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Topology optimization method applied to the design of liquid flow channels with dispersed bubbles

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Abstract. A novel topology optimization formulation for the design of channels considering two-phase flow is developed in the present study. The considered cases comprise steady-state laminar flow, consisting of a liquid phase with dispersed gas bubbles. The optimization problem consists of defining a flow channel of liquid with bubbles that minimizes an objective function defined by the fluid power dissipation and the drag force on the bubbles. The design variables represent the permeability of each discretized region of the computational domain, simulating solid and fluid areas. The presence of bubbles may affect the flow field by modifying the flow characteristics by introducing disturbances. Thereby, changing the gas concentration by optimizing the flow path is useful in many engineering applications, such as turbomachinery, heat exchangers, and chemical reactors. To simulate the dispersed two-phase flow, the two-fluid Euler-Euler model is applied. The governing equations are solved by the finite element method using the FEniCS open-source platform. The optimization formulation is implemented using the same platform, with automatic differentiation to solve the adjoint problem and the interior point optimization algorithm to update the design variables. The preliminary results include the optimization of a bubbly flow channel to demonstrate the applicability of this methodology.

Keywords: Topology optimization, Bubbly-flow, Two-fluid flow, Computational fluid dynamics

1 Introduction

The optimization of channels for liquid-gas two-phase flow is a subject of great relevance for various industrial applications. In sectors such as oil and gas, chemical processing, and power generation, it is common to require the pumping of liquids containing bubbles, which can significantly affect the energy efficiency of these systems. Therefore, the development of methodologies to optimize flow devices while considering multiphase flow can yield substantial benefits in terms of energy consumption and performance.

Topology optimization (TO), in contrast to parametric and shape optimization, is capable of generating optimized configurations that differ significantly from the initial guess and without the designer's intuition at the conceptual design stage. Bendsøe and Kikuchi [1] first introduced the material distribution method for TO, applying it to structural optimization. The TO for fluid flow problems was first developed by Borrvall and Petersson [2], who proposed a fictitious force term to model porous domains, allowing for no-slip boundary conditions by setting a sufficiently high value of fictitious force. Subsequent research has expanded the scope of TO in fluid dynamics, for instance, to laminar steady-state Navier-Stokes flows by Gersborg-Hansen et al. [3], turbulent flow problems by Dilgen et al. [4] and compressible flows by Sá et al. [5]. There has been growing interest in the TO of multiphase flows in recent years. This problem was first addressed by Andreasen et al. [6], who applied the TO method to enhance the mixing performance of inertial microfluidic devices. Prado et al. [7] and Chen and Yaji [8] conducted TO of solid particle systems using the Euler–Euler approach.

The aim of this work is to develop a TO method for the design of flow channels considering liquid with dispersed gas bubbles. Thus, the objective of the proposed optimization method is to determine the flow path within the design domain, to optimize performance parameters under multiphase (gas-liquid) flow conditions, while satisfying design constraints. The flow is characterized by a continuous liquid phase and a dispersed gaseous phase in the form of bubbles, which are already present in the flow stream admitted by the system. This study presents a preliminary implementation of the multiphase TO method applied to the optimization of a steady-state laminar flow channel, addressing the minimization of power dissipation in the design domain.

2 Fluid flow model

In this work, the configuration analyzed is that of a bubbly flow with distinct and immiscible compositions. The multiphase flow setup is solved using the Euler-Euler approach, which solves the set of conservation equations for each phase separately, and the coupling is achieved by sharing the pressure field and using models for their interaction. The description of multiphase flow as an interpenetrating continuum incorporates the concept of phase volume fractions, ϕ, representing the space occupied by each phase. The assumptions adopted for the preliminary exploration are as follows: both fluid phases are modeled as steady, incompressible, and laminar; all bubbles are assumed to have the same diameter; bubble breakup and coalescence are neglected; and the drag force is considered the sole mechanism of momentum exchange between the phases. Accordingly, the mass and momentum balance equations for the liquid (subscript 'l') and gas (subscript 'g') phases can be written, respectively, as

$$\nabla \cdot (\phi_l \mathbf{u}_l) = 0, \tag{1}$$

$$\nabla \cdot (\phi_l \rho_l \mathbf{u}_l \mathbf{u}_l) = -\phi_l \nabla p + \nabla \cdot (\phi_l \mu_l (\nabla \mathbf{u}_l + \nabla \mathbf{u}_l^T)) + \phi_l \rho_l \mathbf{g} - \mathbf{F}_D + \kappa(\alpha)\mathbf{u}_l, \tag{2}$$

$$\nabla \cdot (\phi_g \mathbf{u}_g) = 0, \tag{3}$$

$$\nabla \cdot (\phi_g \rho_g \mathbf{u}_g \mathbf{u}_g) = -\phi_g \nabla p + \nabla \cdot (\phi_g \mu_g (\nabla \mathbf{u}_g + \nabla \mathbf{u}_g^T)) + \phi_g \rho_g \mathbf{g} + \mathbf{F}_D + \kappa(\alpha)\mathbf{u}_g, \tag{4}$$

where $\phi_{l,g}$ is the volume fraction, $\rho_{l,g}$ is the density, and $\mathbf{u}_{l,g}$ is the velocity vector, p is the pressure, $\mu_{l,g}$ is the fluid viscosity, and $\mathbf{g}$ is the gravity acceleration. $\kappa(\alpha)\mathbf{u}_{l,g}$ is the force term that is used in TO to model the solid material as a function of the design variable α. The drag force, $\mathbf{F}_D$, is calculated as proposed by Schiller and Naumann [9] as a function of the bubble diameter d_b.

In this study, the computational fluid dynamics simulation throughout the TO of the channel is performed using the finite element method with the open-source software FEniCS [10]. The flow behavior is calculated by the weak formulation of the Navier-Stokes equations, considering the method of weighted residuals of the Galerkin method. The computational domain is discretized using Taylor–Hood finite elements, where linear interpolation is applied to the pressure and design variable, while a quadratic interpolation is used for the velocity of both phases.

3 Topology optimization formulation

The TO algorithm for fluids generates a pseudo-rigid domain that optimizes the fluid passage region with respect to the objectives and constraints of the problem. This can be done by considering the flow through a porous medium to represent the transition between solid and fluid. In this way, the variation in permeability provided by the porous medium allows fluid blockage in a solid region and flow in a fluid region. The TO design variable α is used to simulate solid (α = 0) or fluid (α = 1) regions. Intermediary (gray) α values are permitted to relax the TO formulation. As defined by Borrvall and Petersson [2], the change between solid and fluid is then controlled by the inverse permeability $\kappa(\alpha)$ as $\kappa(\alpha) = \kappa_{max} + (\kappa_{min} - \kappa_{mx})\alpha(1+q)/(\alpha+q)$, where κ_{max} is the maximum value, and κ_{min} is the minimum value. The penalization parameter q is used for the relaxation of the material model.

The objective is to optimize the gas-liquid channel to minimize energy dissipation (due to viscous losses in the flow) and the energy transferred from the liquid to the bubbles, by minimizing the bubble drag force, while maintaining the flow of liquid with bubbles. The power dissipation for both phases, $\Phi_{l,g}$, and the bubble drag force ζ over the design domain Ω can be evaluated by:

$$\Phi_l = \int_\Omega \left[\kappa(\alpha) \boldsymbol{u}_l \cdot \boldsymbol{u}_l + \frac{\phi_l \mu_l}{2} \nabla \boldsymbol{u}_l : \nabla \boldsymbol{u}_l \right] d\Omega, \tag{5}$$

$$\Phi_g = \int_\Omega \left[\kappa(\alpha) \boldsymbol{u}_g \cdot \boldsymbol{u}_g + \frac{\phi_g \mu_g}{2} \nabla \boldsymbol{u}_g : \nabla \boldsymbol{u}_g \right] d\Omega, \tag{6}$$

$$\zeta = \int_\Omega |\boldsymbol{F}_D| \, d\Omega. \tag{7}$$

The TO problem is formulated as the minimization of an objective function J considering the normalized values of $\Phi_{l,g}$ and ζ based on their values from the first iteration of the optimization, $\hat{\Phi}_{l,g}$ and $\hat{\zeta}$. They are also multiplied by weighting factors, $w_{\Phi_{l,g}}$ and w_ζ, to control the impact of each term. The TO problem, subject to a minimum fluid volume constraint $\bar{V}$ to prevent trivial solutions, and a box constraint on the design variable, is:

$$\begin{aligned} \min_{\alpha} \quad & J = w_{\Phi_l} \frac{\Phi_l}{\hat{\Phi}_l} + w_{\Phi_g} \frac{\Phi_g}{\hat{\Phi}_g} + w_{\zeta} \frac{\zeta}{\hat{\zeta}} \\ \text{s.t.} \quad & V = \int_{\Omega} \alpha d\Omega - \bar{V} \leq 0, \\ & 0 \leq \alpha \leq 1. \end{aligned} \tag{8}$$

The TO method updates the design based on information related to the derivatives (sensitivities) of the objective function with respect to the design variables. In this way, a sensitivity map is generated in each optimization cycle, showing how the objective function changes in relation to the presence of solid material throughout the domain. In this work, the discrete adjoint method is employed for this purpose through the dolfin-adjoint library of the FEniCS software. Additionally, the interior point optimization algorithm is used to update the design variables.

4 Results

The aforementioned TO formulation was applied to the optimization of bubbly flow in the design domain presented in Fig. 1(a), with L=1m. The properties of the liquid and gas phases are defined as densities of 10 kg/m^3 and 1 kg/m^3, and viscosities of 1 kg/(m·s) and 0.01 kg/(m·s), respectively, with a bubble diameter of 0.1 mm. No-slip conditions are imposed for the liquid phase along the domain walls, while a slip condition is applied to the gas phase. A parabolic profile with a maximum velocity of 0.0001 m/s is imposed for the inlet velocities for both phases, and the gas volume fraction is fixed to 0.02 at the inlet. The outlet pressure is set to 0 Pa. The computational domain is discretized with triangular elements with 0.02 m edges. To amplify the influence of the gas phase, the weighting factors are set as $w_{\Phi_g} = 10^7$ and $w_{\Phi_g} = w_{\zeta} = 1$, and the volume constraint is $\bar{V} = 0.38$.

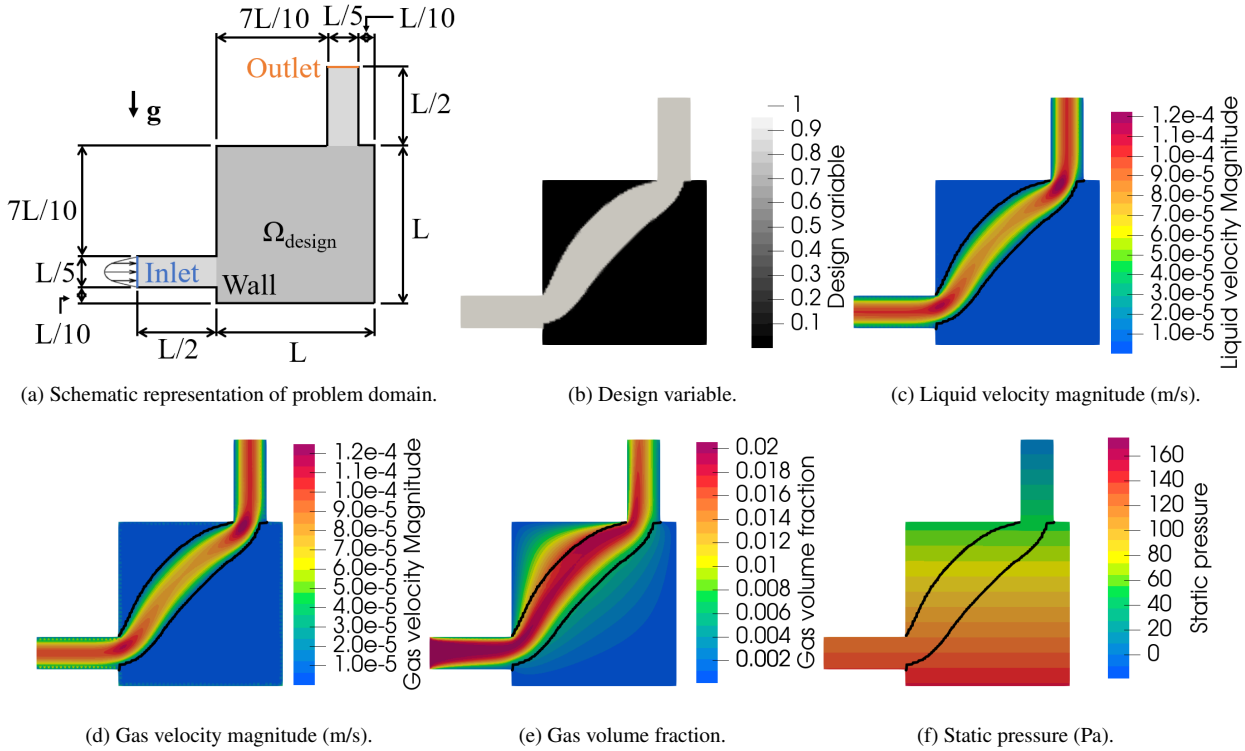
Figure 1(b) presents the optimized design variable field, where the black and white areas denote solid and fluid regions, respectively. The effect of the optimized porosity can be verified in the liquid and gas velocity fields, Figs. 1(c) and 1(d), respectively, where zero velocity is verified in the region of $\alpha = 0$. The optimized flow path generated is a curve developed according to the higher gas volume fraction area, as displayed in the gas volume fraction contour in Fig. 1(e). As a consequence of the density difference between phases and the action of gravity, the bubbles rise, leading to the higher gas volume fraction observed in the upper region. From Fig. 1(f), it can be verified that the pressure field presents a gradient in the vertical direction due to the gravitational force.

5 Conclusions

The developed TO formulation successfully addressed the flow channel problem considering gas-liquid flow, demonstrating that gas phase presence drove geometric enhancements optimizing the fluid flow path to minimize energy dissipation and bubble drag. Further improvements to the current implementation are necessary to address the challenges encountered during the development of the TO framework for bubbly flows, such as convergence difficulties when using real fluid properties and the unintended presence of gas in solid regions. Additionally, future work should incorporate the effects of turbulence and account for variations in bubble size to increase the applicability of the method to practical engineering problems.

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(a) Schematic representation of problem domain.

(b) Design variable.

(c) Liquid velocity magnitude (m/s).

(d) Gas velocity magnitude (m/s).

(e) Gas volume fraction.

(f) Static pressure (Pa).

Figure 1. Design and variable fields for the pipe optimized channel.

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