,Rd}$ (kNcm)
Mathematica	4.356	1352.130
Excel verification	4.379	1345.336
Difference (%)	0.523%	0.505%

It is noted that the results found for both a_y, and $M_{e,Rd}$ are very similar, which validates the two proposed tools. The observed difference between the values (0.523% and 0.505%, respectively) is small and can be explained by the fact that the determination by the Excel tool is made by an approximate method, through the discretization of the volume into rectangular prisms. Thus, considering this, the solidity of the results can be confirmed, in addition to verifying the validity and efficiency of the computational tool for the calculation of the resistant bending moment.

Additionally, by applying the $M_{e,Rd}$ value obtained from the computational tool to the flexo-compression design spreadsheet, the results presented in Figures 5, 6 and 7 were obtained. These results relate to the section verification regarding axial compression, pure bending, and the standard models for combined bending and axial compression. Preliminarily, the good agreement of these results with those obtained by Lai [7] for an identical section is highlighted, reinforcing the reliability of the developed model.

It is noted that, in the example considered, the section is classified as compact for axial compression, which allows its strength to be based on the complete plasticization of the section. The results show that the demand is less than the design resistance. For flexural loads, the section is classified as non-compact, and in this case as well, the demands were below the calculated resistances.

However, since the column in question is subjected to combined bending and axial compression, its verification must be conducted according to the applicable normative models. In this context, considering it is a non-compact tubular section, Models I and III are adopted, since Model II is intended exclusively for compact sections. Thus, from the results obtained, it can be highlighted that, although both models yielded similar results, the design using the more simplified Model I presented a higher utilization ratio of 0.743 (74.3%) and is therefore more conservative than Model III, in which the ratio was 0.480 (48.0%).

Verification of axial compression capacity

Required data:		
γ_a	1.1	
γ_c	1.4	
γ_s	1.15	
f_{yd}	29.82	kN/cm²
f_{cd}	6.57	kN/cm²
f_{sd}	43.48	kN/cm²
α_c	0.58	
$(EI)_e$	131417416	kNcm²

λ	λ_p	λ_r
89.41	91.46	115.85
Section classification		
Seção Compacta		

$N_{pl,a,Rd}$	$N_{pl,c,Rd}$	$N_{pl,s,Rd}$	$N_{pl,Rd}$
239.35	1082.71	0.00	1322.06
$N_{y,Rd}$	$\sigma_{cr,Rd}$	$N_{p,Rd}$	
1037.14	31.52	1322.06	

N_e	$\lambda_{0,m}$	χ	N_{Rd}	Verification
28858.98	0.21	0.98	1296.95	NRd≥NSd. Thus, the section is adequate for the compressive loads.

Figure 5: Verification of axial compression capacity.

Flexural verification

Soliciting moment

$\lambda 0,m$	(EA)e	$\lambda 0,lim$	Situation
0.21	947919.8	1.07	The effects off concrete shrinkage and creep do not need to be considered.

m	e	ecc	Mcc	MSd + Mcc
-	5	0	0	10

Design resistance

λ	λ_p	λ_r	Section classification
89.41176471	54.88	189.02	**Non-Compact Section**

Zc (cm³)	Zcn (cm³)	Zs (cm³)	Zsn (cm³)	Za (cm³)	Zan (cm³)	hn (cm)
546.90	340.78	0	0	38.4047907	7.80	4.79
Mpl,Rd (kNcm)	ay (cm)	Me,Rd (kNcm)	MRd (kNm)	Verification		
1556.04	4.379	1345.34	15.02	Ok! The section is adequate for the bending forces.		

Figure 6: Flexural verification.

Combined bending and axial load verifications

Calculation Model I

NSd/NRd	MSd/MRd	Case study	Optimization	Verification
0.15	0.67	(NSd/2NRd)+(MSd/MRd)<=1	0.743	Ok! The section is adequate for the design forces.

Calculation Model III

csr	cp	cm	Case study	Optimization	Verification
0.17	0.56	1.67	NSd/NRd<cp	0.480	Ok! The section is adequate for the design forces.

Figure 7: Combined bending and axial load verification according to the models in ABNT NBR 8800:2024 [3].

4 Conclusions

This paper addressed the normative principles for the design of non-compact circular tubular composite columns filled with concrete (CFST columns), as established in the new edition of NBR 8800, by means of a practical example verification. Two computational tools were developed, one in Excel and another using the Mathematica software, with the objective of determining the position of the elastic neutral axis and the bending moment corresponding to the onset of yielding of the compressed region of the section, $M_{e,Rd}$. The results obtained from both tools proved to be consistent, allowing for the cross-validation of the methods and the subsequent determination of the section's resistant forces and moments, according to the normative models for flexo-compression verification.

In the context of the analyzed example, the results demonstrated the robustness and reliability of the developed tools. Furthermore, the comparison between the normative calculation models indicated that Model I, a more generalized application, provided more conservative results (utilization ratio of 74.3%) compared to Model III, which is specific for non-compact sections, and presented a ratio of 48.0%. Despite these differences, both models can be used according to ABNT NBR 8800:2024 [3].

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