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A Multimarket Study of Efficient Market Hypothesis in Granger Causality Networks

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Abstract. Granger causality analysis is a statistical tool used to identify predictive relationships between time series. This article applies this methodology to assets that make up indices of six countries—Germany, Turkey, Japan, India, Brazil, and the USA—with the aim of constructing a causality network for each of them that reveals the interdependencies between these assets. Using historical price data of the assets, the analysis aims to identify the directions and strengths of the predictive relationships of the method. By examining the global characteristics of the networks, this study seeks to advance mathematical knowledge about financial markets. Additionally, the analysis explores how the characteristics of the networks compare with the Hurst exponent of the respective assets and indices in order to identify relationships between their interpretations. The methodology applied in this study can also serve as a basis for future research on causality networks in different contexts and markets.

Keywords: Econophysics, Efficient Market Hypothesis, Granger Causality, Hurst Exponent, Networks

1 Introduction

Understanding the interdependencies between global financial markets has become a central issue in contemporary econometric literature, especially given the growing integration of capital markets and the rapid transmission of economic disturbances between different economies. The identification of causal relationships between financial assets can not only contribute to understanding the mechanisms of data transmission in markets, but also provides valuable information for risk management, portfolio diversification, and economic policy formulation.

Financial markets are characterized by complex networks of interactions, where the behavior of one asset can significantly influence the performance of others. This interdependence is particularly relevant when considering markets from different geographical regions and levels of economic development, such as Brazil, the United States, Germany, Japan, India, and Turkey, which represent a diversified sample of developed and emerging economies.

This study proposes an integrated approach that combines Granger causality tests and complex network analysis to investigate relationships between financial assets in six distinct markets. By examining multiple temporal lags and applying thresholds based on the distribution of F-statistics, we seek to identify the most significant causal relationships and construct networks that adequately reflect the structure of interdependencies. The subsequent analysis of the topological properties of these networks, together with their correlation with the Hurst exponent, may contribute to a deeper understanding of information transmission mechanisms and efficiency in the studied financial markets.

2 Theoretical Framework

2.1 Granger Causality

Granger causality is a statistical concept that allows for the analysis of causal relationships in temporal data and is used to determine if one time series helps predict another. Granger causality is based on a statistical method-

ology that allows inferring directions of influence between variables: a variable X "Granger-causes" a variable Y if knowledge of X's past significantly improves the prediction of Y, compared to what would be possible using only Y's history, as shown in Granger [1].

The Granger causality test allows determining if one variable precedes another temporally, providing evidence about possible causal relationships, as stated in Geweke [2]. The implementation of Granger causality involves an estimation process in which each variable is modeled as a linear function of its own lagged values and the lagged values of other variables in the system, as shown in Sims [3]. The statistical test consists of verifying whether the inclusion of lagged values of an explanatory variable significantly improves the predictive power of the model.

Formally, to test if a variable X Granger-causes a variable Y, the autoregressive model that incorporates the lagged values of both variables is estimated:

$$Y_t = \alpha + \sum_{i=1}^{p} \beta_i Y_{t-i} + \sum_{j=1}^{q} \gamma_j X_{t-j} + u_t, \tag{1}$$

where p and q represent the number of lags of Y and X, respectively, α is the intercept and u_t is the error term. The null hypothesis of no Granger causality ($H_0 : \gamma_1 = \gamma_2 = ... = \gamma_q = 0$) is tested to verify if the coefficients of the lagged values of X are jointly significant. If the null hypothesis is rejected by some test, it is concluded that X Granger-causes Y, indicating that knowledge of X past significantly improves the prediction of Y beyond what would be possible using only Y history.

2.2 Efficient Market Hypothesis

The Efficient Market Hypothesis (EMH) posits that financial asset prices reflect, in a rapid and automatic manner, all relevant available information, making pricing, in theory, perfect and instantaneous. Under this premise, originally proposed by Samuelson [4] and founded on the idea of completely rational investors, price fluctuations would follow a random course, eliminating the possibility of obtaining consistent abnormal returns, that is, returns above those adjusted for the asset's risk, and making the practice of arbitrage unfeasible. Consequently, buying or selling a security at the prevailing market price would never be a transaction with a positive Net Present Value (NPV), as shown by Mussa [5].

However, this theoretical conception of the EMH is idealized and does not find complete support in reality, where markets operate as approximations of a perfectly efficient system, as discussed in Mantegna [6]. Deviations like access to privileged information, financial crises, and the irrationality of agents themselves corroborate the idea that the EMH does not perfectly describe global markets. To classify the different degrees of information assimilation, the theory is divided into three levels: the weak form, in which prices reflect only the history of past quotations; the semi-strong form, which incorporates all available public information; and the strong form, which encompasses the entire set of existing information, whether public or private.

2.3 The Hurst Exponent

The Hurst Exponent represents an indicator that allows measuring persistence characteristics and long-range correlations in time series, widely employed in the evaluation of behavioral patterns and randomness in chronologically arranged information.

This parameter, represented by the notation h, habitually oscillates in the interval between 0 and 1. For $h = 0.5$, a random dynamic is identified, without long-range correlations or patterns in the observation series. For $h > 0.5$, the data possess persistence properties and tend to present long-range correlations, which suggests that behaviors identified at a specific instant tend to perpetuate subsequently. Cases where $h < 0.5$ demonstrate that the series manifests mean reversion or anti-correlated properties, revealing that the series tends to converge to its average or reference levels over time, as proposed in Hurst [7].

In the research developed by Cajueiro [8], developing and already consolidated markets were examined. The study employed information sets covering a four-year period, contemplating indicators from emerging economies from Asian continents and the Latin American region, in addition to indicators originating from two developed economies. It was found, through the determination of respective Hurst exponents, that consolidated economies demonstrate greater approximation with directional neutrality, presenting exponents close to the value 0.5, behavior not identified in emerging economies. According to the inferences of this work, this disparity results from the speed in data transmission, optimization in transaction mechanisms, and mobility of financial resources. The concept underlying this conclusion is founded on the principle that these three mentioned elements suffer diminution of

their effects through market rivalry, originated predominantly by the activity of large investor institutions. Hurst exponents that deviate from 0.5 indicate, therefore, a departure from the efficient market hypothesis.

2.4 Stock Markets Description

This study analyzes the market behavior of six different countries, selecting three developed countries (Germany, Japan and USA) and three developing, or emerging countries (Brazil, India, and Turkey). Thirty of the main assets were chosen for each of the analyzed stock markets.

Brazil: As the largest financial market in Latin America, Brazil's market is operated by B3 (Brasil, Bolsa, Balcão) in São Paulo. The main index is the Ibovespa. The market is heavily influenced by commodity companies, such as mining and oil, in addition to having a robust banking sector. It is classified as an emerging market, frequently impacted by domestic policies and fluctuations in global commodity prices, according to Clemente [9].

Turkey: The Turkish financial market, centered on Borsa Istanbul (BIST), is a dynamic emerging market whose main index is the BIST 100. It reflects a diversified economy with prominent industrial, financial, and service sectors. However, it is also characterized by significant volatility, influenced by geopolitical issues, as shown by Karime [10].

India: India's financial market is one of the fastest-growing in Asia, with two major exchanges, the National Stock Exchange (NSE) and the Bombay Stock Exchange (BSE). It is characterized by a large number of listed companies and a very high trading volume in derivatives. Growth has been driven by the information technology, financial services, and consumption sectors, according to Bala [11].

United States: As shown by Hafer [12], the U.S. financial market is the largest and most influential in the world, primarily composed of the New York Stock Exchange (NYSE) and the NASDAQ, with the latter focusing on technology. Dominated by a wide range of sectors and featuring high liquidity, its main indices, such as the S&P 500 and the Dow Jones, are global benchmarks.

Germany: The German financial market is the largest in the Eurozone, centered on the Frankfurt Stock Exchange (Deutsche Börse). Its main index, the DAX, is composed of the 40 largest "blue-chip" companies in the country. The market is characterized by a strong industrial and export-oriented base, with major influence in the automotive, chemical, and engineering sectors, as described by Döpke [13].

Japan: The Japanese financial market, operated by the Japan Exchange Group (which includes the Tokyo Stock Exchange), is one of the largest and most developed in the world. The most well-known benchmark index is the Nikkei 225. The market is home to many of the largest global companies in the automotive, electronics, and industrial sectors, according to Abhyankar [14].

3 Methodology

To investigate the relationships between financial assets over time in the six markets studied, we adopted an approach based on Granger causality, a statistical tool used to identify directional relationships between time series. This type of analysis allows us to verify whether the historical information of one asset contributes significantly to forecasting the future movements of another.

The Granger causality test was applied to assess whether the return series X_t Granger-causes the return series Y_t, that is, whether the past values of X_t and Y_t help predict Y_t better than the past values of Y_t alone. This test was performed for the Brazilian, U.S., German, Japanese, Indian, and Turkish markets, for all possible combinations of asset pairs in the datasets. The tests were conducted considering different lags, ranging from 1 to 10 time steps t. In this way, each lag investigates the predictive capacity of one series in relation to another when lagged by t. For each asset pair, the test was conducted in both directions: whether asset X Granger-causes Y, and whether Y Granger-causes X. Different lags were tested, and the corresponding F-statistics were recorded.

After obtaining the results of the Granger causality tests, Granger-causality networks were constructed for each lag and market. In these networks, financial assets represent the nodes, and the directed edges between nodes indicate the existence of a Granger-causal relationship. To determine these connections, we considered a threshold on the F-statistic for each lag and market, including only those relationships where the F-statistic value was in the third quartile among all calculated values. This approach uses the F-statistic threshold as a proxy for statistical significance, since lowering it would increase the associated critical F value. Thus, the resulting network reflects the most significant interactions between assets, according to the Granger test results.

Initially, the interest was in analyzing topological parameters of the Granger-causality networks and comparing them with other tools used in econometrics. The focus was directed toward analyzing the metrics of the assets (nodes) in each of the selected markets, enabling the search for features in common with the Hurst exponent, estimated using the multifractal detrended fluctuation analysis method, as described in Kantelhardt [15], an

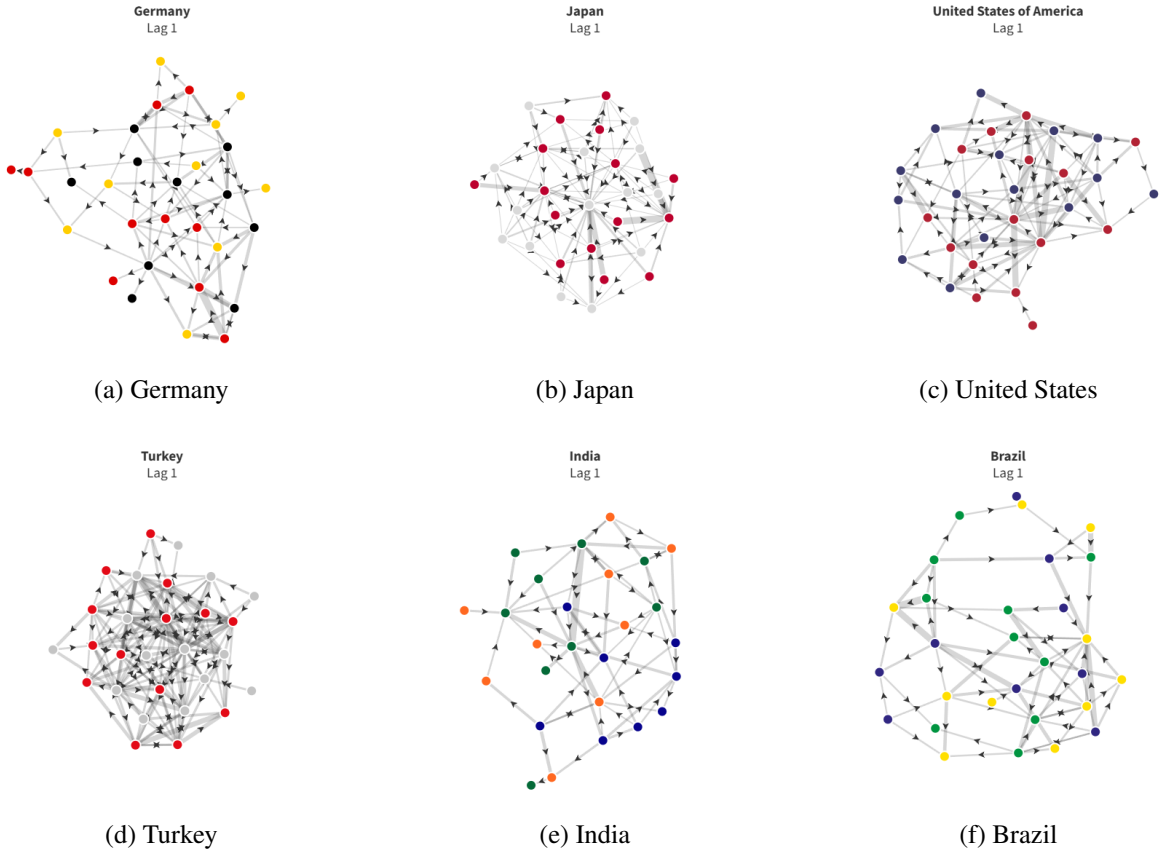
important tool in the study of the efficient market hypothesis. The node-level topological metrics studied were PageRank, degree centrality, betweenness centrality, and closeness centrality. Pearson correlations between the results for each of the above metrics and the canonical Hurst exponent ($q = 2$) were calculated in order to provide interpretations regarding the Granger-causal asset networks.

4 Results

4.1 Data Description

In this study, the input data consisted of closing price series of market assets from six different countries: Brazil, the United States, Germany, Turkey, Japan, and India. To perform the Granger causality tests, the first difference of the price series was used, which corresponds to the return series of the assets. This transformation is necessary to ensure the stationarity of the series, a *sine qua non* condition for the application of the Granger test.

The data collected and used correspond to 1-minute candle closing prices of 30 of the main assets from multiple stock exchanges, within their respective trading hours, over the period from July 7 to 15, 2025. In certain networks, some nodes may be disconnected—and therefore absent—due to the lack of edges that meet the selected criteria. However, in none of the cases did the networks experience significant loss of nodes. Figure 1 shows the networks obtained for lag 1.



(a) Germany (b) Japan (c) United States

(d) Turkey (e) India (f) Brazil

Figure 1. Lag 1 Granger-causality networks. Labels (a)–(f) represent the networks for each market analyzed.

4.2 Hurst Exponents, Centralities and Coreness

The Hurst exponents of each asset were estimated, and the average Hurst exponent for each of the analyzed markets was calculated. Table 1 presents the results obtained.

For the networks generated for each asset at each of the analyzed lags, the topological metrics PageRank, degree centrality, betweenness centrality, and closeness centrality were calculated. Each of these metrics was compared with the Hurst exponent calculated for the same asset using a Pearson correlation test. The results were

not statistically significant at the 10% level for the Brazilian, German, and Japanese markets at any lag. In the Indian market, on the other hand, statistically significant results at the 5% level were found only at lag two and only for PageRank ($r = 0.3694$ with p-value $= 0.0486$) and betweenness centrality ($r = 0.4850$ with p-value $= 0.0077$).

Table 1. Average Hurst exponents by market

Market	Average	Standard Deviation
Brazil	0.4654	0.0539
Germany	0.4761	0.0320
India	0.4744	0.0421
Japan	0.4999	0.0530
Turkey	0.4578	0.0423
USA	0.5190	0.0395

The results for the Turkish and U.S. markets, in turn, show statistically significant correlations for multiple lags. Table 2 presents the results obtained for these two markets.

Table 2. Pearson coefficients r and p-values for EUA and Turkey by lag for Hurst-centrality/coreness measure

Lag	Measure	EUA (r, p)	Turkey (r, p)
1	PageRank	(0.3765, 0.0403)	(-0.0717, 0.7169)
	Degree Centrality	(0.4629, 0.0100)	(-0.0822, 0.6774)
	Betweenness Centrality	(0.3293, 0.0756)	(-0.0948, 0.6313)
	Closeness Centrality	(0.3478, 0.0596)	(0.0014, 0.9943)
2	PageRank	(0.4015, 0.0279)	(-0.1256, 0.5243)
	Degree Centrality	(0.4582, 0.0109)	(-0.1669, 0.3959)
	Betweenness Centrality	(0.3301, 0.0748)	(-0.1100, 0.5775)
	Closeness Centrality	(0.4463, 0.0134)	(-0.0967, 0.6245)
3	PageRank	(0.3025, 0.1042)	(-0.1625, 0.4087)
	Degree Centrality	(0.4086, 0.0250)	(-0.1772, 0.3671)
	Betweenness Centrality	(0.3394, 0.0665)	(-0.1659, 0.3990)
	Closeness Centrality	(0.4592, 0.0107)	(-0.1129, 0.5673)
4	PageRank	(0.3092, 0.0964)	(-0.2233, 0.2535)
	Degree Centrality	(0.4018, 0.0278)	(-0.2509, 0.1979)
	Betweenness Centrality	(0.3914, 0.0324)	(-0.1971, 0.3146)
	Closeness Centrality	(0.3984, 0.0292)	(-0.2177, 0.2658)
5	PageRank	(0.3018, 0.1051)	(-0.2623, 0.1775)
	Degree Centrality	(0.4213, 0.0204)	(-0.2779, 0.1522)
	Betweenness Centrality	(0.3201, 0.0846)	(-0.2400, 0.2186)
	Closeness Centrality	(0.4478, 0.0131)	(-0.2398, 0.2190)
6	PageRank	(0.1929, 0.3072)	(-0.3956, 0.0411)
	Degree Centrality	(0.2692, 0.1503)	(-0.4229, 0.0280)
	Betweenness Centrality	(0.1987, 0.2926)	(-0.4567, 0.0166)
	Closeness Centrality	(0.2582, 0.1684)	(-0.3857, 0.0470)

Lag	Measure	EUA (r, p)	Turkey (r, p)
7	PageRank	(0.1374, 0.4691)	(-0.3884, 0.0411)
	Degree Centrality	(0.1816, 0.3369)	(-0.4002, 0.0349)
	Betweenness Centrality	(0.1328, 0.4843)	(-0.5055, 0.0061)
	Closeness Centrality	(0.2130, 0.2584)	(-0.3809, 0.0455)
8	PageRank	(0.1430, 0.4510)	(-0.4295, 0.0225)
	Degree Centrality	(0.1469, 0.4386)	(-0.4401, 0.0191)
	Betweenness Centrality	(0.1501, 0.4285)	(-0.5593, 0.0020)
	Closeness Centrality	(0.1462, 0.4408)	(-0.4093, 0.0306)
9	PageRank	(0.0705, 0.7113)	(-0.4342, 0.0210)
	Degree Centrality	(0.0465, 0.8074)	(-0.4422, 0.0185)
	Betweenness Centrality	(0.0617, 0.7462)	(-0.5408, 0.0030)
	Closeness Centrality	(0.0240, 0.8999)	(-0.4190, 0.0265)
10	PageRank	(0.0799, 0.6749)	(-0.4560, 0.0147)
	Degree Centrality	(0.0358, 0.8511)	(-0.4574, 0.0144)
	Betweenness Centrality	(0.0655, 0.7310)	(-0.5881, 0.0010)
	Closeness Centrality	(0.0219, 0.9087)	(-0.4314, 0.0219)

The results above reveal statistical significance for most measures in the U.S. market for lags one through five, where positive correlations are observed. In the Turkish market, on the other hand, statistical significance is found at lags six through ten, with negative correlations observed. Several hypotheses can be proposed in an attempt to explain the presence of correlations at different lags for both markets. One of these is the asymmetry in information absorption, which varies from market to market and is generally faster in developed markets. Trading in the Turkish market may, therefore, experience a delay. Another hypothesis that possibly partially explains the phenomenon is the greater persistence of long-range correlations in the time series related to emerging markets. Many other hypotheses can be raised, and most likely, none of them alone would fully explain the phenomenon, which is probably explained by an amalgamation of factors.

Table 1 shows that the results found for the Hurst exponent in the U.S. market are predominantly above 0.5, while those found for the Turkish market are, on average, below 0.5. Comparing these results with the statistically significant findings in Table 2 suggests that, for Hurst exponent values below 0.5, increasing the Hurst exponent causes the topological metrics to decrease, whereas for values above 0.5, increasing the Hurst exponent causes the topological metrics to increase as well. In other words, approaching perfect efficiency, where the Hurst exponent equals 0.5, leads to a decrease in the analyzed topological metrics of the networks.

The above interpretation leads to specific conclusions for each of the topological metrics. For degree centrality, the results indicate that markets with assets less connected to other assets tend to exhibit higher efficiency. For betweenness centrality, the results indicate that markets with assets less likely to mediate paths between other randomly selected assets also tend to be more efficient. For closeness centrality, the results point to the idea that markets with assets having a lower capacity to connect directly with any other randomly selected asset will be more efficient. Finally, the results for PageRank suggest that markets with assets that have few connections—especially to other high-PageRank assets—tend to exhibit higher efficiency. In summary, the results suggest that markets with highly connected and interdependent assets tend to deviate from the efficiency proposed by the efficient market hypothesis.

5 Conclusions

The present study aimed to investigate Granger–causality networks of asset markets in terms of their topological characteristics and to validate these networks as tools capable of advancing our understanding of markets, especially with respect to the efficient market hypothesis. By comparing these networks with instruments used in other studies of the hypothesis, such as the Hurst exponent, we obtained results indicating a certain efficacy of Granger–causality networks in the analysis of assets and markets.

The search for correlations between other methods and the proposed networks yielded results that, although not present in every test conducted, provided interpretations for the behavior of both emerging and developed markets. The attributes of these markets, of the assets themselves, or even the specific features of these networks that give rise to the observed outcomes remain unknown and may be explored in future work on the subject. However, the correlations found between the canonical Hurst exponent and the topological metrics of market networks indicate behavior consistent with the nature of the exponent regarding market efficiency, since the same pattern is observed as one approaches full efficiency.

The analysis suggests that, as a market approaches perfect efficiency, where the Hurst exponent equals 0.5, the centrality and connectivity metrics of assets within the causality network tend to decrease. Therefore, we conclude that markets characterized by highly connected and interdependent assets, which act as important intermediaries, tend to deviate from market efficiency. This indicates that efficiency is associated with a lower degree of interconnection in the causality network. Such an interpretation corroborates the efficient market hypothesis, as a market participant may hedge against adverse shocks by selecting less connected assets, thereby reducing risk but concomitantly choosing assets that approach efficiency, which are generally associated with fewer profit opportunities. The observed differences in lag structures can be partially explained by hypotheses such as asymmetric speeds of information absorption across markets, although this phenomenon likely results from a combination of multiple factors.

Despite the positive findings from the Granger–causality network analysis, the method remains experimental, and future work subjecting the tool to more robust tests is warranted.

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