

# Numerical Simulation of Non-Newtonian Multiphase Flow in Porous Media Using a Generalized Newtonian Model in OpenFOAM

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**Abstract.** The simulation of non-Newtonian fluid flow in porous media remains a significant challenge due to the complex rheology of such fluids and the structural heterogeneity of porous materials. This work presents a numerical approach for modeling multiphase flows of non-Newtonian fluids using a Generalized Newtonian (Power-law) rheological model, implemented within the open-source computational framework OpenFOAM. The strategy employs advanced discretization techniques and extends existing OpenFOAM libraries originally designed for Newtonian fluid simulations. The numerical implementation was validated through comparisons with analytical solutions, demonstrating the accuracy and robustness of the developed code in simulating Power-law fluid flow in porous media. These results highlight the viability of the proposed approach for more complex studies involving the transport of non-Newtonian fluids in natural or synthetic porous structures.

**Keywords:** Non-Newtonian Fluids, Generalized Newtonian model, Power-law, Porous Media, OpenFOAM

## 1 Introduction

Simulating the flow of non-Newtonian fluids through porous media is a significant research area with extensive practical applications. These include enhanced oil recovery, the removal of residues in porous media through fluid injection [1], separation and reaction processes in fixed and fluidized beds, the mitigation of liquid pollutants in subsurface environments and the filtration of polymeric solutions [2] [3]. Unlike Newtonian fluids, which have a constant viscosity, non-Newtonian fluids exhibit complex, nonlinear rheological behaviors where apparent viscosity depends on the shear rate and time. The inherent complexity of both the fluid rheology and the heterogeneous structure of porous media presents a considerable modeling challenge.

Several factors contribute to this challenge. First, the non-linear rheology of non-Newtonian fluids, characterized by shear-rate-dependent viscosity and viscoelasticity, necessitates sophisticated constitutive equations. Models with multiple parameters can be unwieldy, and predicting non-zero relaxation times is particularly difficult. Second, the microscale heterogeneity of porous media, including pore size distribution and interconnectivity, profoundly impacts flow [4]. This often leads to deviations from the traditional Darcy's Law and requires approaches that account for non-uniform pore distribution. Third, the coupling of multiple phenomena, such as the multiphase flow of immiscible fluids, non-Darcian effects at high velocities, and phase compressibility, adds another layer of complexity. From a numerical standpoint, solving the governing equations frequently leads to convergence difficulties and time-step limitations. Furthermore, the lack of analytical solutions for most complex scenarios hinders the validation of these numerical models [5].

To address these issues, researchers have employed various methodologies. Continuum models, such as extensions of Darcy's Law, are macroscopically simple but may neglect crucial pore-scale physics and show limitations with transient or yield-stress phenomena [4]. Techniques like volume averaging are used to derive generalized non-Darcian models that incorporate pore structure properties. In contrast, pore network models represent the porous medium as a collection of interconnected tubes, offering a more detailed description of pore-scale physics at a higher computational cost. The Effective Medium Theory (EMT) can be coupled with pore network simulations to homogenize the medium, though its accuracy may decrease for broad pore size distribution [6] [3].

This work contributes to the multiphase simulation of non-Newtonian fluids in porous media using the generalized Power-Law rheological model. A numerical approach, implemented in the OpenFOAM environment that extends existing libraries for Newtonian fluid simulations, is presented. The objective is to compare numerical results for non-Newtonian fluids (Power-Law model) against analytical solutions. The paper is organized as follows: the "Modeling" section details the mathematical and numerical models; the "Results" section presents the findings, including comparisons with analytical solutions; and the "Conclusion" summarizes the main findings and suggests directions for future work.

## 2 Modeling

Due to the microscopic complexity of porous geometries, a direct application of the Navier-Stokes equations for reservoir-scale problems is not feasible [5]. Consequently, the continuum approach is widely adopted to model fluid flow in porous media. This methodology conceptualizes the porous medium as a continuous entity where microscopic properties are represented by averaged values, such as porosity ( $\phi$ ) and permeability ( $K$ ), defined over a Representative Elementary Volume (REV) [1] [3] [5] [7].

Fluid flow in these media is governed by the fundamental laws of mass and momentum conservation. For slow flow regimes ( $Re \ll 1$ ), the momentum balance equation is often simplified to the Darcy law, which establishes a linear relationship between pressure gradient and velocity [7]. However, the original Darcy law has limitations for non-Newtonian fluids because it assumes a constant fluid viscosity. To extend its applicability to multiphase flows, the concept of relative permeability ( $k_{r,i}$ ) is introduced, reflecting the capacity of each phase to flow in the presence of others [5].

### 2.1 Governing Equations

The viscosity of non-Newtonian fluids is directly dependent on the flow potential to which they are subjected. Consequently, the Darcy law cannot be applied in its original form. This limitation is addressed in the literature by defining an apparent viscosity  $\mu_{nn}$ , which allows the application of the Darcy law as follows:

$$\mathbf{U} = -\frac{K}{\mu_{nn}} \nabla \Phi, \quad (1)$$

where the index  $nn$  is an acronym for the non-Newtonian fluid,  $\mathbf{U}$  is the velocity vector,  $K$  is the permeability of the porous media,  $\mu_{nn}$  is the viscosity of the non-Newtonian fluid that is dependent on  $\Phi$  that is the flow potential and can be defined as:

$$\Phi = \rho_{nn} \left( \int_{P_0}^P \frac{dP}{\rho_{nn}(P)} - \mathbf{g} \right), \quad (2)$$

where  $P_0$  represents the reference pressure,  $\rho_{nn}$  represents the fluid density,  $P$  is the pressure to which the non-Newtonian fluid is submitted and  $\mathbf{g}$  is the gravitational force.

For multiphase flows, it is necessary to incorporate the relative permeability  $k_{r,i}$  of the phase to determine the Darcy velocity,

$$\mathbf{U} = -\frac{K k_{r,i}}{\mu_{nn}} \nabla \Phi, \quad (3)$$

where the relative permeability was calculated using the Brooks and Corey [8] model defined as:

$$k_{r,i} = k_{r,i(\max)} \left( \frac{\alpha_i}{\alpha_v} \right)^\eta, \quad (4)$$

where  $\alpha_i$  is the fluid phase fraction,  $\eta$  is a power coefficient associated with the porous media properties,  $\alpha_v$  is the void volumetric fraction, and  $k_{r,i(\max)}$  is the maximal relative permeability.

Defining the saturation  $S_i$  as

$$S_i = \frac{\alpha_i}{\alpha_v}, \quad (5)$$

then, the apparent viscosity also becomes dependent on the saturation of the non-Newtonian phase:

$$\mu_{nn} = \mu_{nn}(\nabla \Phi, S_{nn}). \quad (6)$$

For fluids that adhere to the Power-law model, as analyzed by Wu [9], the apparent viscosity can be determined using the following expressions:

$$\mu_{nn} = \mu_{eff} \left( \frac{\mathbf{K}k_{r,i}}{\mu_{eff}} |\nabla\Phi| \right)^{\frac{n-1}{n}}, \quad (7)$$

with

$$\mu_{eff} = \frac{H}{12} \left( 9 + \frac{3}{n} \right)^n [150\mathbf{K}k_{r,i}\phi(S_{nn} - S_{nn,r})]^{\frac{1-n}{2}}. \quad (8)$$

Here,  $H$  is the consistency index of the Power-law model,  $n$  is the exponent of the Power-law model,  $\phi$  is the porosity of the porous media,  $S_{nn}$  is the saturation of the non-Newtonian fluid, and  $S_{nn,r}$  is the residual saturation. To simplify implementation, the potential gradient is eliminated by isolating it in Equation 3:

$$\nabla\Phi = -\frac{\mathbf{U}\mu_{nn}}{\mathbf{K}k_{r,nn}}. \quad (9)$$

This allows for substitution into Equation 7, which simplifies to:

$$\mu_{nn} = \mu_{eff} |\alpha\mathbf{U}|^{n-1}. \quad (10)$$

For shear-thinning fluids ( $0 < n < 1$ ) the apparent viscosity of the fluid becomes infinite as the flow potential gradient tends to zero, like in the shock front. That leads to numerical difficulties in a numerical simulation. To overcome this difficulty, a linear interpolation is used when the potential gradient is very small [10]:

$$\mu_{nn} = \mu_1 + \frac{\mu_1 - \mu_2}{\delta_1 - \delta_2} (|\nabla\Phi| - \delta_1) \quad (11)$$

when  $|\nabla\Phi| \leq \delta_1$ , for which the interpolation parameters are  $\delta_1$  ( $\approx 10$  Pa/m) and  $\delta_2$  ( $\delta_1 - \delta_2 = 10^{-7}$  Pa/m) and the  $\mu_1$  and  $\mu_2$  values are:

$$\mu_j = \mu_{eff} \left( \frac{k_{r,i}}{\mu_{eff}} \delta_j \right)^{\frac{n-1}{n}} \quad j = 1, 2. \quad (12)$$

## 2.2 Numerical Implementation

The numerical simulations were performed using the open source OpenFOAM software, a C++ library specialized in Computational Fluid Dynamics (CFD) that operates on the Finite Volume Method (FVM). A fully implicit coupled solver based on coupledMatrixFoam [11] was employed, which utilizes a Euler-Euler model and Darcy law to simulate multiphase flow of Newtonian fluids within the porous medium. This solver linearizes the governing equations through Taylor series expansions and uses a block matrix structure to implicitly couple the pressure and phase fraction fields. This approach improves numerical stability and computational efficiency, particularly for simulations involving complex geometries.

For the non-Newtonian rheology, the equations for the Power-Law model were integrated directly into the numerical framework. The implementation of rheological models was guided by the codes and methodologies presented in the rheoTool package [12]. A key component of this implementation is the calculation of the apparent viscosity of the non-Newtonian fluid ( $\mu_{nn}$ ), which is a function of the flow potential gradient and phase saturation. The computational relationships for both the apparent viscosity ( $\mu_{nn}$ ) and the effective viscosity ( $\mu_{eff}$ ) were implemented following the established approach presented in Equations (7) and (8).

The accuracy and validity of the numerical implementation were verified by comparing the simulation results with the analytical Buckley-Leverett solution for a one-dimensional immiscible displacement [5]. This comparison serves as a rigorous reference point to confirm the accuracy of the integrated rheological model and the overall numerical scheme.

The implemented model considers only incompressible fluids and neglects the effects of capillary pressure gradients, which were assumed to be negligible. Furthermore, the simulations are restricted to homogeneous and isotropic porous media, and do not account for inter-phase mass transfer, hydrodynamic dispersion, or fluid adsorption onto the porous matrix.

### 3 Results

The efficacy of the proposed numerical formulation was evaluated by comparing its results with an analytical solution. This solution, developed by Wu [9], builds upon the Buckley and Leverett [13] framework to account for non-Newtonian fluid behavior in two-phase flow through homogeneous porous media.

For the test case, a 5 m one-dimensional porous domain, initially saturated with a Newtonian fluid (denoted by  $n$ ), was considered. A non-Newtonian fluid (denoted by  $nn$ ) was then injected at  $1 \times 10^{-5}$  m/s from the left side, while the pressure on the right boundary was maintained at 0.1 MPa. Both phases were characterized by the Brooks and Corey relative permeability model with  $\eta = 3$  and  $k_{r,i(\max)} = 1$ . Fluid properties were set as follows:  $\rho_n = 1000$  kg/m<sup>3</sup>,  $\rho_{nn} = 800$  kg/m<sup>3</sup>,  $\mu_n = 0.006$  Pa · s. For the non-Newtonian fluid a Power-law rheological model was used with the parameters  $H = 0.01$  Pa · s <sup>$n$</sup>  and  $n = 0.6$ . The porous medium itself had an absolute permeability  $K = 9.87 \times 10^{-13}$  m<sup>2</sup> and a porosity  $\phi = 0.2$ .

Figure 1 illustrates the resulting non-Newtonian fluid saturation profile at  $t = 10$  h, calculated using 2000 computational cells. The clear correspondence between the numerical and analytical solutions validates the accuracy of the simulation approach.

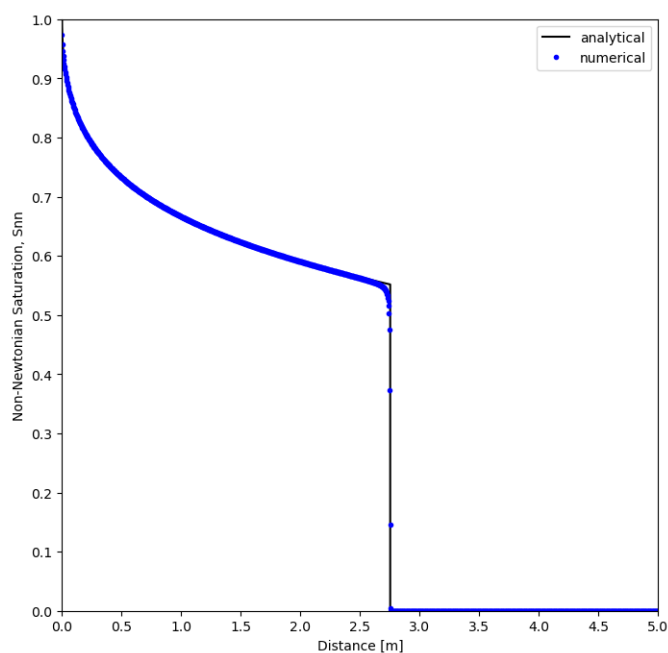


Figure 1. Non-Newtonian Fluid Injection Saturation: Analytical and Numerical

### 4 Conclusions

A numerical model for multiphase flow of non-Newtonian fluids in a porous media, using a Power-law rheological model, was implemented in the OpenFOAM framework. The implementations extend OpenFOAM libraries to account for non-Newtonian rheology based on the continuum approach and an apparent viscosity formulation using fully implicit, block-coupled solution for the governing equations. Validation against an analytical solution for a one-dimensional immiscible displacement case showed good agreement between the numerical results and the analytical saturation profile. This correspondence confirms the accuracy and robustness of the code developed for simulating the flow of Power-law fluids through porous media under the tested conditions.

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