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Parallel Thread Processing for Performance Optimization in a Multilevel Finite Element Method (FE2) Analysis

Hugo M. Leão[1], Eduarda M. Ferreira[1], Roque L. S. Pitangueira[1], Lapo Gori[1], Ramon P. Silva[1]

[1]*Structural Engineering Department, Federal University of Minas Gerais*
Av. Antônio Carlos, 6627, Pampulha, Belo Horizonte, 31270-901, Minas Gerais, Brazil
hugomleao@yahoo.com.br, eduardaferreira@ufmg.br, roque@dees.ufmg.br, lapo@dees.ufmg.br, ramon@dees.ufmg.br

Abstract. Heterogeneous materials are widely used in civil engineering, and studying these materials is essential to ensure the safety and reliability of structures. One approach to analyzing these complex materials is multiscale analysis, which involves dividing the problem into two distinct observation scales: the macro scale and the micro scale. At the macro scale, the material behaves as if it were homogeneous, while at the micro scale, it is possible to identify and analyze the interactions between the material's different constituents. From these interactions arise the formation of microcracks and other failure mechanisms. This paper employs the Multilevel Finite Element Method (FE2) to perform multiscale analysis on quasi-brittle materials. This strategy consists of using a finite element model to represent the macro scale and others finite element models to represent the micro scale, enabling the capture of the complexities at both scales. Each integration point at the macro scale is associated with a micro scale model, and the macro scale parameters are obtained from a homogenization process based on the micro scale information. However, one of the main challenges of this approach is the high computational cost, as micro scale models typically require significant refinement to correctly describe the different constituents of the medium. This is particularly relevant in phase-field models, the constitutive model adopted at the micro scale. Given the complexity of processing all microscale models, the multiscale analysis becomes computationally expensive. Therefore, the objective of this article is to investigate the potential performance gains achieved through parallel processing, utilizing threads to optimize execution time. Preliminary results will be presented. All implementations were carried out in INSANE (Interactive Structural Analysis Environment), an open-source software developed by the Department of Structural Engineering at UFMG.

Keywords: Phase-field, multiscale analysis, multithreading, computational homogenization

1 Introduction

Widely used in civil engineering, heterogeneous materials exhibit complex behavior and demonstrate various failure mechanisms due to the random arrangement of their constituents within the mixture. The study of these materials is a challenging task that has been undertaken for a long time [1–6].

With the increasing processing capabilities of computers, Multilevel Finite Element Method (FE2) analysis has become feasible and has emerged as an alternative for conducting computational tests of engineering structures [7–9]. FE2 analysis involves dividing the model into two scales: the larger scale, known as the macroscale, which represents the average behavior of the material, and the smaller scale, called the microscale, where all the constituents of the material mixture are represented. Each integration point at the macroscale is associated to a corresponding microscale model, with the macroscale parameters being obtained through a homogenization process based on the information upscaled from the microscale. It is at the microscale that microinteractions between different constituents lead to the cracks emerging.

This process of having one Finite Element Model for the macroscale and n models for the microscale, one for each macroscale integration point, introduces significant computational costs and represents one of the key limitations of the FE2 method. To mitigate this, the present work proposes the use of multithreading to speed up the analysis and reduce the overall computational time.

Furthermore, phase-field models will be employed as they provide excellent results in refined meshes and, due to their ability to emerge and describe the crack path without the need for explicit crack tracking, offer a more efficient and accurate approach to modeling damage [10].

2 Phase-field models

Phase-field models are based on the variational approach of Griffith's theory [11–13] and involve spreading a sharp crack over a smoothed region, regularized by a length scale parameter l_0 (see Fig. 1). To achieve this, an additional equation is introduced to the problem in order to calculate the phase-field variable ϕ, ranging from 0 to 1, to represent material damage.

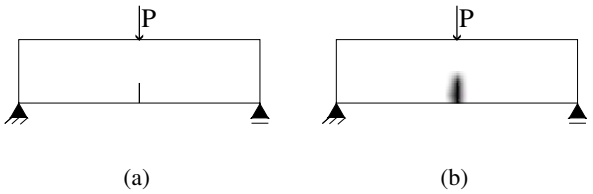


Figure 1. Comparison between (a) Griffith crack and (b) Phase-field diffuse crack

The residual form of the phase-field problem is given by:

$$\bar{r}_I^u = \int_\Omega [\mathbf{B}]_I^{u,T}\, \bar{\sigma}\, \mathrm{d}\mathcal{V} - \bar{f}_I = \bar{0}_I \tag{1a}$$

$$\bar{r}_I^\phi = -\int_\mathcal{B} [\mathbf{N}]_I^{\phi,T} \left(g'(\phi)\bar{Y} + \frac{1}{C_0 l_0}\alpha'(\phi) G_c \right) \mathrm{d}\mathcal{V} - \int_\mathcal{B} \frac{2l_0}{C_0} G_c\, [\mathbf{B}]_I^{\phi,T}\, \nabla\phi\, \mathrm{d}\mathcal{V} \leq \bar{0}_I \tag{1b}$$

where eq. (1a) is the equation responsible for calculating the nodal displacements u, while eq. (1b) comes from the additional equation that governs the phase-field variable ϕ. The parameters the equations represents the external force vector $\bar{f}_I$, the stress tensor $\bar{\sigma}$, the energetic degradation function $g(\phi)$, the geometric crack function $\alpha(\phi)$, the length scale parameter l_0 and C_0 is a constant calculated by $C_0 = 4\int_0^1 \alpha^{-1/2}(\phi)\, d\phi$.

3 Multilevel finite element method (FE2)

The multilevel finite element method (FE2) consists of splitting the problem into two scales, in such way that the constitutive equations are written only on the microscopic scale, and macroscopic parameters are obtained through homogenization techniques [7–9].

In the present work, macroscale strains are imposed on the microscale boundaries via Dirichlet boundary conditions, and the homogenized stress tensor is upscaled from the microscale to the macroscale (see Fig. 2). The evolution of the phase-field damage variable is computed exclusively at the microscale. A failure zone

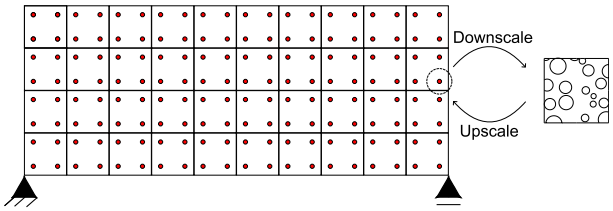


Figure 2. Illustrations of the multiscale process. Macroscale information is transferred to the microscale through a downscale process, while information from the microscale is transferred back to the macroscale through an upscale process.

homogenization process is employed to transfer information between the two scales [14–16].

4 Computational implementation

Due to the large number of microscale models and the required mesh refinement needed to accurately represent all mixture phases, the computational cost often becomes a limiting factor in performing multiscale analyses. To address this, the analysis was implemented such that the macroscale model runs on the main thread, controlling the entire process, while the microscale models are executed in parallel using multithreading. This approach ensures the reliability of the process and improves computational efficiency. Fig. 3 illustrates a single iteration of the Newton-Raphson method employing multithreading. To compute the internal forces and update the residual vectors, all microscale models must be updated using the current strain state from the macroscale model. At this stage, the main thread initiates the multithreading process to run all microscale models in multithreads and waits for all of them to complete their updates before continue.

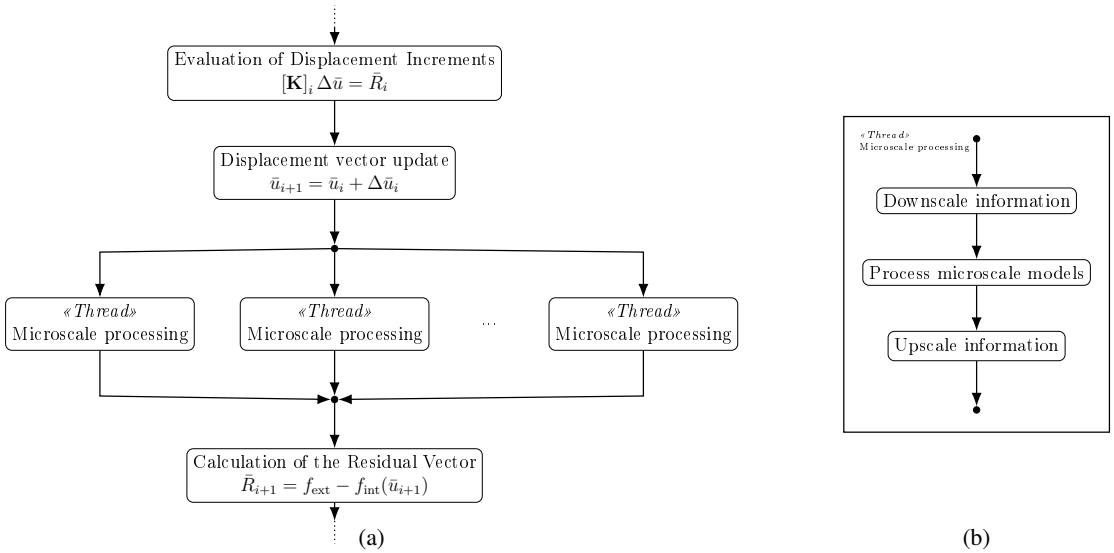


Figure 3. (a) Schematic representation of a single iteration of Newton-Raphson method. (b) Schematic representation of a single iteration of thread process.

5 Results

To perform the multiscale analysis, a bar with a width of 4000 mm and a height of 1000 mm subjected to axial loading was considered, as illustrated in Fig. 4. The geometry was discretized using square quadrilateral Q4 elements with a mesh size of 1000 mm.

Additionally, the microstructure was constructed with dimensions of 65x65 mm, using the "take and place" method [17, 18] based on the curve of the second granular fraction of NBR 7211:2005 [19], until reaching a 30% volume fraction of inclusions. The materials used for the matrix and aggregate in the microstructure are in Table 1.

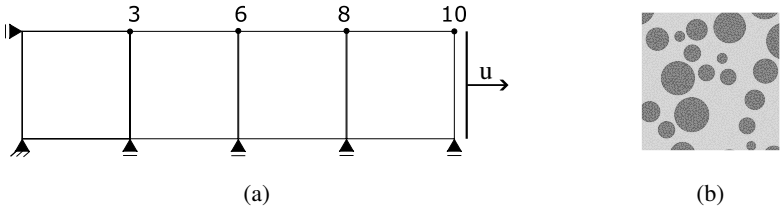


Figure 4. Schematic representation of the analysis. (a) Macroscale model. (b) Microscale model.

To induce crack localization, the first element was assigned a thickness equal to 90% of that of the remaining of the domain. The analysis was performed using the displacement control method, in which a total of 185 loading

Table 1. Material parameters for microscale model

Material	Elasticity Modulus (E_0) [$\mathrm{N/mm^2}$]	Poison's rate (ν) [$-$]	Fracture Energy (G_c) [$\mathrm{N/mm}$]	Tensile strength (f_t) [$\mathrm{N/mm^2}$]
Aggregate	100000.0	0.20	0.1300	16.0
Mortar	21876.0	0.18	0.0018	3.48

steps were applied. The right face of the model was subjected to a prescribed displacement with increments of 2.5×10^{-3} mm per step. All simulations were carried out on a computational system equipped with 16 virtual CPUs (vCPUs) and 64 GB of RAM.

5.1 Performance Analysis: Single-Thread vs. Multithread Execution

The analysis was performed multiple times using different numbers of threads in the process illustrated in Fig. 3.a, in order to evaluate the performance of the multithreading in running multiscale analysis.

To evaluate the influence of multithreading on the performance of the multiscale analysis, the simulation was executed five times using 1, 2, 4, 8, and 16 threads.

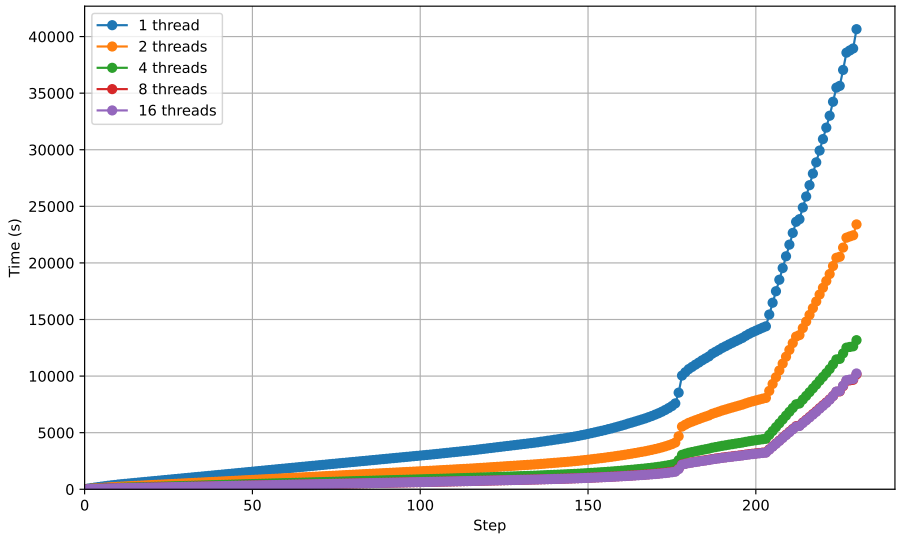


Figure 5. Analysis Time for Different Numbers of Threads. The x-axis represents the analysis step, while the y-axis indicates the elapsed time from the beginning of the simulation until the convergence of each step.

The results presented in Fig. 5 demonstrate a significant reduction in computational time as the number of threads increases. When using only a single thread, the analysis requires over 40,000 seconds (approximately 11 hours). In contrast, employing 16 threads reduces the computational time to around 3 hours for the same analysis. It is also evident from the plot that there is a change in the slope of the curve after step 170, which corresponds to the onset of crack propagation. This change in slope highlights the greater computational effort needed during crack growth simulation. From Fig. 5, it can be observed that the performance improvement (marginal gain) decreases as the number of threads increases, illustrating the saturation effect of parallelization, which is common in algorithms with sequential components or synchronization bottlenecks [20, 21].

From a structural analysis perspective, it is important to emphasize that, as expected, the results obtained for all analyses, with varying numbers of threads, were the same. In other words, the increasing in analysis efficiency was achieved without compromising the quality of the results.

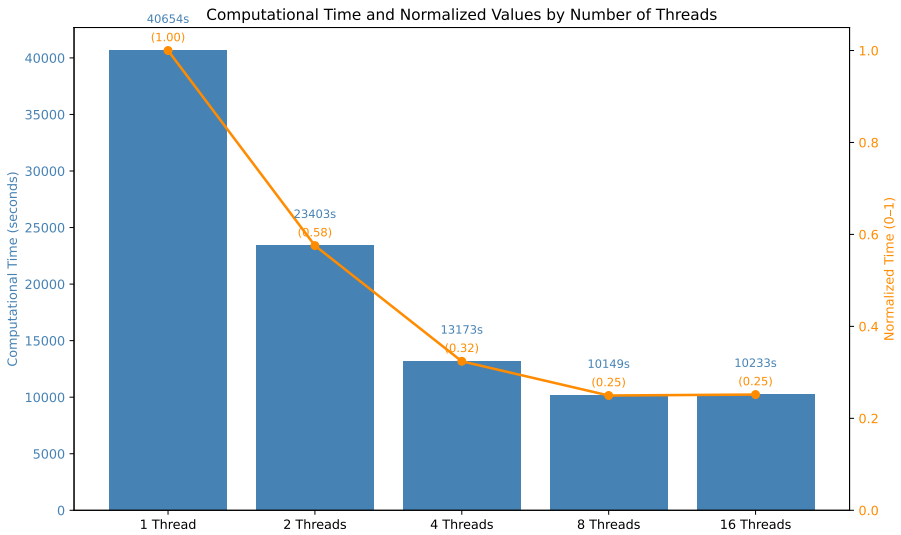


Figure 6. Analysis Time for Different Numbers of Threads. The x-axis represents the analysis step, with the primary y-axis (in blue) indicating the elapsed time from the beginning of the simulation to the convergence of each step. The secondary y-axis (in orange) shows the normalized time, referenced to the single-thread analysis. The curves for 8 and 16 threads overlap.

5.2 Structural simulation analysis

Fig. 7 presents the contour plots of: (a) the strain component in the x-direction at the final step of the macroscale analysis; and (b) the phase-field variable in one of the microscales corresponding to the first macroscale element, whose thickness was reduced to induce crack localization. Fig. 8.a presents the stress–strain curves obtained from a representative integration point of each macroscale element. Only one integration point per element is shown, as the responses at other points are similar. Fig. 8.b shows the load–displacement curves corresponding to selected macroscale nodes (see Fig. 4.a).

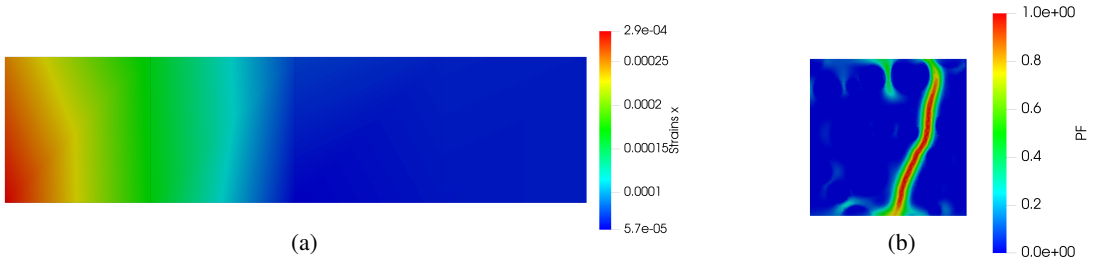
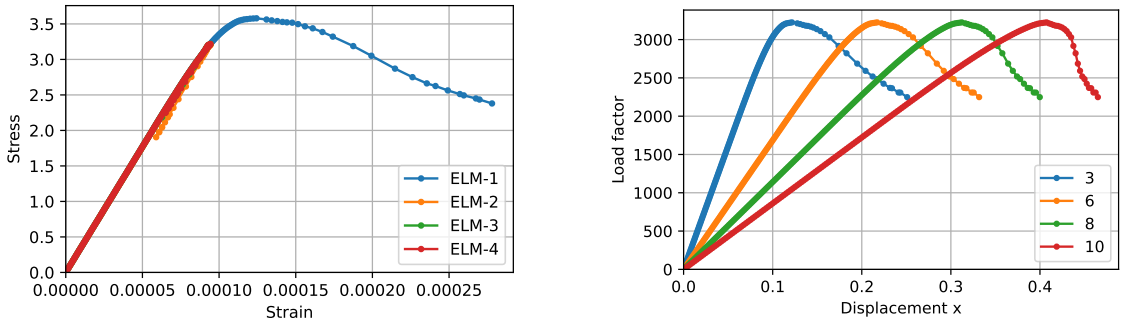


Figure 7. Contour plots of: (a) Macroscale strain component in the x-direction (ε_x); (b) Microscale phase-field distribution within the first macroscale element intersected by the propagating crack.

Fig. 7.a illustrates that the strain is concentrated on the left side of the beam, while lower values are observed on the right side. This behaviour is consistent with the stress–strain responses presented in Fig. 8.a, which demonstrate that once the crack initiates in the leftmost element, it dominates the structural response, causing progressively unload to adjacent elements. It can be observed from Fig. 7 that the crack propagates exclusively through the mortar matrix, circumventing the aggregates. This indicates that the aggregates act as obstacles to crack growth, effectively redirecting the fracture path along the direction of least energy dissipation.

6 Conclusions

The results presented in this work demonstrate the effectiveness of using multithreading to reduce the computational cost of multiscale finite element (FE2) analyses using microscale phase-field models. By parallelizing the



(a) (b)

Figure 8. Charts for: (a) Stress–strain curves obtained at an integration point for each macroscale element; (b) Load factor versus displacement curves for four selected macroscale nodes (refer to Fig. 4.a).

solution of microscale problems, a significant reduction in total analysis time was achieved, from over 11 hours in single-thread execution to approximately 3 hours with 16 threads, without compromising the accuracy or stability of the simulation. Despite the performance gains, a saturation effect was observed, indicating that increasing the number of threads no longer effectively reduces the computational cost beyond a certain point. It is important to note the increasing in analysis efficiency was achieved without compromising the quality of the results.

From the structural simulation perspective it could be seen that the crack initiation and propagation occurred in the region with reduced thickness, following a path through the mortar and circumventing the aggregates which acted as obstacles to crack growth. This behaviour illustrates the capability of phase-field models to capture complex fracture mechanisms in heterogeneous microstructures.

6.1 Possible future developments in the research

- Conduct experiments on alternative structural models, such as L-panels or bending tests, in order to validate the presented benchmark.
- Perform experimental studies to assess the accuracy of the multiscale implementation in comparison with previously reported results in the literature.
- Apply the multiscale model exclusively in the region of interest, and use a linear elastic model outside this region to enhance computational efficiency.

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