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Using the Arruda-Boyce Elasto-Viscoplastic Constitutive Model to Predict the Monotonic Compressive Response of the PC/ABS 80:20

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Abstract. This study evaluates the capability of the Arruda–Boyce elasto-viscoplastic model to predict the monotonic compressive response of the PC/ABS 80:20 polymer blend. A finite element model of a standard compression specimen, following ASTM D695, was developed in the commercial package ABAQUS. The model incorporates geometric, contact, and material nonlinearities via a custom VUMAT routine. It effectively captured the material's nonlinear response, including strain softening, and showed good agreement with experimental stress–strain data. The work emphasizes the superiority of force–displacement data for constitutive calibration and demonstrates the potential of population-based optimization methods using a one-dimensional viscoplastic model implementation. These findings establish a solid basis for further extension of the model to more complex loading scenarios.

Keywords: Viscoplasticity, Constitutive Modeling, Finite Element Analysis (FEA), Polymer Mechanics, Force-Displacement Calibration

1 Introduction

1.1 Context and motivation

In mechanical and materials engineering, predictive understanding of material behavior under service conditions is fundamental to informed design decisions. Questions regarding component geometry, structural integrity, lifetime, and cost-efficiency are ultimately grounded in the ability to anticipate how a material deforms, yields, or fails under prescribed loads. While physical prototyping provides a direct route to such knowledge, it is often costly and time-consuming. In contrast, computational simulations—when paired with robust constitutive models—offer a powerful alternative for evaluating mechanical performance in a virtual environment, with reduced development time and cost.

Thermoplastics have gained widespread relevance due to their favorable mechanical properties, recyclability, and versatility in applications where metals are either suboptimal or unusable. These include biomedical devices such as cranial implants [1], drug delivery systems [2], cartilage scaffolds and polymeric valves [3] as well as engineering components like gears [4] and protective casings. To faithfully reproduce the mechanical response of such components in simulation, it is essential to characterize the behavior of thermoplastics under realistic and often extreme loading conditions—particularly in the large deformation regime, where the material's nonlinearities become more pronounced and symmetry between tensile and compressive responses is lost [5-6].

A growing body of literature has addressed the development and calibration of constitutive models for polymers under finite strains, incorporating hyperelasticity, viscoplasticity, and damage mechanics [7-12]. Nevertheless, there remains a need for efficient strategies to tailor these models to specific polymer blends and loading scenarios. This work focuses on a PC/ABS 80:20 polymer blend, composed of 80% polycarbonate (PC) and 20% acrylonitrile-butadiene-styrene (ABS), which combines impact resistance with thermal and mechanical stability. This blend is employed across a range of structural and consumer applications, including electronic housings, helmets, and rapid prototyping.

To model such materials under complex loading conditions, data-driven surrogate modeling offers a promising path. Artificial Neural Networks (ANNs) have demonstrated excellent capability in learning nonlinear input–output relations from experimental or numerical datasets, enabling fast and reliable predictions with minimal computational cost [13-15]. However, the effectiveness of such machine learning models critically depends on the quality and physical relevance of the training data. This, in turn, requires the construction of a well-calibrated, physically based constitutive model that accurately captures the material's mechanical response across the relevant loading scenarios. By ensuring that the training dataset reflects the true underlying physics, one improves both the predictive power and the generalizability of the surrogate model. Thus, rather than replacing traditional modeling approaches, data-driven tools complement them—serving as computational accelerators or design exploration tools built on top of validated mechanistic frameworks.

1.2 Objective

This study aims to demonstrate the prediction capability of the Arruda-Boyce model, implemented via a VUMAT subroutine on the commercial software ABAQUS, for this unique polymer blend, laying the foundations needed to develop more complex studies, such as parameter identification problems through inverse modeling, comparison between the Arruda-Boyce and other rheological-based viscoplasticity models and data-driven description of the compressive mechanical behavior, all under monotonic or cyclic loadings.

2 Numerical implementation

2.1 Compressive Test Reproduction by Finite Element modeling

The compression test analyzed in this study adheres to the specifications outlined in the ASTM D695 standard [16]. Consequently, the formulation of the Finite Element (FE) model was carefully designed to comply with the standard's requirements while accurately representing the key physical phenomena involved in the test. The FE model incorporates geometric nonlinearity to capture the barreling effect, a typical deformation pattern in polymeric specimens under compression. Boundary condition nonlinearity is addressed through the implementation of frictional contact between the specimen and the compression platens. Additionally, material nonlinearity is considered via a user-defined VUMAT subroutine implementing the Arruda-Boyce viscoplastic constitutive model. The VUMAT employed in this work was adapted from the implementation by M. Flávio [13], which originally included the more complex Mulliken-Boyce viscoplastic model. The Mulliken-Boyce formulation features an additional parallel network in the computation of the back-stress tensor, resulting in a larger number of material parameters to calibrate. In contrast, the Arruda-Boyce model was chosen for this study due to its relatively simpler structure, while still providing robust predictive capability across a broad range of loading conditions and polymeric materials. The FE problem is formulated using a displacement-controlled approach: the upper compression platen is subjected to a prescribed vertical displacement. Frictional contact is enforced between the specimen and the platens, and a symmetric boundary condition is applied on the mid-edge of the specimen (zero displacement in the XY plane) to enhance numerical stability without compromising accuracy. Figure 1 illustrates the implemented FE model configuration.

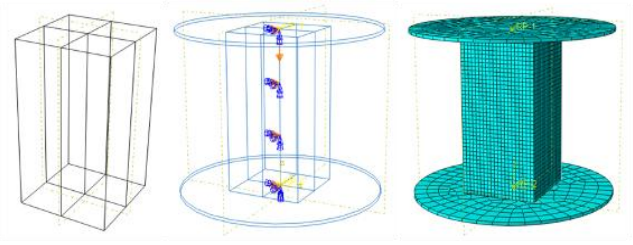


Figure 1. Visual details of the FE model implementation: a) Model's partitions. b) BCs. c) Mesh.

2.2 Constitutive modelling

The Arruda–Boyce (AB) elasto-viscoplastic constitutive model [17] is grounded in the micromechanical interpretation of the deformation behavior of polymeric materials. It assumes that the material's resistance to deformation arises from two parallel contributions—commonly referred to as networks "A" and "B". These two networks capture distinct physical mechanisms related to the rearrangement and alignment of polymer macromolecules under strain. Network A primarily accounts for the resistance due to the *rotation* and *uncoiling* of macromolecular chains at small to moderate strains. Network B represents the *strain-hardening* behavior associated with the alignment and stretch of chains as deformation progresses into the large-strain regime.

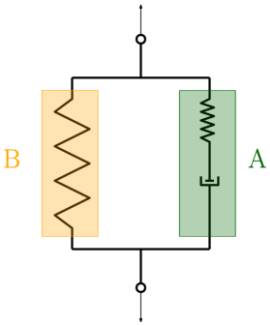


Figure 2. One-dimensional rheological interpretation proposed by the AB model. Source: Adapted from [18].

Mathematically, the total Cauchy stress tensor, T, is the additive combination of both networks:

$$\mathbf{T} = \mathbf{T_A} + \mathbf{T_B}\,, \tag{1}$$

where the contribution from network A is given by:

$$\mathbf{T_A} = \frac{1}{J^e}(2\mu\mathbf{E^e} + \lambda^e \mathrm{tr}[\mathbf{E^e}]\mathbf{I}), \tag{2}$$

and the contribution from network B, capturing strain hardening, is:

$$\mathbf{T_B} = \frac{C_R}{3}\frac{\lambda_{lock}}{\lambda_{chain}}\mathcal{L}^{-1}\left(\frac{\lambda_{chain}}{\lambda_{lock}}\right)\mathbf{B}^*_{dev}\,. \tag{3}$$

Here, $\boldsymbol{E}^e = ln(\boldsymbol{V}^e)$ is the Hencky deformation tensor, which is evaluated from the elastic portion of the deformation gradient, $\boldsymbol{F}^e_{\boldsymbol{A}} = \boldsymbol{F_A}\boldsymbol{F}^{p-1}_{\boldsymbol{A}}$. Superscripts "$e$" and "$p$" denote the elastic and plastic components, respectively. Plastic evolution in the model is governed by a flow rule that defines the evolution of the plastic multiplier as:

$$\dot{\gamma}_p = \dot{\gamma}_0 exp\left[-\frac{\Delta G}{k_R\theta}\left(1-\frac{\tau}{s+\alpha p}\right)\right]. \tag{4}$$

Finally, μ^e and λ^e are the Lamé constants relative to the material's elastic properties, $\boldsymbol{B}^*_{dev}$ is the distortional-deviatoric version of the left Cauchy-Green tensor, λ_{lock} and λ_{chain} are the macromolecules extensibility factors from the Eight-Chain hyperelastic model, C_R is the elastomer module, k_B *is the Boltzmann Constant*, θ is the absolute temperature, ΔG is the activation energy, $\dot{\gamma}_0$ is called the pre-exponential factor, α_p is the pressure coefficient, $\mathrm{p} = 1/3\ \mathrm{tr}[T_A]$ is the hydrostatic pressure. The internal shear resistance "s" evolves toward a steady-state value, S_{ss}, according to the following kinetic rule:

$$s_{i+1} = h\left(1-\frac{s_i}{S_{ss}}\right)\dot{\gamma}_p\,, \tag{5}$$

$$\mathrm{s_0} = \frac{0.077\mu^{\mathrm{e}}}{1-\nu}, \tag{6}$$

Where h and S_{ss} are constants and ν is the Poisson ratio. This formulation enables the model to capture both rate-dependence and nonlinearity in polymer deformation, making it especially suitable for large-strain simulations.

2.3 Experimental data

The experimental dataset used to assess the predictive accuracy of the constitutive models consists of a true stress–strain curve derived from force–displacement measurements obtained during uniaxial compression tests. This test adheres to the ASTM D695 standard and was performed on polymer specimens of PC/ABS 80:20. The raw force–displacement data were converted into engineering stress–strain values, and subsequently transformed into the true stress–strain domain using the following standard relationships:

$$\varepsilon_{true} = ln\big(1 + \varepsilon_{eng}\big), \tag{7}$$

$$\sigma_{true} = \sigma_{eng}\big(1 + \varepsilon_{eng}\big). \tag{8}$$

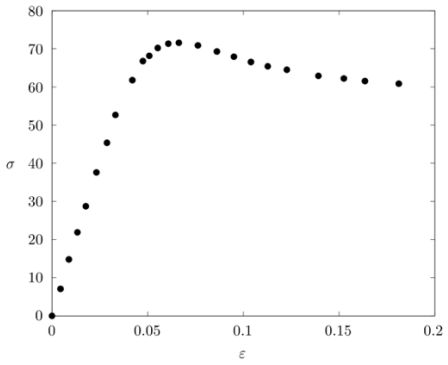


Figure 3. Experimental data translated from experimental force-displacement data through the eqs. (7) and (8).

These transformation formulas are derived under the assumptions of incompressible and homogeneous deformation. However, such assumptions become less valid in the presence of barreling, frictional effects, and non-uniform strain localization, which are typical in large-strain compressive behavior of polymers. As a result, the stress–strain data obtained through this method is not fully representative of the entire specimen volume, especially near the boundaries and in high-deformation zones.

Therefore, when using these data for constitutive model evaluation, caution must be exercised. A strain threshold must be defined to ensure the comparison remains within a regime where the assumptions of uniform deformation hold reasonably well. This restriction helps avoid spurious conclusions about model performance that could arise from comparing against data collected under kinematic and boundary-condition distortions.

From a model calibration standpoint, using true stress–strain data derived from force–displacement measurements is not ideal. In regions affected by barreling, contact-induced friction, or strain localization, the local stress and strain fields become highly heterogeneous. As a result, constitutive parameters tuned against such data might not represent the intrinsic material behavior but rather an artifact of the testing configuration. For more accurate and generalizable calibration, alternative approaches—such as inverse finite element techniques using force–displacement data directly—are more appropriate.

Nonetheless, the experimental curve (Fig. 3) offers valuable insights into the distinct compressive mechanical response of the PC/ABS 80:20 polymer blend. The material exhibits an initial elastic regime followed by a strain-softening response and eventually transitions into a strain-hardening regime. This behavior stems from the internal microstructural mechanisms of the polymer. Initially, the entangled macromolecular chains resist deformation through rotational and configurational entropy. As strain increases, these chains begin to reorient and uncoil, leading to a temporary reduction in stiffness (softening). Eventually, as the chains align in the direction of maximum stretch—corresponding to the barreling direction under compression—molecular alignment contributes to increased material resistance, resulting in strain hardening. This complex three-phase mechanical signature highlights the necessity of using physically informed constitutive models that can capture both entropic elasticity and viscoplastic flow.

3 Results and discussion

The results presented in this section build upon an initial sensitivity study conducted by M. Flávio [15] on the Mulliken-Boyce elasto-viscoplastic model. The Arruda-Boyce (AB) model was subsequently adapted and manually tuned to align with the experimental data obtained for the PC/ABS 80:20 blend. Fig. 4a illustrates the AB model's predictive performance against the experimental true stress–strain data. It is noteworthy that the model successfully captures the strain-softening behavior occurring at the end of the elastic regime and maintains an absolute error below 10% across the targeted strain range (Fig. 4b).

To further contextualize the AB model's capabilities, a comparison was made with a classical plasticity model featuring isotropic hardening, defined by the following polynomial expression:

$$\sigma_{yield}(\varepsilon^p) = (P_1 + P_2\varepsilon^p)[1 - P_3\, exp(-P_4\varepsilon^p) + P_5]. \tag{9}$$

This model was calibrated using a gradient-based optimization algorithm, employing the experimental true stress–strain data as the target. The comparison, shown in Fig. 5, reveals that while the isotropic hardening model fits post-yield hardening behavior, it fails to capture the nonlinear elasticity observed in the initial deformation regime. This is expected, given that classical plasticity models assume linear elastic behavior prior to yielding, thereby disregarding any pre-yield nonlinearities. In contrast, the AB viscoplastic framework accommodates both pre-yield stiffness variation and post-yield flow characteristics.

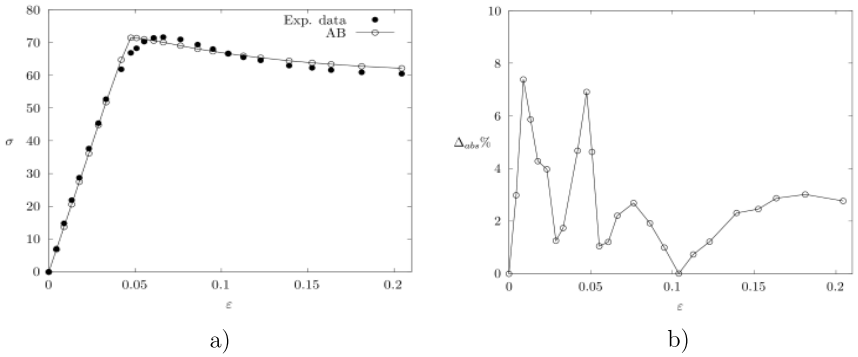


Figure 4. a) AB Stress-Strain prediction compared to experimental data. b) Absolute percentage error between the AB prediction and the experimental stress value for each strain level.

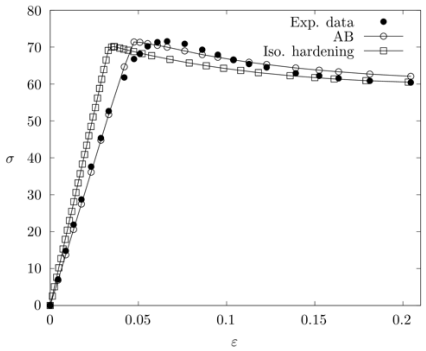


Figure 5. Comparison between the AB viscoplastic constitutive model and a plasticity model with isotropic hardening.

These findings emphasize the superior representational capability of the AB model under monotonic compressive loading and validate its theoretical framework for capturing the distinctive mechanical signature of thermoplastic blends. Moreover, this sets the stage for future extensions of the model to simulate more complex scenarios, including cyclic loading, strain rate dependence, and loading–unloading cycles.

For more robust material characterization, particularly under large deformations, future work should pivot toward

parameter identification based on raw force–displacement (f–d) data rather than derived stress–strain curves. Force–displacement data offer the most direct and least manipulated representation of experimental observations, capturing boundary effects, barreling, and contact interactions that are otherwise smeared out or misrepresented in stress–strain transformations.

To explore this idea, the Bergström–Boyce (BB) viscoplastic model was implemented in a one-dimensional format and embedded within a genetic algorithm (GA) optimization scheme. This approach allowed for global search over the parameter space without requiring a precise initial guess—often a limitation of gradient-based methods. Seven model parameters were initialized with random values in the range [0, 500], chosen arbitrarily. Figure 6 demonstrates the resulting BB model prediction, which was able to fit the target data with satisfactory accuracy. This parameter set could then be used as a robust initial guess for local refinement using gradient-based techniques.

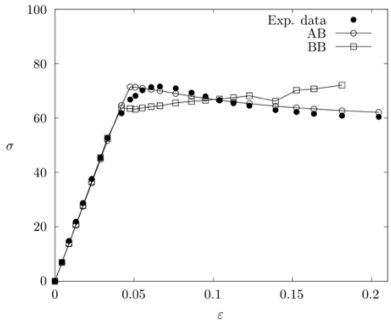


Figure 6. GA tuned BB constitutive model comparison with the VUMAT implemented AB constitutive model prediction.

4 Conclusions

This study explored the ability of the Arruda–Boyce elasto-viscoplastic constitutive model to predict the monotonic compressive response of the PC/ABS 80:20 polymer blend, using finite element simulations grounded in the ASTM D695 standard. By incorporating geometric nonlinearity due to barreling, frictional contact boundary conditions (BCs), and advanced material behavior via a custom VUMAT subroutine, the finite element framework was tailored to replicate realistic test conditions.

The results demonstrated that the Arruda–Boyce model, even when manually calibrated, accurately captured the nonlinear features of the stress–strain response, including the distinct strain-softening that characterizes the transition between molecular rotation and alignment in thermoplastics. Compared to simpler plasticity models with isotropic hardening, the AB model showed clear superiority in modeling the pre-yield nonlinearities and viscoplastic hardening mechanisms inherent to the PC/ABS blend.

Despite these promising results, the study also emphasized the limitations of using transformed stress–strain data for constitutive model calibration, given its sensitivity to assumptions such as homogeneous and incompressible deformation. Instead, the use of force–displacement data was advocated as a more robust and physically representative foundation for parameter identification, particularly in the presence of barreling and localized deformation. A preliminary genetic algorithm implementation for the Bergström–Boyce model further demonstrated the viability of data-driven parameter optimization, capable of supplying high-quality initial guesses for gradient-based methods.

In conclusion, this work validates the Arruda–Boyce model as a powerful tool for simulating the compressive behavior of the PC/ABS 80:20 polymeric blend and lays the groundwork for future studies aimed at refining the parameter identification process through hybrid optimization strategies and the direct use of raw force–displacement data.

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