A system with asymmetric stability analyzed with optimization techniques

Reyolando M.L.R.F. Brasil[1]

[1]*Dept. of Structural and Geotechnic Engineering, University of São Paulo*
Address, Zip-Code, State/Province, Country
reyolando.brasil@usp.br

Abstract. We consider a discrete model composed of two horizontal rigid articulated equal length massless bars, initially collinear, with an inclined linear spring connected to the bars at their central pinned joint. We consider a hinged fixed support at the left end of the system and a mobile articulated support at the right end. A horizontal compressive axial load is applied to the mobile support in the direction of their original primary stable horizontal configuration. As this force grows, a critical buckling load may be reached so that this primary straight configuration becomes instable and the system assumes a new secondary inclined position of the bars. In this model, for positive inclination angles, the secondary configuration is stable, but for negative ones it is instable, a potentially dangerous asymmetric bifurcation stability problem that may arise in aerospace applications. In our analysis, we compute the Total Potential Energy (TPE) of the system. According to Lagrange's theorem, equilibrium points correspond to stationary TPE, being stable if it is a point of minimum and instable if a point of maximum. We use optimization algorithms to find stable and instable equilibrium points for very close axial load values near the critical one.

Keywords: numerical optimization; asymmetric bifurcation stability; Lagrange's Theorem

1 Introduction

This paper revisits a classical asymmetric bifurcation stability problem, Brasil [1], Bazant and Cedolin [2], Croll and Walker [5], from an optimization perspective.

We consider a discrete model composed of two horizontal rigid articulated bars of length L, massless, initially collinear, with a linear spring of stiffness K inclined connected to the bars at their central pinned joint. We consider a hinged fixed support at the left end of the system and a mobile articulated support at the right end.

A horizontal compressive axial load P is applied to the mobile support in the direction of their original primary stable horizontal configuration. As this force grows, a critical buckling load may be reached so that this primary straight configuration becomes instable and the system assumes a new secondary inclined position of the bars that perform a rotation angle θ with respect to their original positions.

In this model, for positive valued θ angles the secondary configuration is stable, but for negative valued angles it is instable, a potentially dangerous asymmetric bifurcation stability problem that may arise in aerospace applications.

The Lagrange-Dirichlet Theorem, Brasil [1], Bazant and Cedolin [2], Croll and Walker [5] establishes that the equilibrium of a discrete system is given by the Total Potential energy being stationary, this equilibrium being stable if it is a minimum point and unstable if it is a maximum point.

Numerical optimization techniques, Brasil and Silva [3], Arora [4], are used to find the displaced stable configurations for prescribed loads.

2 The mathematical model - Lagrange's Theorem

Consider the discrete model in Fig. 1, composed of two horizontal rigid articulated bars of length *L*, massless, initially collinear, with a linear spring of stiffness *K* inclined connected to the bars at their central pinned joint. We consider a hinged fixed support at the left end of the system and a mobile articulated support at the right end.

Angle θ is the displacement of the bars from their initial position. It will be adopted as our only generalized coordinate in the following development.

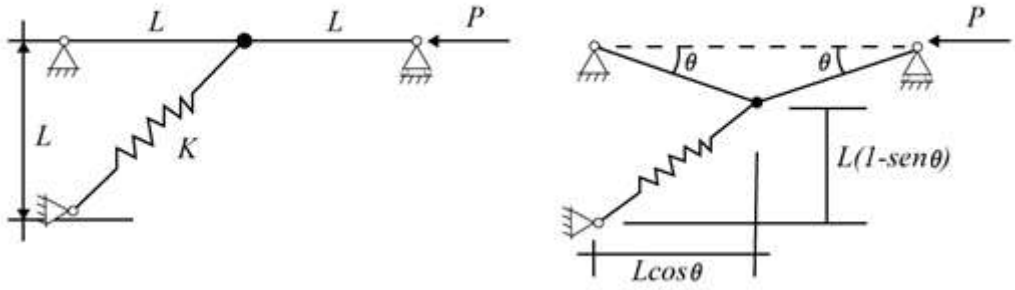


Figure 1: the mathematical model

The Lagrange-Dirichlet Theorem establishes that the equilibrium of a discrete system is given by the Total Potential energy being stationary, this equilibrium being stable if it is a minimum point and instable if it is a maximum point, Brasil [1], Bazant and Cedolin [2], Croll and Walker [5].

2.1 Total Potential Energy

The change of the length of the spring is

$$\Delta_s = L\sqrt{2}\left(1-\sqrt{1-sen\,\theta}\right) \tag{1}$$

so that the strain energy is

$$U = \frac{1}{2}K\,\Delta_s^2 = \frac{1}{2}K\left[L\sqrt{2}\left(1-\sqrt{1-sen\,\theta}\right)\right]^2 \tag{2}$$

The displacement of the application point of load P is

$$\Delta_P = 2L(1-cos\,\theta) \tag{3}$$

so that the work of the external loads of the system is

$$W = P\,\Delta_s = 2PL(1-cos\,\theta) \tag{4}$$

and the Total Potential Energy is

$$V = U - W \tag{5}$$

$$V(\theta) = \frac{1}{2}K\left[L\sqrt{2}\left(1-\sqrt{1-sen\,\theta}\right)\right]^2 - 2PL(1-cos\,\theta) \tag{6}$$

2.2 Equilibrium points and their stability

Next, we apply Lagrange's Theorem to access the equilibrium configurations, by making zero the derivative of the Total Potential Energy, Eq. (6), with respect to the adopted generalized coordinate.

$$\frac{dV}{d\theta} = 0 = KL^2\,cos\,\theta\left(\frac{1}{\sqrt{1-cos\,\theta}}-1\right) - PL\,sin\,\theta \tag{7}$$

As per the mentioned Lagrange's Theorem, an equilibrium point is stable if the second derivative of the Total Potential Energy, Eq. (6), is positive. Otherwise, if negative the equilibrium point is unstable. These conclusions are shown in Fig. 2. that displays the equilibrium trajectory of this problem, as a function of a parameter $p = \frac{4P}{KL}$. Stable segment is plotted in solid line and instable segment in dashed line.

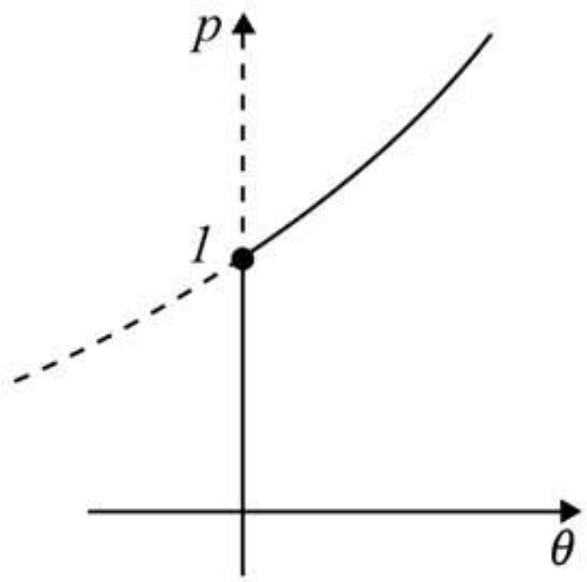


Figure 2: stability trajectories

3 Application of numerical optimization techniques

We now adopt numerical values L = 1 m and K = 1 kN/m. Function ***fminbnd*** of the Matlab Optimization Toolbox, for optimization of nonlinear unconstrained problems, is used to minimize the Total Potential Energy, Eq. (6), in order to check stability for two values of load P, one at the stable region and the other in the instable one.

For P = 0.275 kN, we obtain $\theta = 0.1304$ rad, a positive value in the stable region of Fig. 2.

For P = 0.225 kN, we obtain $\theta =$ - 0.1363 rad, a negative value, in the instable region of Fig. 2.

The transition point $p = 1$ in Fig. 2 is, of course, correspondent to P = 0.25 kN.

These results agree perfectly with those available in the literature, Croll and Walker [5].

4 Conclusions

This paper revisits a classical asymmetric bifurcation stability problem, from an optimization perspective. The ***fminbnd*** algorithm of Matlab Optimization Toolbox, for optimization of nonlinear unconstrained problems, was successfully used for the solution of two equilibrium problems via minimization of the Total Potential Energy of the mathematical model. First, a load in the stable region of the problem was applied. Next, the same is presented for a load in the instable region.

The results agree perfectly with those available in the literature.

Acknowledgements. The author acknowledges support by CNPq, Grant 303529/2021-0, a Brazilian research funding agency.

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