Calibration of the Resistance Factor for Beam-Columns in Cold-Formed Steel Members

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Abstract. The Direct Strength Method (DSM), used for the design of cold-formed steel structural members, is based on the analysis of local, distortional, and global elastic buckling of the cross-section, combined with empirically derived direct expressions to estimate ultimate strength. This method is widely accepted and incorporated into national design standards, as it enables a unified, robust, and flexible approach to structural design. Specifically, for beam-columns, current DSM-based design specifications still rely on simplified linear interaction expressions, which combine axial and bending strength responses separately. With the advancement of computational tools—such as the finite strip method—it is now feasible to accurately determine local, distortional, and global elastic buckling under any combination of axial load and bending moment. This allows a more realistic stability assessment that directly accounts for combined actions. In this context, Torabian and Schafer (2018) proposed new DSM expressions specifically tailored for the design of beam-columns, incorporating the effects of combined compression and bending directly into the stability evaluation. The authors also conducted an extensive experimental program, complemented by data from the literature, to validate the performance of the proposed formulations. The development of the DSM for beam-columns represents a significant step forward in achieving a more realistic and mechanically consistent design approach, while also reducing the excessive conservatism inherent in traditional interaction-based expressions. This study presents a structural reliability analysis based on the First-Order Reliability Method (FORM), applied to the enhanced DSM model proposed by Torabian and Schafer for stiffened channel beam-columns under combined axial compression and bending. Using the experimental and theoretical dataset provided by the authors, a statistical analysis of model error was performed, followed by the calibration of the resistance factor, assuming a target reliability index of 2.5. The aim is to contribute to the refinement of design codes through statistically grounded safety calibration, aligned with structural reliability principles.

Keywords: reliability, beam-column, DSM, FORM, cold-formed.

1 Introduction

Current design codes, such as AISI S100 [1] and NBR 14762 [2] present the Direct Strength Method (DSM) to determine axial load and bending capacities of cold-formed steel members. DSM uses elastic stability analysis results, together with experimentally derived expressions to predict the resistance, allowing a unified, robust and flexible approach.

The combined load effects acting on a cold-formed steel member are taken into account using a simple linear combination (eq. (1)) of the pure axial load and bending capacities, previously calculated via DSM or Effective Width Method (EWM). The non-linear responses of yielding, inelastic reserve and stability of the member under combined compression and bending, if considered, show that the use of this interaction expression is highly simplified and conservative (Torabian and Schafer [3]).

$$\frac{M_{1Sd}}{\phi_b M_{1Rn}} + \frac{M_{2Sd}}{\phi_b M_{2Rn}} + \frac{N_{Sd}}{\phi_c N_{Rn}} \leq 1. \quad (1)$$

The subscript *Sd* indicates the required load effects due to the factored loads, while *Rn* indicates the nominal resistance. ϕ_b and ϕ_c are beam and column resistance factors respectively.

Torabian and Schafer [3] proposed an extension of DSM to elements under combined compressive load and bending, incorporating the already known non-linear behavior in a complete and self-consistent design methodology. Experimental results were used to evaluate the accuracy of the proposed method, that proved to have good agreement with the actual resistance values.

This paper presents a reliability-based study using the First-Order Reliability Method (FORM) to evaluate the applicability of Torabian and Schafer's [3] formulation for beam-columns on the design guided by AISI S100 [1] LRFD and NBR 14762 [2]. The classical First-Order Second Moment (FOSM) method and the Monte Carlo simulation (MCS) technique were also employed for a comparative analysis. By the comparison of the experimental resistance to the nominal results from the DSM-based method, a statistical evaluation of the model error factor was conducted, followed by a reliability analysis and a later calibration of resistance factor for pre-defined target reliabilities. The study aimed to contribute to the enhancement of design codes, by presenting a statistically founded calibration aligned with the principles of structural reliability.

2 Structural reliability

2.1 Reliability methods

The safety or failure of a structure has traditionally been quantified in terms of its probability of failure P_f, in other words, the probability of the effects of the loads *S* overcome the structural resistance *R*. This probability can be expressed as $P_f = P(R \leq S)$. When dealing with the limit state function $G(.) = R - S$, P_f can be calculated as $P(G \leq 0)$. The reliability is defined as a complement of the probability of failure, and is typically expressed in terms of a reliability index β. Design code provisions usually specify a target reliability index β_0 as a minimum value for structures' β to reach when designed under its requirements. The relation between β and P_f is given by eq. (2), where Φ^{-1} is the inverse of cumulated density function Φ of the standard normal random variable.

$$\beta = -\Phi^{-1}(P_f). \quad (2)$$

The calibration process of design codes is based on the definition of safety criteria that ensure the structures to resist to the predicted actions throughout its design life, keeping a probability of failure under reasonable limits. In LRFD, is established a β_0 from which the partial safety factors applied to loads and resistance are calculated. A simplified way to perform this calibration is to adopt a previously defined load combination and adjust the resistance factor in a way that β approaches the target β_0.

Although the FOSM method is widely used to calibrate the partial factors for cold-formed steel, due to the simplicity of its formulation, the consideration of the first and second statistical moments only might compromise the precision of the results, especially for highly non-linear limit state functions or non-symmetric probability distributions (Haldar and Mahadevan [4]). The FORM, incorporating the distribution characteristics of the variables have shown to be effective, offering consistent results even with moderately non-linear limit state functions, which promotes greater rigor on the reliability-based calibration process.

The reliability derivation routine via FORM demands the conversion of the variables into a standard normal space (equivalent normal transformation can be employed for non-normal variables [5]). The design point – the most probable point of failure in the standard normal space – can be derived solving an optimization problem, using algorithms such as the HLRF(Hasofer and Lind [6], Rackwitz and Fiessler [7]). The reliability index is the distance from this point to the origin of the standard normal space. The Monte Carlo simulation (MCS) technique is frequently used as a benchmark to validate the results found via FOSM method or FORM, allowing the quality of the first-order approximation.

2.2 Limit state function

The limit state function $G(.)$ can be re-written in terms of the nominal resistance (R_n) and nominal dead (D_n)

and live loads (L_n). The dimensionless normalized random variables $\boldsymbol{R} = R/R_n$, $\boldsymbol{D} = D/D_n$ e $\boldsymbol{L} = L/L_n$ were employed. In order not to compromise the precision of the results using a single variable R for the resistance, this variable was described in a format $\boldsymbol{R} = \boldsymbol{MFP}$ (Ellingwood *et al*. [8]). For convenience, the normalized random variables are presented in a bolted format. In eq. (3), R depends on uncertainties related to material properties, cross-section geometry and professional factor, respectively represented by the normalized variables $\boldsymbol{M}$, $\boldsymbol{P}$ and $\boldsymbol{F}$.

$$G(.) = R - S = \boldsymbol{MFP} - (D_n\boldsymbol{D} + L_n\boldsymbol{L}). \tag{3}$$

The AISI S100 [1] and NBR 14762 [2] normative criteria's can be expressed by eq. (4), When the member is subjected on the effects of gravity loads only ($D + L$).

$$\phi R_n \left(\text{or } \frac{R_n}{\gamma}\right) \geq \gamma_D D_n + \gamma_L L_n. \tag{4}$$

where:

ϕ or γ = resistance factor ($\phi \leq 1$ in LRFD format, or $\gamma \geq 1$ in NBR format);
γ_D = load factor for dead load;
γ_L = load factor for live load.

The design at the limit state for a member with a nominal resistance R_n considers the application of the factors ϕ (or γ), γ_D and γ_L, an eq. (4) transformed into an equality. Table 1 brings the partial factors of loads and resistance from AISI S100 [1] and NBR 14762 [2] design codes. The subscripts *b* and *c* at the resistance factors refer to columns and beams respectively, which have different prescribed values.

Table 1. Resistance factors for columns and beams and load factors

Code	Compression	Bending	γ_D	γ_L
AISI S100 (LRFD)	$\phi_c = 0.85$	$\phi_b = 0.90$	1.20	1.60
NBR 14762	$\gamma_c = 1.20$	$\gamma_b = 1.10$	1.35	1.50

2.3 Random variables

The statistical parameters of the variables $\boldsymbol{M}$, $\boldsymbol{F}$, $\boldsymbol{D}$ and $\boldsymbol{L}$, are shown in Table 2 and were obtained from Ellingwood et al. [8]. In reliability studies, it is usual to use the coefficient of variation, a measure of the dispersion of the variable in relation to the average value, in the analyses performed.

Table 2. Statistical data on the normalized random variables (with the exception of $\boldsymbol{P}$)

Random variable	Mean	CoV	Probability distribution
$\boldsymbol{M}$	1.05	0.10	Lognormal
$\boldsymbol{F}$	1.00	0.05	Lognormal
$\boldsymbol{D}$	1.05	0.10	Normal
$\boldsymbol{L}$	1.00	0.25	Gumbel

3 DSM-based procedure for the design of beam-columns

Torabian and Schafer [3] formulated an extension of DSM to calculate the resistance of beam-columns based on the actual stresses generated by the demanding load condition. The proposal is compatible with the DSM routine for purely compressive or bending members, offering a design mechanically coherent with the traditional methods. Both resistance and the demanding actions on the beam-column are described in terms of the magnitude *B*. For the axial (N_{Sd}) and major (M_{1Sd}) and minor axis bending (M_{2Sd}) load effects due to the factored loads, the generalized nominal load effect B_{Sn} is given by eq. (5).

$$B_{Sn} = \sqrt{\left(\frac{M_{1Sd}}{M_{1y}}\right)^2 + \left(\frac{M_{2Sd}}{M_{2y}}\right)^2 + \left(\frac{N_{Sd}}{N_y}\right)^2}. \tag{5}$$

For LRFD, the design must satisfy the inequation $\phi B_{Rn} \geq B_{Sn}$ ($B_{Rn}/\gamma \geq B_{Sn}$ for the Brazilian code), where B_{Rn} is the nominal generalized resistance, calculated by the proposed method. As the method is derived from the DSM framework, the nominal resistance is given by $B_{Rn} = \min(B_{nG}, B_{nL}, B_{nD})$, with B_{nG}, B_{nL}, B_{nD} being respectively the nominal global, local and distortional buckling strengths. The global buckling strength is calculated using eq. (6), as a combination of B_{nGN} and B_{nGM}, given by the proposed equations for columns and beams respectively. ϕ_{NM} is the angle between the normalized axial load (N_{Sd}/N_y) axis and the plane formed by the normalized moments (M_{1Sd}/M_{1y} and M_{2Sd}/M_{2y}), calculated as: $\phi_{NM} = \cos^{-1}(N_{Sd}/N_y B)$.

$$B_{nG} = B_{nGN} + (B_{nGM} - B_{nGN}) \sin \phi_{NM}. \tag{6}$$

The slenderness parameter for beam columns is $\lambda_G = \sqrt{B_y/B_{crG}}$. For compressive members, that have $0 \leq \phi_{NM} \leq \pi/2$, B_{nGN} is given by eq. (7). For tensile members, with $\pi/2 < \phi_{NM} \leq \pi$, $B_{nGN} = B_y$.

$$B_{nGN} = \begin{cases} 0.658^{\lambda_G^2} B_y, & \text{for } \lambda_G \leq 1.5 \\ 0.877\lambda_G^{-2} B_y, & \text{for } \lambda_G > 1.5. \end{cases} \tag{7}$$

The B_{nGM} are defined by equations for beams with or without inelastic reserve. B_{nGM} without inelastic reserve is calculated by eq. (8), while B_{nGM} with inelastic reserve is calculated by eq. (9).

$$B_{nGM} = \begin{cases} B_y, & \text{for } \lambda_G \leq 0.6 \\ \dfrac{10}{9}\left(1 - \dfrac{10}{36}\lambda_G^2\right) B_y, & \text{for } 0.6 \leq \lambda_G \leq 1.34 \\ B_y \lambda_G^{-2}, & \text{for } \lambda_G > 1.34. \end{cases} \tag{8}$$

$$B_{nGM} = \begin{cases} B_p, & \text{for } \lambda_G \leq 0.23 \\ B_p - (B_p - B_y)\dfrac{\lambda_G - 0.23}{0.37}, & \text{for } 0.23 \leq \lambda_G \leq 0.6 \\ \dfrac{10}{9}\left(1 - \dfrac{10}{36}\lambda_G^2\right) B_y, & \text{for } 0.6 \leq \lambda_G \leq 1.34 \\ B_y \lambda_G^{-2}, & \text{for } \lambda_G > 1.34. \end{cases} \tag{9}$$

The local slenderness is given by $\lambda_L = \sqrt{B_y/B_{crL}}$, if $B_{nG} > B_y$, or $\lambda_L = \sqrt{B_{nG}/B_{crL}}$ if $B_{nG} \leq B_y$. Equation (10) gives the local buckling strength without inelastic reserve, while eq. (11), the strength with inelastic reserve. In eq. (11), B_{y2c} is the strength at which yielding initiates in compression (after yielding in tension).

$$B_{nL} = \begin{cases} B_{nG}, & \text{for } \lambda_L \leq 0.776 \\ \left(1 - 0.15\lambda_L^{-0.8}\right)\lambda_L^{-0.8} B_{nG}, & \text{for } \lambda_L > 0.776 \end{cases} \tag{10}$$

$$B_{nL} = \begin{cases} \text{symmetry about axis of bending or first yield in compression:} \begin{cases} B_y + \left(1 - C_{yL}^{-2}\right)\left(B_p - B_y\right) \\ \text{with: } C_{yL} = \sqrt{0.776/\lambda_L} \leq 3 \end{cases} \\ \text{first yield in tension:} \begin{cases} B_{y2c} + \left(1 - C_{yL}^{-2}\right)\left(B_p - B_{y2c}\right) \leq B_{yt3} \\ \text{with: } B_{yt3} = B + \left(1 - C_{yt}^{-2}\right)\left(B_p - B_y\right),\ C_{yt} = 3 \end{cases} \\ \left(1 - 0.15\lambda_L^{-0.8}\right)\lambda_L^{-0.8} B_{nG}, \end{cases} \quad \begin{matrix} \\ \\ \\ \text{for } \lambda_L \leq 0.776 \\ \\ \\ \text{for } \lambda_L > 0.776 \end{matrix} \tag{11}$$

Finally, the slenderness for the distortional buckling strength is given by $\lambda_D = \sqrt{B_y/B_{crD}}$. Equations (12) and (13) give the distortional buckling strength without and with inelastic reserve respectively.

$$B_{nD} = \begin{cases} B_y, & \text{for } \lambda_D \leq 0.561 + 0.112 \sin \phi_{NM} \\ (1 - c_1 \lambda_D^{c_2})\lambda_L^{c_2} B_y & \\ \text{with: } c_1 = 0.25 - 0.03 \sin \phi_{NM}, & \text{for } \lambda_D > 0.561 + 0.112 \sin \phi_{NM} \\ \quad c_2 = 0.2 \sin \phi_{NM} - 1.2 & \end{cases} \tag{12}$$

$$B_{nD} = \begin{cases} \text{symmetry about axis of bending or first yield in compression: } \begin{cases} B_y + \left(1 - C_{yD}^{-2}\right)\left(B_p - B_y\right) \\ \text{with: } C_{yD} = \sqrt{(0.561 + 0.112)/\lambda_D} \leq 3 \end{cases} & \text{, for } \lambda_D \leq 0.561 + 0.112 \sin\phi_{NM} \\ \text{first yield in tension: } \begin{cases} B_{y2c} + \left(1 - C_{yD}^{-2}\right)\left(B_p - B_{y2c}\right) \leq B_{yt3} \\ \text{with: } B_{yt3} = B + \left(1 - C_{yt}^{-2}\right)\left(B_p - B_y\right),\ C_{yt} = 3 \end{cases} & \\ (1 - c_1 \lambda_D^{c_2}) \lambda_L^{c_2} B_y & \\ \text{with: } c_1 = 0.25 - 0.03 \sin\phi_{NM}, & \text{for } \lambda_D > 0.561 + 0.112 \sin\phi_{NM} \\ \quad c_2 = 0.2 \sin\phi_{NM} - 1.2 & \end{cases} \tag{13}$$

The yield strength B_y, needed in eqs. (6) to (13) can be derived as $B_y = \left(F_y / \sigma_{Sn\text{-max}}\right) B_{Sn}$. $\sigma_{Sn\text{-max}}$ is the maximum absolute required axial stress σ_{Sn} from eq. (14), as a function of the distances y_{c1} and y_{c2} from an arbitrary point to the centroidal axes. B_{yt} and B_{yc}, needed for inelastic reserve calculations are given by eqs. (15) and (16).

$$\sigma_{Sn}\left(y_{c1}, y_{c2}\right) = \frac{N_{Sd}}{A} + \frac{M_{1Sd} y_{c2}}{I_1} + \frac{M_{2S,d} y_{c1}}{I_2}. \tag{14}$$

$$B_{yt} = \left\{-F_y / \min\left[\sigma_{Sn}\left(y_{c1}, y_{c2}\right)\right]\right\} B_{Sn}. \tag{15}$$

$$B_{yc} = \left\{F_y / \max\left[\sigma_{S,n}\left(y_{c1}, y_{c2}\right)\right]\right\} B_{Sn}. \tag{16}$$

The plastic strength B_p, however, needs to be determined by numerical methods such as fiber element analysis. A fiber analysis modulus was implemented in CUFSM (Schafer and Ádany [9]), and was suggested by Torabian and Schafer [3] to be used to derivate B_p. More details about B_p derivation can be found at Torabian and Schafer [3]

Experimental programs conducted by Torabian, Zheng and Schafer [10] characterized the failure modes and the bearing capacity of fifty-five lipped channel prototypes under combined axial load and bending. The prototypes had three different lengths: 305 mm (short), 610 mm (medium) and 1,219 mm (long). The nominal dimensions of the cross-sections were 152.4 mm for web depth, 34.9 mm for flange width, 9.5 mm for edge lip and 1.37 plate thickness. The professional factor statistics were derived for this database, and further used to perform the reliability analyses.

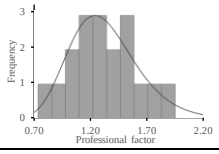
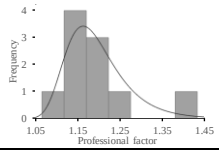
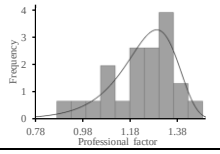
4 Results

4.1 Statistical analysis of the professional factor

The professional factor (model error) ***P*** represents the ratio between the nominal and predicted resistance of the beam-column, serving as a measure of the precision of the proposed resistance calculation method. In most cases, the higher the mean of ***P*** and the lower its coefficient of variation (CoV), the higher the reliability index will be. Their stochastic models were defined by Anderson-Darling adherence test [11], applicated in groups separated by three load conditions: minor axis bending + axial load, major axis bending + axial load and biaxial bending + axial load. The proper distribution was selected among the test results considering normal, lognormal, Type I of largest, Type I of smallest, Type III of smallest ang Gamma. Table 3 shows the results of the probability description of the professional factor, with values of the AD statistics. It measures how well the data follow the adjusted distribution. The lowest AD was used to select the proper distribution for each group.

Results showed that each load condition had a specific statistical description of the professional factor. The employment of the Anderson-Darling adherence test allowed the identification of suitable probability distributions, all with high statistical significance ($p > 0.25$). None of the evaluated distribution was rejected to the significant level of 5%, indicating acceptable adherences. The histograms visually corroborate the quality of the adjusts.

Table 3. Statistical parameters of the professional factor

Information	Minor axis bending + axial load	Major axis bending + axial load	Biaxial bending + axial load
Prototypes	18	10	24
Mean	1.237	1.202	1.219
CoV	0.200	0.070	0.112
Distribution	Lognormal	Type I – largest	Type I – smallest
p value	0.995	>0.25	>0.25
AD	0.098	0.267	0.180
Histogram			

4.2 Reliability Index

Figure 1 show the calculated reliability indices β with the current resistance factors indicated for LRFD and NBR designs, for three different load conditions. A load ratio of $\rho = L_n/D_n = 5$ was considered in the calculation; a typical value for cold-formed steel structures reliability studies. The target Reliability index $\beta_0 = 2.5$ is highlighted in a marked dash line. The results indicated that the obtained reliability indices β are, in general meeting the LRFD safety requirements, mainly for the load condition "Minor axis bending + axial load", which values are close to the target $\beta_0 = 2.5$. This behavior is consistent with the statistical data of the professional factor seen in section 4.1, that for this group, has the higher coefficient of variation (CoV = 0.200) – higher dispersion of the individual values of $\boldsymbol{P}$. For NBR, the indices are also satisfactory, with exception of "Minor axis bending + axial load" condition, which β's are under the target $\beta_0 = 2.5$. These lower reliabilities are again related to the higher variability of $\boldsymbol{P}$. The higher value seen in Figure 1 is $\beta = 3.40$, calculated via FOSM method for "Major axis bending + axial load", that was the condition with the lower CoV (0.070) among the groups.

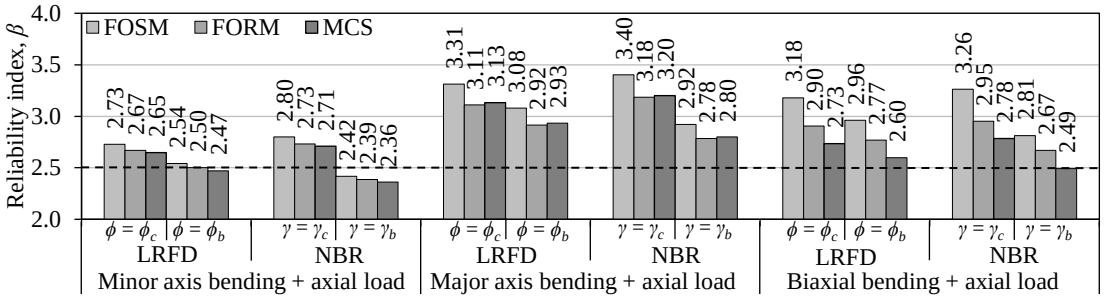


Figure 1. Reliability indices calculated with the current resistance factors

In general, the FOSM method presented the highest β values; reflect of its simplifications. The FORM results were generally closer to those from MCS, reflecting its precision. For the case "Biaxial bending + axial load", FOSM method and FORM significantly differ from MCS. A second-order reliability method (SORM – see Haldar and Mahadevan [4] for more details) was employed and supported the values found via MCS. This finding may be related to the high non-linearity of the limit state function for this load condition, which probability distribution adjusted for $\boldsymbol{P}$ was the Type I of smallest value.

4.3 Resistance Factors

Table 4 have the calibrated resistance factors for $\rho = 5$, aiming at a target reliability index of $\beta_0 = 2.5$. The resistance factor results are consistent with the previous reliability levels seen in the previous analyses. In the load case "Minor axis bending + axial load", the results are close to those already indicated in LRFD and NBR design frameworks. In the remaining cases, the calibration indicated that the use of the current resistance factors is conservative for the adapted DSM for beam-columns, especially in the case "Major axis bending + axial load".

Table 4. Calibrated resistance factors

Condition	Method	ϕ	γ	Condition	Method	ϕ	γ	Condition	Method	ϕ	γ
Minor axis	FOSM	0.91	1.14	Major axis	FOSM	1.04[(a)]	1.00	Biaxial	FOSM	1.01[(a)]	1.02
bending +	FORM	0.90	1.16	bending +	FORM	1.02[(a)]	1.02	bending +	FORM	1.00	1.04
axial load	MCS	0.89	1.17	axial load	MCS	1.02[(a)]	1.02	axial load	MCS	0.93	1.11

[(a)]In the case of $\phi > 1$, suggest $\phi = 1$

5 Conclusions

A reliability-based calibration of resistance factors was performed for an adapted version of the DSM for beam-columns. The results showed that for the load condition "Minor axis bending + axial load", the calibrated factors (ϕ and γ) are close to the actual normative values for beam design. For the remaining load conditions, especially "Major axis bending + axial load", the calibrated values are higher than the actual normative values for LRFD (and lower for NBR), indicating that the use of the current values for the proposed method is significantly conservative.

In the hypothesis of the incorporation of the method in future versions of the design codes, the actual beam resistance factor is suitable only for the case "Minor axis bending + axial load". In the other two load conditions, a unitary value can be adopted – unreduced nominal resistance. For a general value of resistance factor for any load condition, consistent with the actual design philosophies, the values of $\phi = 0.90$ and $\gamma = 1.16$ can be adopted.

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