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A 3D Crack Propagation Model Based on XFEM/GFEM with a Hybrid Strategy for Crack Path Computation

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Abstract. Modeling three-dimensional crack propagation remains a challenging task in the context of computational methods, even for the widely used eXtended Finite Element Method/Generalized Finite Element Method (XFEM/GFEM) and its variants. Predicting the crack path for curved and non-planar crack surfaces is a subject of ongoing research and continuous improvement in 3D simulations. This work presents an XFEM/GFEM model for 3D crack propagation involving planar, curved, and non-planar cracks, in which a hybrid methodology for computing the crack direction is analyzed. The proposed approach for evaluating the crack direction combines the Linear Elastic Fracture Mechanics (LEFM) criterion with the Maximum Principal Stress (MPS) criterion, leveraging the strengths of each approach to address the inherent limitations of the model. The stress intensity factor (SIF) for each crack opening mode is calculated using the Displacement Correlation Method (DCM), and the proposed model can be applied to predict the crack paths of both pure and cohesive cracks in three dimensions.

Keywords: crack propagation, XFEM/GFEM, 3D, Displacement Correlation Method, SIFs

1 Introduction

The eXtended/Generalized Finite Element Method (XFEM/GFEM) (see [1], [2] for GFEM and [3], [4] for XFEM) has been used in the last decades for the simulation of a large number of mechanical phenomena. Among these, the prediction of crack coalescence and crack propagation is one of the most studied and published in literature. Although many of the models proposed in the context of XFEM/GFEM for crack propagation are 3D models, the modeling of three-dimensional crack propagation is still a very arduous task which remains opened to new technics capable of improving the performance of the model and the accuracy of results.

In the problems described by the Linear Elastic Fracture Mechanics (LEFM), some of the more important parameters are the stress intensity factors (SIFs), which are used mainly for prediction of the crack growth direction of a cracked elastic body under a provided loading. Amongst the numerical methods used for SIFs computation, the Displacement Correlation Method (DCM) is adopted here. The DCM is based on the asymptotic expansion of the elasticity solution in the neighborhood of a crack, thus its computation of SIFs relies upon the crack displacements developed in the vicinity of crack tip. More details of the methods used for SIFs computation in the XFEM/GFEM context and the performance of the DCM can be seen in Gupta et al [5].

Even being less accurate than other methods used for SIFs computation, like the energy release methods, the DCM is adopted here as a feasible alternative for SIFs extraction and its use for the crack path prediction have been proved to be advantageous from the perspective of ease of implementation and low computational cost. Additionally, the crack path direction computed with the DCM presents reasonable results and avoids some drawbacks presented by more accurate methods (see [5], [6]).

The use of different criteria for the determination of the crack growth direction has been presented in

literature and the principal stress criterion based on the averaged stresses around the crack tip is very popular and one of the more simple and intuitive criterion. Unfortunately, this criterion has limitations and is not sufficiently accurate in most cases, resulting in a deviation of the correct crack path (see, e.g., [7], [8], [9]). On the other hand, there are very popular criteria for crack propagation direction based on LEFM. A criterion used by various authors is the maximum circumferential stress criterion, based upon the evaluation of the stress intensity factors in 2D crack propagation simulations (see [10], [11]). Here, we adopt the Schöllmann's criterion [12], a LEFM criterion for crack direction computation which is proper to 3D problems, similar to the maximum circumferential stress criterion for 2D. Complementing the previous criterion, a Maximum Principal Stress (MPS) criterion is employed when the former is not suitable or is not able to compute accurately the crack propagation direction.

2 Proposed model

The crack propagation model here presented is implemented in the INSANE [13] system, an open-source software implemented in Java, an Object Oriented Programming (OOP) language. In order to simulate 3D crack propagation, we adopted an explicit representation of the crack surface, in which the surface is explicitly meshed independently of the volume mesh. The presence of the crack is inserted into the XFEM/GFEM analysis by a topological enrichment. Thus, the adopted enrichment applies the shifted Heaviside enrichment function at nodes of elements completely cut by the crack surface. Additionally, asymptotic crack-tip functions are employed for the enrichment only at nodes of elements that are intersected by the crack front. More details related to the crack surface representation, the adopted enrichment and the implementation of the DCM can be seen in [6]. For the prediction of crack front propagation direction the Schöllmann's criterion [12] is adopted, which relies upon the SIFs values, K_I, K_{II} and K_{III}. Additionally, the MPS criterion is adopted in order to complement the former criterion based on SIFs, as explained hereafter.

2.1 Schöllmann's criterion

The three-dimensional criterion proposed by Schöllmann [12] is based on the maximum principal stress. According to this criterion, the crack surface will grow radially from the crack front into the direction which is perpendicular to maximum principal stress σ_1'. In turn, the maximum principal stress σ_1' depends on the stress components σ_z, σ_φ and $\tau_{\varphi z}$ (see Fig. 1). The beginning of crack growth will occur if the maximum principal stress σ_1' in the vicinity of the crack front reaches a critical material parameter σ_{1C}'.

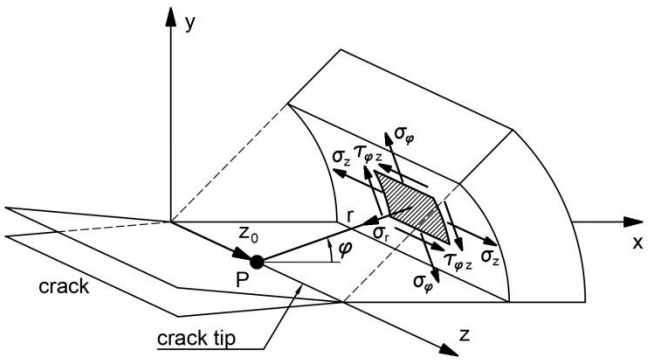


Figure 1. Stress components and cylinder coordinates at the crack tip [12]

With the maximum principal stress σ_1' and the stress components σ_z, σ_φ and $\tau_{\varphi z}$ a formulation for the crack deflection angles φ_0 and ψ_0 is developed (see Fig. 2). This formulation depends on the three-dimensional mixed-mode loading condition as a function of the SIFs K_I, K_{II} and K_{III}. The explicit formulas for angles φ_0 and ψ_0 as well its deductions can be seen in Schöllmann et al. [12].

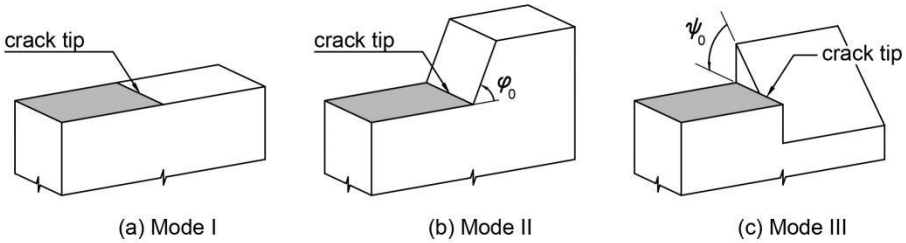


Figure 2. Crack growth for the basic fracture modes: (a) Mode I, (b) Mode II and (c) Mode III [12]

2.2 Hybrid strategy for crack direction computation

The adopted DCM formulations are based on displacements measured in any point located at the vicinity of the crack tip, not necessarily located at the nodal points of the finite elements. The approximations for the SIFs are evaluated at each vertex j located at crack front, using the components of the displacement jump across the crack faces, $[\![v(r)]\!] = v(r,\pi) - v(r,-\pi)$ computed at a distance r behind the crack front. The crack front coordinate system at a vertex j, can be seen in Fig. 3.

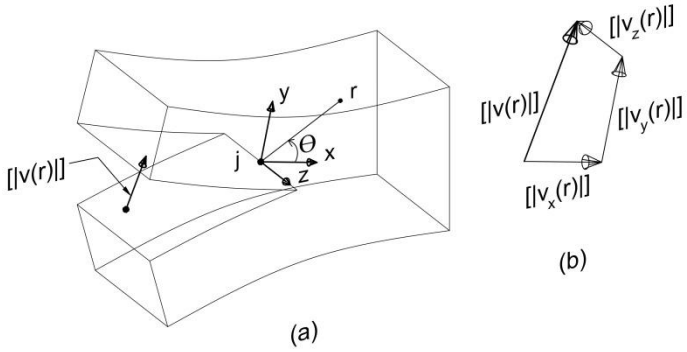


Figure 3. Displacement jump $[\![v(r)]\!]$ at a crack front vertex j: (a) $[\![v(r)]\!]$computed on crack faces at a distance r behind the crack tip (b) components of $[\![v(r)]\!]$ on the vertex coordinate system [6]

Several aligned points behind the crack tip vertex are selected and the displacement jump $[\![v(r)]\!]$ is computed for each one of them. Afterwards, the SIFs K_I, K_{II} and K_{III} are computed for groups of two subsequent points. Thus, the procedure of computing several sampling points for computing several SIFs values behind a crack front vertex is complemented by a linear least square extrapolation procedure, which is used for improving the results of K_I, K_{II} and K_{III}. The complete formulation and procedures can be seen in [5] and [6].

This DCM approach for SIFs computing relies on a minimal number of points located behind the crack tip vertices. Usually, the more points there are behind the crack front, the more accurate are the resultant SIFs. However, if the crack propagation starts from a region located at the boundaries of the specimen studied, as frequently occurs, there will be no points enough to compute the SIFs accurately. Consequently, with poor or nonexistent SIFs, the crack direction cannot be computed by any LEFM criterion.

On the other hand, the MPS criterion generally provides good results for crack direction when the numerical simulation is in its early stages, i.e., when the applied load factor is low and the crack length is short. Furthermore, when the crack length approaches its maximum length and the load factor reaches high values, the MPS fails to provide accurate results for crack direction. (see [8]).

Therefore, the presented model simply binds these criteria by using both along the full crack propagation: the MPS criterion and the LEFM criterion, formed by Schöllmann's criterion and supported by the DCM criterion for SIFs computation. In this way, the MPS is applied first, at the beginning of numerical simulation; when the DCM fails in provide good values for SIFs. Afterwards, when the crack has a sufficient length and the number of the sample points used for DCM computations is large enough, the SIFs are computed and the crack direction is evaluated with the Schöllmann's criterion.

This hybrid criterion, formed by the addition of a principal stress criterion with a LEFM criterion, is capable of provide very reasonable results for the crack direction values along crack propagation. The MPS criterion is always applied first and the shift for the LEFM criterion based on DCM for SIFs computation is triggered when the values of SIFs become consistent enough. In fact, the SIFs are computed by the DCM from the beginning of simulation and are monitored at each incremental step, without being applied. Meanwhile the crack direction is being computed by MPS results. As soon as the SIFs present consistent values, the criterion is changed definitively to the LEFM criterion (SIFs computed by DCM and Schöllmann's criterion for crack direction computation).

3 Numerical simulations

3.1 L-shaped panel

The geometry and boundary conditions are shown in Fig. 4. A mesh with 5851 linear tetrahedrons was used (see Fig. 4). The initial crack is positioned at the corner of the panel, where $x = 0$. The material parameters are: $E = 18.5\ GPa,\ \nu = 0.18,\ f_t = 2.5\ MPa,\ G_c = 0.13\ N \cdot mm^{-1}$. A cohesive propagation was applied to the specimen and the crack length was increased by $25\ mm$ at the end of step when the stresses ahead of crack tip reached the tensile strength f_t .

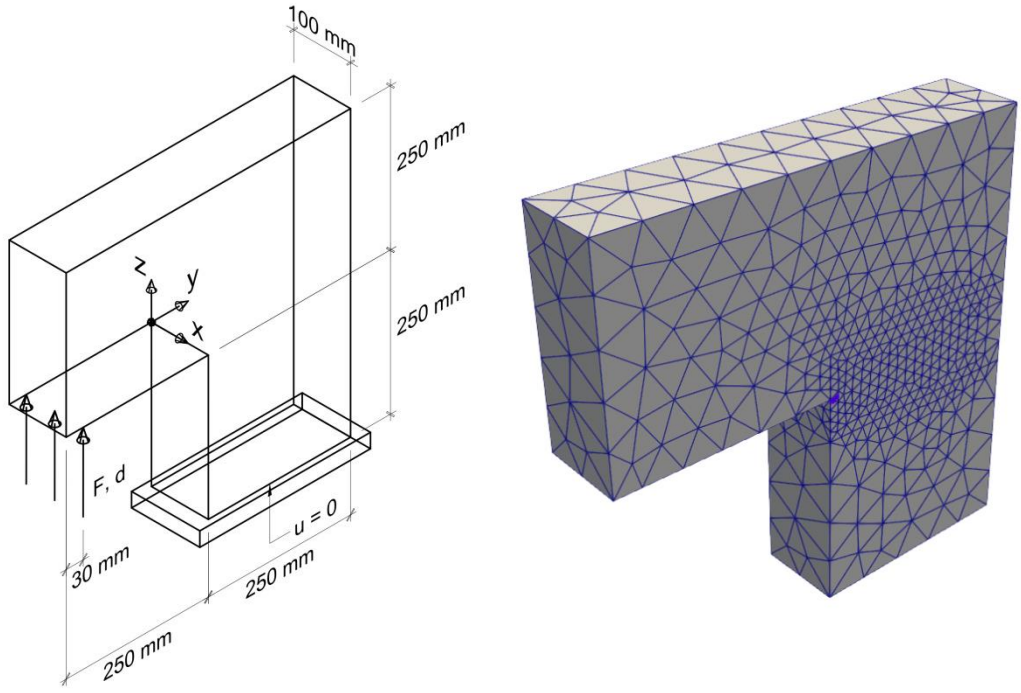


Figure 4. Geometry, boundary conditions and mesh with initial crack for L-shaped panel

In Fig. 5 the crack surface and the deformed mesh are shown, after 26 steps. Using the coordinate system shown in Fig. 4 (for x = 50), the comparison of numerical and experimental crack paths for L-shaped panel is represented in Fig. 6. The numerical results obtained by Ferté et al. [14] and the experimental results presented by Winkler [15] are compared with the results computed by INSANE.

Ferté et al. [14] state that the use of SIF-based criteria in cohesive models is not new and has been used by some authors in cohesive propagation only for the purpose of crack direction computation. Möes and Belytschko [10] used the SIF-based criteria by computing SIFs straight from the cohesive results. The same procedure was adopted here and the crack path obtained was based on the displacements results generated with cohesive forces included.

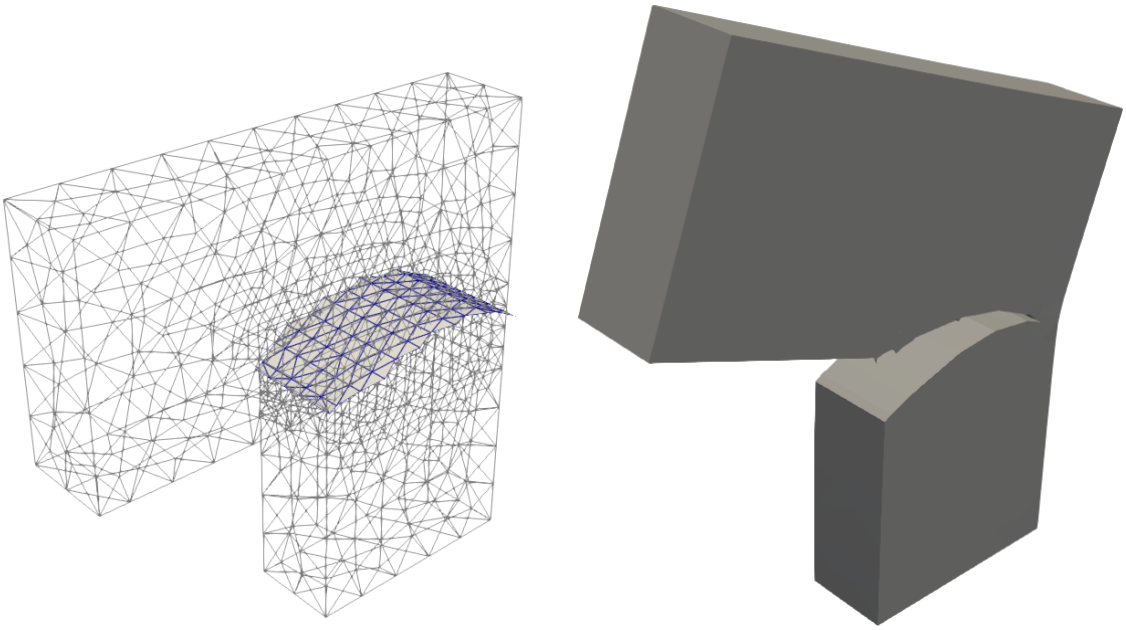


Figure 5. Crack surface and displaced mesh at step 26

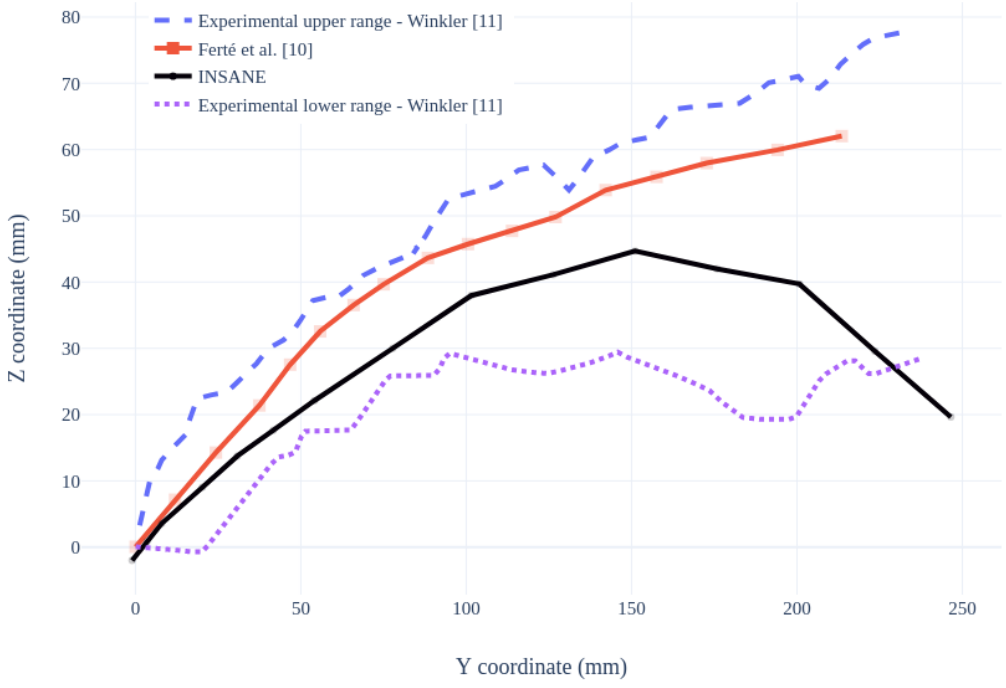


Figure 6. Numerical and experimental crack paths for L-shaped panel

4 Conclusions

The crack path of the L-shaped panel, resultant from the use of the hybrid strategy, presented great adherence to the results of the referenced authors. The crack deviation at the end of analysis can be attributed to the accumulated errors in crack direction computation, at each step, along the propagation. The accuracy of the SIFs computed with the DCM is usually poor, but even so, the use of the SIFs computed in that way, for the purpose of crack propagation direction, is very reasonable and can be acceptable for the prevision of the crack path both in cohesive and cohesionless crack propagation.

The hybrid criteria for crack direction computation yielded good results and is presented here as a feasible alternative for 3D crack propagation, presenting low time consuming for the crack direction computations and no interference in numerical stability.

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