XLVI CILAMCE
VITÓRIA - ES - BRAZIL - 2025

Reliability analysis of pultruded GFRP I-sections subjected to lateral torsional buckling ultimate limit state

Felchak L. W.[1], Machado R. D.[2], Kroetz H. M.[2]

[1]*Postgraduate Program in Civil Engineering, Federal University of Paraná (UFPR)*
leonardo.wendler@ufpr.br
[2]*Dept. of Civil Structures, Federal University of Paraná (UFPR)*
Polytechnic Center of UFPR, PO Box 19.011, CEP: 81531–980, Curitiba - PR, Brazil
rdm@ufpr.br, henrique.kroetz@ufpr.br

Abstract. Pultruded Fiber-Reinforced Polymers (FRP) profiles are innovative structural elements with outstanding mechanical properties, combined with economic viability. As some interesting properties of that material, it is possible to mention: its corrosion resistance, thermal insulation, high impact resilience, superior strength to-weight when compared to conventional civil industry materials like steel and aluminum, and cost-effective manufacturing processes. Nevertheless, studies regarding the effects of the heterogeneity of elastic and mechanical properties, due to the pultruded FRP profiles (PFRPP) manufacturing process, on the reliability of those structural members under common loading conditions are scarce on the literature. So, in this sense the present work compares reliability analysis results of global buckling ultimate limit state of I-section pultruded beams, considering fiber-resin content uncertainty under homogeneous distribution and by considering the spatial variability of these properties using gaussian random fields. First Order Reliability Analysis (FORM) method and Monte Carlo Simulation (MCS) with Latin Hypercube Sampling were employed to calculate failure probabilities of pultruded FRP profiles under global buckling limit state. Buckling loads estimates were obtained by eigenvalue buckling finite element analysis. In order to reduce computational cost of the FEM models gaussian process metamodels were employed.

Keywords: Pultruded Fiber-Reinforced Polymers; spatial variability; gaussian random fields; finite element model.

1 Introduction

Fiber-reinforced polymer (FRP) structural elements have gained prominence as a viable and attractive alternative to traditional materials used in the construction industry. Due to FRP composites' promising properties, such as high strength-to-weight ratio, durability, and resistance to corrosion, fatigue, and wear, this material's applications in civil engineering and infrastructure are wide spreading Kaw [1], Bank [2]. Among these applications, pultruded glass fiber-reinforced polymer (GFRP) profiles have stood out and are increasingly being adopted in civil construction.

Pultruded profiles are manufactured through pultrusion, a continuous and automated fabrication process in which fibrous materials, such as glass or carbon fibers, are impregnated with resin and pulled through a heated die, where shaping and curing take place.

The increasing adoption of fiber-reinforced composite materials across various industries has led to the development of numerous standards and guidelines aimed at establishing design procedures and standardizations Clarke [3], Council et al. [4], Ascione et al. [5].

In pultruded profiles the dominant failure modes are generally associated with buckling, due to their high strength-to-stiffness ratio and slender, thin-walled geometries, which make them particularly susceptible to such phenomena. This is especially true for open-section profiles, such as "U", "I", or angle shapes, where additional risks of distortional or lateral torsional buckling (LTB) arise.

Given the inherent variability of FRP structures stemming from their complex manufacturing and assembly process, in addition to another various significant sources of uncertainties related to mechanical properties, geometric and loading parameters, imperfections, and mathematical modeling, it becomes imperative to adopt a probabilistic approach for structural analysis.

The present work addresses the reliability problem of glass fiber-reinforced pultruded profiles with respect to buckling ultimate limit state (ULS). Uncertainties associated with the fiber/resin content were incorporated into the analysis through numerical modeling, wherein the elastic properties of the material were computed using deterministic micromechanical formulations, parameterized by the local fiber/resin volume fraction.

Initially, the uncertainty in the fiber/resin ratio was modeled as a random variable, under the assumption of homogeneous material property distributions across the entire cross-section of the profile. This approach was subsequently refined by accounting for the spatial variability of the fiber/resin content, which was modeled through the application of random fields.

2 Data acquisition of resin content distribution and elastic properties formulation

The database related to experimental measurements of resin content distribution, used for random variable and random field modelling, the geometric parameters of the I-section analyzed, and also the formulations for deriving the mechanical properties using the resin content as input, were based on Feng et al. [6] article. In this study a total of 6 I-sections were subjected to a calcination test procedure to evaluate their resin content. The model was built based only on the geometry of one sample so far, this profile has 200 mm of overall height, 100 mm of flange width and 9,5 mm of web and flange thickness.

The experimental measurements results obtained by Feng et al. [6] are shown in Table 1 for nine sample locations, three in the flange (with equal spacing), two at the joints between the web and the flange, and four at the midpoints of the top and bottom flanges, for more details about the testing methodology, geometries, resin and fiber properties employed in this study, the reader is invited to read the refered article. In the last column the mean and the coefficient of variation (COV) obtained for each sample is shown.

Table 1. Resin contents of I-section

Sample	Resin content (%)									Mean (COV)
	Top flange			Web			Bottom flange			
	TF-1	TF-M	TF-2	W-1	W-2	W-3	BF-1	BF-M	BF-2	
1	20.25	21.60	20.28	20.22	20.36	19.89	20.04	21.74	20.96	20.59 (0.03)
2	23.96	28.80	23.23	32.42	33.74	33.43	22.15	27.65	23.39	27.64 (0.16)
3	23.99	27.00	23.78	29.28	29.29	30.00	25.56	27.54	25.64	26.90 (0.08)
4	25.66	27.75	21.55	33.70	33.99	31.90	28.68	30.05	28.82	29.12 (0.13)
5	30.52	33.06	30.07	29.92	30.16	30.72	26.81	25.05	31.72	29.78 (0.08)
6	26.07	27.44	27.77	30.28	30.16	29.75	20.76	24.72	21.93	26.54 (0.12)

Source: Adapted from Feng et al. [6].

As mentioned before, in this work the resin content parameter is used as the basis for determining the elastic properties for the material. This methodology was used in Feng et al. [6] work, and complies with most existing design standards. An assumption adopted was that possible variations in material properties in the longitudinal direction will be neglected. In the mentioned article the resin content is initially converted into fiber volume, to posteriorly apply the rule of moisture to obtain the longitudinal elastic modulus E_1 and the major Poisson's ratio ν_{12}.

Once the longitudinal elastic modulus of the fiber E_f; The longitudinal elastic modulus of the matrix (resin) E_m; The Poisson's ratio of the fiber ν_f, and ν_m the Poisson's ratio of the resin, the Huang [7] model can be employed to calculate the transversal elastic modulus E_2 and the shear modulus G_{12}. This study focuses only on pultruded glass fiber-reinforced epoxy resin profiles, so only the glass fiber and epoxy resin properties presented in Feng et al. [6] were considered.

3 Numerical modeling

Aiming to predict the structural behavior of the profiles under buckling, a linear eigenvalue buckling analysis was conducted by finite element method. A I-section beam 4m long was modeled in ANSYS MECHANICAL APDL 2024 R1 [8] with the geometry and boundary conditions indicated in Figure 1. The referred span was chosen after several tests to ensure a high slenderness ratio, and force global buckling as first buckling mode, removing possible local modes from the analysis.

SHELL281 elements were used with an 8mm size and aspect ratio of 1:1, a mesh judged very acceptable after some convergence tests. The two end sections edges of the beam are fixed (translations and rotations in all direction were set equal to zero), and a uniform distributed line load is applied at the beam top flange.

It was assumed that the loading is of a permanent nature, originating from gravitational forces. As commonly adopted in probabilistic modelling of this class of loads, its model corresponds to a gaussian random variable, characterized by low to moderate variability. A coefficient of variation (COV) of 10% was assumed, according to standard assumptions given by European Committee for Standardization (CEN) [9] for permanent gravitational loading. Two different values were considered for the random variable mean: 3000 N/m and 3300 N/m.

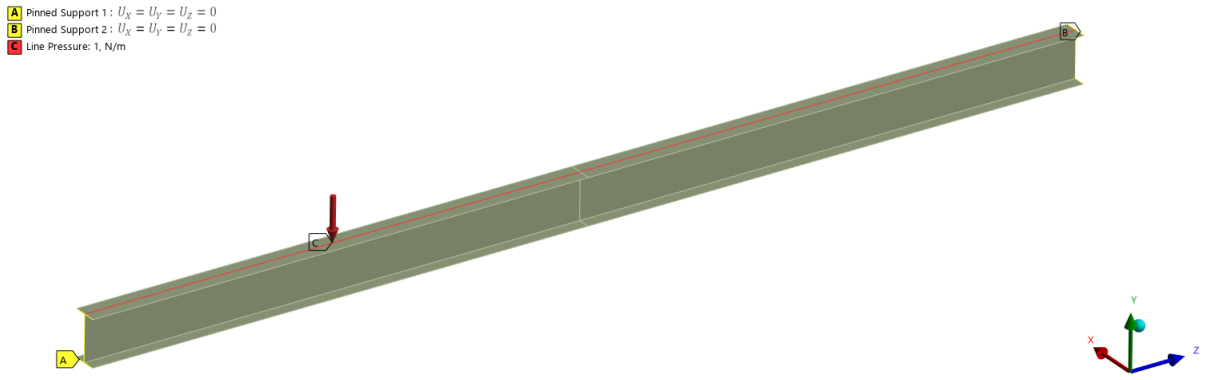


Figure 1. Finite element model

In the first approach for modeling material properties uncertainty, it was assumed that the material is homogeneous, and the resin content was modeled as a gaussian random variable with mean and standard deviation calculated considering all sample points shown in Table 1.

The second approach considers the spatial variability of the properties by modeling the resin content distribution over the cross-section as a gaussian homogeneous random field. The field mean and standard deviation were determined the same way as described for the random variable modeling. The chosen correlation function for representing the field was the exponential ellipsoidal correlation function, given by the expression:

$$R(h) = \exp(-h) \tag{1}$$

with:

$$h = \frac{1}{\theta}\left[\sum_{i=1}^{M}\left(x_i - x_i^{'}\right)^2\right]^{0.5} \tag{2}$$

where $M = 2$, since the variability is only considered in the 2D cross section plane. x_i and $x_i^{'}$ are the coordinates of the point analyzed, and θ is commonly referred in the literature as correlation length or scale parameter [10].

The exponential correlation function was selected based on Sriramula and Chryssanthopoulos [11] studies. According to these authors, this function showed the best fit for experimental measurements of longitudinal modulus of glass fiber reinforced composite panels. The assumed values for correlation length were also based on this study, which recommends lengths of approximately 20 mm for exponential functions. Values of $\theta = 90$ mm and $\theta = 1000$ mm were also assessed to analyze the effects of this parameter in failure probability.

The random field was discretized using the discrete Karhunen-Loève expansion Schenk and Schuëller [12]. In this method, the correlation matrix is directly determined by means of the element mesh centroids, given by the finite element method discretization, and the spectral decomposition of that matrix gives the eigenvalues and eigenfunctions to compute the Karhunen-Loève expansion approximation. A minimum energy ratio (also known as explained variance) of $\delta_e \geq 0.98$ was set to define the minimum number of terms for the truncated series expansion.

4 Reliability problem formulation

In the structural reliability problem formulation considering homogeneous resin content distribution, the limit state function depends on two random variables: the resin content R_C, and the applied load S:

$$g(M_m, S) = \lambda_{i,crit}(M_m) - S \leq 0 \tag{3}$$

where $\lambda_{i,crit}$ is the first eigenvalue, or applied load factor to obtain the first buckling mode of the structure under analysis (N/m); S is a gaussian random variable representing a uniform line load applied to the beam along its length (N/m).

To solve this particular problem, the First-Order Reliability Method (FORM) was employed, using the improved Hasofer-Lind-Rackwitz-Fiessler (iHLRF) algorithm to search for the Most Probable Point (MPP), also referred to as the design point. During the iterative procedure, the finite element (FEM) model must be evaluated at each iteration to compute the limit state function $g\left(M_m, S\right)$, and its corresponding gradient vector. Since the limit state function defined in Equation 3 lacks a closed-form analytical expression, its gradient is not available explicitly and must be approximated numerically. In this sense, a forward finite difference scheme was adopted to evaluate the gradient components. The algorithm was initialized at the mean values of the input random variables, and the iterations step size was determined using the Armijo rule.

The limit state equation for the reliability analysis which considers material properties spatial variability is described by a gaussian random field denoted as $H\left(x\right)$, and a gaussian random variable S representing the applied load:

$$g\left(H\left(x\right), S\right) = \lambda_{i,crit}\left(H\left(x\right)\right) - S \leq 0 \tag{4}$$

Due to the non-linearity and higher dimension of that problem, Monte Carlo simulation method with Latinhypercube sampling was employed to calculate the beam failure probability for three different length scale parameter values. Also three samples were generated for each Monte Carlo simulation to verify results sampling independence. A total number of 150 samples was generated for each simulation.

5 Results and discussion

Table 2 presents the FORM algorithm number of iterations required for convergence, the design point and the failure probability obtained while solving the reliability problem given by Equation 3.

Table 2. FORM results

Case	$S(\mu)$	Iterations	S^*	M_m^*	p_f
1	3000	4	2898.210	25.166	6.928E-01
2	3300	5	3054.450	23.237	8.662E-01

Figure 2 and Figure 3 shows results for the proposed analysis considering fiber-resin spatial variability. In those figures three curves are shown, each one represents the average of MCS samplings for a fixed correlation length θ as indicated in the legend, also a horizontal line is presented, which corresponds to the result for reliability analysis considering fiber-resin uniformity, obtained with FORM method. Table 3 presents the failure probability estimates and associated coefficient of variation.

Table 3. MCS probability of failure and coefficient of variation results

Case	S	θ	n_S	p_f	CoV (%)
1	3000	20	150	0.742	4.803
2	3000	90	150	0.707	5.259
3	3000	1000	150	0.682	5.572
4	3300	20	150	0.927	2.295
5	3300	90	150	0.916	2.471
6	3300	1000	150	0.858	3.318

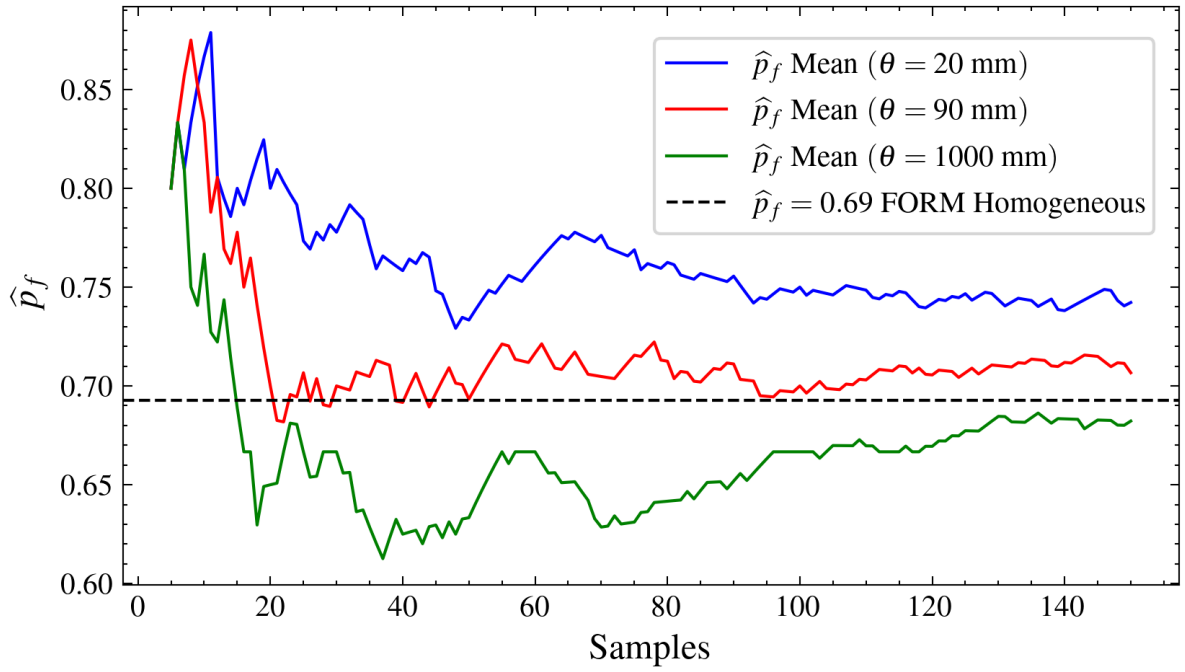


Figure 2. FORM and MCS analysis results for $S(\mu = 3000$ N/m)

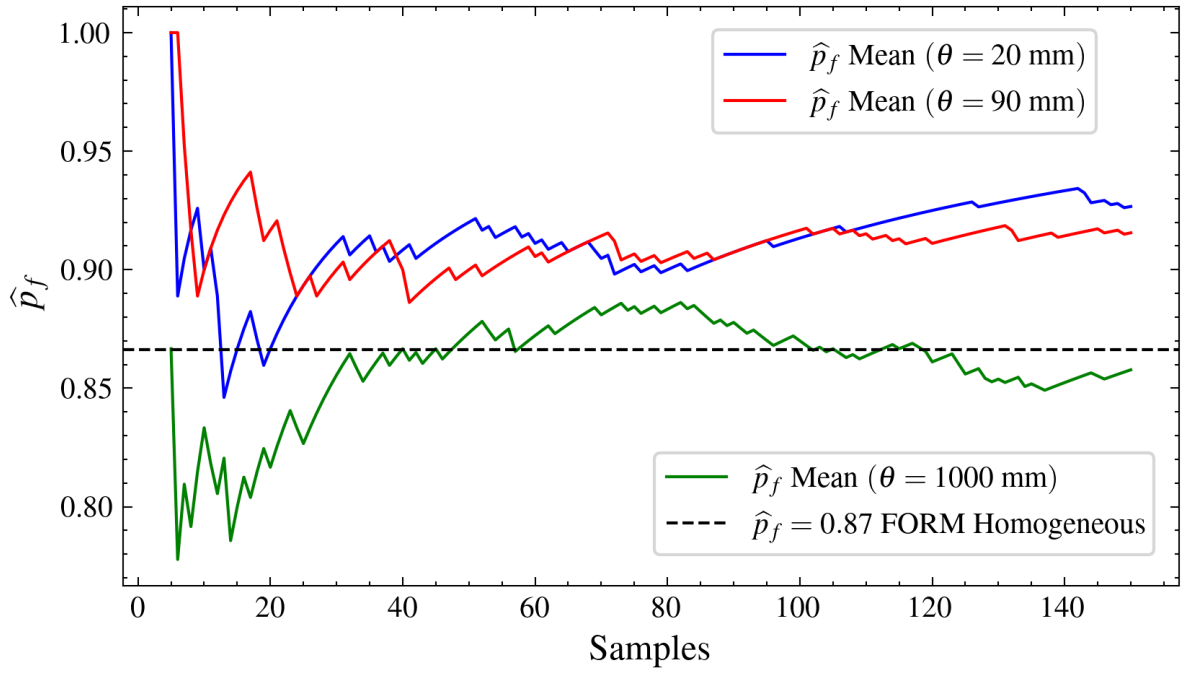


Figure 3. FORM and MCS analysis results for $S(\mu = 3300$ N/m)

For both cases is possible to observe that lower correlation lengths leads to higher probability of failures. For gaussian random fields with exponential correlation functions, a short correlation length means the field decorrelates quickly with increased distances, and points separated by small distances become nearly independent, resulting in high spatial gradients, oscillations or variations. As can be seen in Figure 2 and Figure 3, the highest

failure probabilities $p_f = 74.22\%$ and $p_f = 92.67\%$ were obtained for the reliability heterogeneous problems related to the shortest correlation length.

The field with $\theta = 1000$ mm presented in both analyzed cases a probability of failure very similar to the one derived from FORM method, for a homogeneous distribution of fiber-resin. It can also be explained by the lower dispersion, or "noisy" presented by the random field realizations when its modelled with high correlation lengths. For the problem with $S(\mu = 3000$ N/m), an increase in failure probability of 6% was observed, when comparing $\widehat{p}_f$ for $\theta = 20$ mm and p_f determined by FORM analysis. In the case with $S(\mu = 3300$ N/m), an increase of 6,9% in failure probability was noticed.

6 Conclusions

Based on the results of the numerical simulations and their comparison, the following conclusions can be summarized:

1. FEM is built to analyze the influence of non-uniform material on compressive member, a parametric model is set up at ANSYS enabling the implementation of iterative schemes. FORM and MCS results show that fiber-resin uniformity assumption increased the overall section resistance to global buckling, leading to lower failure probabilities when compared to profiles with non-uniform fiber-resin distribution.
2. Fiber-resin heterogeneity is a type of material uncertainty in pGFRP, which may reduce global buckling resistance, and the member reliability related to that structural mode of failure. MCS analysis results shown that profiles with short correlation length, which are associated to members with higher material heterogeneity and variability, presented high probability of failure when compared to beams under material homogeneity assumption, for the specific boundary conditions assessed;

Acknowledgements. The authors thank Federal University of Paraná, National Council for Scientific and Technological Development (Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPQ)), and Coordination for the Improvement of Higher Education Personnel (Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES)) for the financial support and infrastructure.

Authorship statement. The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

References

[1] A. K. Kaw. *Mechanics of composite materials*. CRC press, 2005.
[2] L. C. Bank. *Composites for construction: structural design with FRP materials*. John Wiley & Sons, 2006.
[3] J. L. Clarke. *Structural design of polymer composites: Eurocomp design code and background document*. CRC Press, 2003.
[4] N. R. Council and others. Guide for the design and construction of structures made of frp pultruded elements. *CNR-DT*, vol. 205, pp. 25–31, 2007.
[5] L. Ascione, J.-F. Caron, P. Godonou, van K. IJselmuijden, J. Knippers, T. Mottram, M. Oppe, M. G. Sorensen, J. Taby, L. Tromp, and others. Prospect for new guidance in the design of frp, 2016.
[6] P. Feng, Y. Wu, and T. Liu. Non-uniform fiber-resin distributions of pultruded gfrp profiles. *Composites Part B: Engineering*, vol. 231, pp. 109543, 2022.
[7] Z. M. Huang. Micromechanical strength formulae of unidirectional composites. *Materials Letters*, vol. 40, n. 4, pp. 164–169, 1999.
[8] Ansys. *Ansys 2024 R1 Release Documentation, Theory and Modelling Guide*. ANSYS, Inc. Canonsburg, PA, 2024.
[9] European Committee for Standardization (CEN). En 1990: Eurocode - basis of structural design, 2002.
[10] C. Lataniotis, D. Wicaksono, S. Marelli, and B. Sudret. UQLab user manual– Kriging (Gaussian process modeling). Technical report, Chair of Risk, Safety and Uncertainty Quantification, ETH Zurich, Switzerland. Report UQLab-V2.1-105, 2024.
[11] S. Sriramula and M. K. Chryssanthopoulos. Quantification of uncertainty modelling in stochastic analysis of frp composites. *Composites Part A: Applied Science and Manufacturing*, vol. 40, n. 11, pp. 1673–1684, 2009.
[12] C. A. Schenk and G. I. Schuëller. *Uncertainty assessment of large finite element systems*, volume 24. Springer Science & Business Media, 2005.