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Topology optimization of vortex-induced vibrations using a geometry projection approach

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Abstract. Fluid-structure interactions are present in many engineering applications, occasionally as a desired effect for energy haversting and, more often, as a non-desired effect, for structures that are subjected to fluid flow. Within this field there are several different phenomena, due to the type of structure, type of flow and dynamics involved. One of the most studied topics is vortex-induced vibrations, in which the natural frequency of a structure under crossflow is similar to the natural frequency of vortex shedding, which excites the structure due to resonance and leads to instability. These methods have been extensively studied experimentally and computationally, although there is still a lack of gradient-based optimizations that deal with both solid and fluid equations coupled. Therefore, the present work proposes a topology optimization study of systems subjected to vortex-induced vibrations using the geometry projection method. The optimization algorithm combines the numerical simulations performed in Nektar++ with a Python script that performs the geometry projection and the optimization with SciPy. The sensitivities are determined in a continuum formulation. Preliminary results validates the sensitivity calculation according to the literature of fluid-structure interactions, and further work will implement the geometry projection methodology in the sensitivity and optimize passive controls positioning.

Keywords: Vortex-Induced Vibrations, Topology Optimization, Sensitivity, aeroelasticity.

1 Introduction

Li et al. [1] proposed a formulation for Fluid-Structure Interactions (FSI) numerical simulations using a non-inertial reference frame, analyzing transverse and angular motions separately. Robertson et al. [2] compared transverse and rotational galloping with quasi-static solutions, noting partial agreement despite inaccuracies. In the field of FSI, one of the most studied phenomena is Vortex-Induced Vibrations (VIV), which is characterized by resonance that occurs when the frequency of vortex shedding due to the presence of the bluff body in the flow is close to the natural frequency of the structure. For a circular cylinder, the vortex shedding frequency is approximately equal to $f_s = 0.2$. for this case, VIV can be observed for frequencies between approximatelu $0.1 \leq f_n \leq 0.3$, with a peak where the frequencies meet. These values can change slightly for different fluids and Reynolds. Linear stability simulations and adjoint sensitivity analysis of VIV were performed by Dolci [3].

Topology optimization (TO) is a branch of mechanical optimization that can lead to solutions that resemble nothing the original geometry, allowing as many changes as necessary, as well as the addition or removal of holes, in order to optimize an objective function. This process is characterized by a complete discretization of a domain and, allied with numerical simulations that replicate the loads and conditions of the system, this discretization is transformed into addition or removal of material. After successive iterations, the final topology of the system is obtained.

For FSI systems, the developments started with Yoon [4], which developed the first concepts and applied them to laminar flows, understanding how the coupling between the fluid and solid mechanics should work. Subsequently, Jenkins and Maute [5] amplified the methodology for a clamped structure under fluid forces. Picelli et al. [6] Introduced a binary topology optimization algorithm denoted TOBS. This methodology is suited for fluid problems since the discretization between solid and fluids are well defined. Recently, Siqueira et al. [7] applied these techniques together with automatic differentiation in order to optimize a splitter plate attached to a cylinder, seeking for an optimized geometry that minimized displacements, allied to natural frequency constraints of the system.

Another approach to topology optimization, based on geometry projections, was developed for solid mechanics by Norato et al. [8]. It adopts standard geometries (SGs) in a user-defined initial condition and optimizes their lenght, positioning and number of these standard geometries. This can lead to the genetration of different topologies, as many as possible by the amount of initial geometries. However, with the design variables being the SGs positioning and quantity, which are parameters, this formulation resembles a parametric optimization. Currently this approach has been developed for different approaches, but mainly in structural mechanics, by works such as Smith and Norato [9], Zhao et al. [10].

Although relevant for solid mechanics, there still lacks developments of the geometry projection method for fluid applications. Since the boundaries are well defined and the computational cost is lessen (due to the fact that only parameters are optimized), the optimization of FSI could benefit from it. Therefore, the present study aims at developing the framework for using topology optimization for systems under VIV, seeking to optimize its behaviour.

2 Formulation

2.1 Fluid-Structure System

Equation 1 defines the coupled fluid-structure system for the vortex-induced vibration analysis. This system is composed of the dimensionless form of the Navier-Stokes equation, as well as the ordinary differential equation of the system. Similar developments of these equations and definition of boundary conditions can be found on Dolci [3], Peron [11].

$$\begin{cases} \nabla \cdot \hat{\mathbf{u}} = 0, \\ \lambda \hat{\mathbf{u}} + \mathbf{U}\left(\hat{\mathbf{u}} \cdot \nabla\right) + \hat{\mathbf{u}}\left(\mathbf{U} \cdot \nabla\right) + \nabla \hat{p} - \nu \nabla^2 \hat{\mathbf{u}} = 0, \\ \lambda \hat{y} - \hat{y_1} = 0, \\ \dfrac{\pi m^*}{4} \lambda \hat{y_1} + \dfrac{\pi^2 \zeta m^*}{U_r} \hat{y_1} + \dfrac{\pi^3 m^*}{U_r^2} \hat{y} = L \end{cases} \tag{1}$$

Where $\hat{\mathbf{u}}$ and $\hat{p}$ are the normal mode approximation of the velocity vector field and pressure, respectively, ν is the fluid kinematic viscosity and $\hat{y}$ is the approximation for the displacement field. By the modal approximation, the eigenvalue corresponds to the derivative with respect to time. Also in Eq. 1, ζ corresponds to the damping coefficient and U_r and m^* are the fluid-structure parameters denoted reduced velocity and mass ratio, respectively. The first corresponds to the difference between the fluid velocity in the far stream to a relative velocity of oscillation on the solid. The mass ratio corresponds to the relation between the mass of the solid and a corresponding mass of fluid. For a more throrough analysis and derivation of all these parameters and assumptions, the reader is encouraged to read Peron [11].

2.2 Adjoint FSI system

The adjoint system for the FSI problem is considered by using the Lagrange Multipliers approach in which λ is the objective function and the system are the constraints. Then, defining the Lagrange multipliers as the adjoint variables, it is possible to reach (deriving the Lagrangian with respect to the state variables)

$$\begin{cases} -\lambda \widehat{\mathbf{u}}^\dagger - \mathbf{U}\nabla\widehat{\mathbf{u}}^\dagger + \nabla\mathbf{U}\widehat{\mathbf{u}}^\dagger - Re^{-1}\nabla^2\widehat{\mathbf{u}}^\dagger - \nabla p^\dagger = \mathbf{0} \\ \nabla \cdot \widehat{\mathbf{u}}^\dagger = 0 \\ \lambda \hat{\mathbf{y}}^\dagger + \frac{\pi^3 m^*}{U_r^2} \hat{\mathbf{y}}_1^\dagger - \int_{\partial\Omega_w} \nabla\mathbf{U} \cdot \sigma \cdot \mathbf{n} dS_w = \mathbf{0} \\ -\frac{\pi m^*}{4} \lambda \hat{\mathbf{y}}_1^\dagger + \frac{\pi^2 \zeta m^*}{U_r} \hat{\mathbf{y}}_1^\dagger - \hat{\mathbf{y}}^\dagger - \int_{\partial\Omega_w} \sigma \cdot \mathbf{n} \mathbf{d} S_w = \mathbf{0} \end{cases} \tag{2}$$

Where σ is the stress tensor and the supperscript $\dagger$ indicates that it corresponds to the adjoint variable. For more details on the derivation and boundary conditions, the reader is encouraged to read Dolci [3].

2.3 Sensitivity Analysis

As proposed by Dolci [3], There are two sensitivity analysis that can be done in order to evaluate the relation of the eigenvalue with respect to an external forcing. The first, denoted wavemaker, does not consider changes in the baseflow, only in the perturbed system. It is characterized with the Lagrangian, considering

$$\frac{\partial \lambda}{\partial \hat{\mathbf{f}}} \delta \widehat{\mathbf{f}} = \left\langle \hat{\mathbf{u}}^{\dagger}, \delta \hat{\mathbf{f}} \right\rangle = \left\langle \hat{\mathbf{u}}^{\dagger}, C(\mathbf{x}) \widehat{\mathbf{u}} \right\rangle \left\| \hat{\mathbf{u}}^{\dagger} \right\| \|\hat{\mathbf{u}}\| \mathbf{C}_0 = \left\| \hat{\mathbf{u}}^{\dagger} \right\| \|\hat{\mathbf{u}}\|$$

Therefore, the wavemaker regions are only dependant on the velocity magnitude regions of the direct and adjoint problems

2.4 Topology optimization approach

Norato et al. [8] proposes a topology optimization approach based on geometry projection, where the first step is to characterize a standard geometry and its positioning. Fig. 1 defines it, also defining its positioning and distance in relation to any specific point in the design domain. Finally, the sensitivities are calculated by characterizing its relation to the pseudodensities (which are implemented into the Navier-Stokes Equations as a forcing term) and its coordinates. This leads to the ability to characterize the sensitivity of the entire fluid-structure system as a chain rule, using the wavemaker defined in 2.3 together with the sensitivities from the geometry projection. For more details on this subject, Norato et al. [8] and Smith and Norato [12] provides a clear understanding of the methodology.

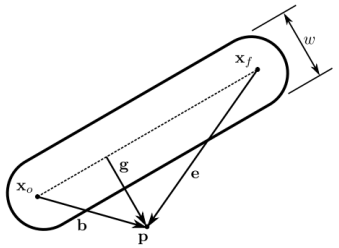


Figure 1. Standard geometry proposed by Norato et al. [8].

3 Results

Figure 2 provides a validation of the wavemaker region as defined by Dolci [3]. It is possible to see that the regions are mapped similarly, which indicates that the first part of the sensitivity calculation is well defined (the small differences in the scale is due to the field normalization). Further results will explore sensitivity calculations considering the chain rule in order to validate the geometry projection algorithm, as well as the implementation in an optimization algorithm in which the system tries to optimize the geometry of passive control addenda in the mitigation of VIV.

4 Preliminary Conclusions

The present study aims at defining an optimization algorithm based on the geometry projection method for Fluid-Structure Interaction Problems. It uses the direct and adjoint linearized Navier-Stokes equations, coupled with an ordinary differential equation of the system, as well as analytical definitions of the sensitivity and the adjoint system.

Preliminary results show coherence with the literature, meaning the entire fluid-structure sensitivity is validated. Further steps include the validation of this methodology considering the geometry projection, as well as an application in the mitigation of VIV in a system.

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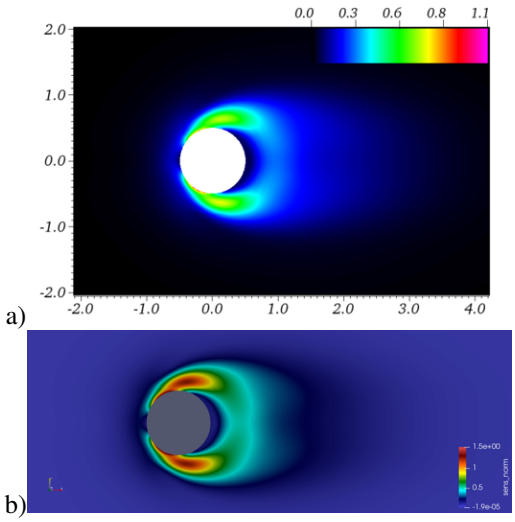


Figure 2. Validation of the Wavemaker regions for $Re = 33$,$m^* = 50$, $\zeta = 0$ and $U_r = 8$. a) Literature reference from Dolci [3]; b)current result

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