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Continuum model for erosion and deposition in porous media

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Abstract. Flow-induced channelization in porous media emerges from the complex interplay between fluid dynamics and the evolving porous structure, driven by erosion and deposition. We propose a continuum model that captures these interactions at the Darcy scale, and demonstrate how even weak heterogeneities in porosity can lead to permanent flow localization. By varying the strength and nature of the disorder, we uncover a smooth localization for porosity fluctuations. Our approach bridges pore- and continuum-scale dynamics and is validated by simulations.

Keywords: Upscaling, Porous Media, Lattice Boltzmann Method.

1 Introduction

The formation of preferential channels in porous media is a complex and fascinating phenomenon that occurs across a wide range of systems, including river networks, eroding landscapes, filters, fuel cells, and subsurface reservoirs. It is governed by a strong nonlinear feedback loop: fluid flow alters the microstructure through erosion and deposition, which in turn modifies the flow paths [1, 2]. Understanding this process is critical not only from a fundamental physics perspective but also for numerous technological applications such as groundwater management, enhanced oil recovery, soil erosion control, and filtration system optimization [3–6].

Channelization is commonly observed in disordered porous materials, where small heterogeneities in structure or material properties can be amplified over time, leading to the emergence of narrow, high-permeability flow pathways [7, 8]. At the pore scale, various models have been developed to explore the intricate interplay between hydrodynamic forces and solid matrix evolution. These include network models, discrete element methods, and lattice Boltzmann simulations that can reproduce channel formation and clogging [4, 6]. However, such models are computationally expensive and often limited to small domains.

To overcome these limitations, continuum-scale models provide a promising alternative, enabling efficient simulation of large systems while still capturing key physical processes. In this work, we present a continuum model derived from the physics of capillary-scale Poiseuille flow coupled to an erosion-deposition mechanism. We investigate how the disorder in the porosity field—affect the long-term flow structure and whether they trigger permanent localization.

We show that disorder in the initial porosity results in smooth but persistent channel formation. Our findings demonstrate that even weak initial heterogeneities can lead to strong flow localization and suggest a critical role for spatial disorder in determining the morphology of erosion-driven systems.

2 Continuum model

We model the porous medium at the Darcy scale with spatially-dependent porosity $\phi(\mathbf{r}, t)$ and velocity $\mathbf{u}(\mathbf{r}, t)$. The fluid dynamics are governed by a generalized Navier-Stokes equation:

$$\nabla \cdot \mathbf{u} = 0, \tag{1}$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\left(\frac{\mathbf{u}}{\phi}\right) = -\frac{1}{\rho}\nabla(\phi p) + \nu \nabla^2 \mathbf{u} + \mathbf{F}, \tag{2}$$

where ν is the kinematic viscosity, ρ is the density, and $\mathbf{F}$ includes body forces and Darcy drag:

$$\mathbf{F} = -\frac{\phi \nu}{k}\mathbf{u} + \phi \mathbf{G}. \tag{3}$$

The permeability k follows the Kozeny-Carman relation for capillary-like pores:

$$k = \frac{a^2}{18}\frac{\phi^3}{(1-\phi)^2}, \tag{4}$$

where a is the local capillary radius. The evolution of a is governed by shear-induced erosion and deposition. Assuming a threshold model:

$$\dot{a} = \begin{cases} +\kappa(\tau - \tau_w), & \tau_w < \tau \\ 0, & \tau_w = \tau \\ -\kappa(\tau_w - \tau), & \tau_w > \tau \end{cases} \tag{5}$$

with wall shear stress τ_w approximated by:

$$\tau_w = \frac{6\rho\|\mathbf{u}\|^2}{\mathrm{Re}}, \tag{6}$$

and Re the local Reynolds number. The porosity evolution follows $\dot{\phi}/\phi = \dot{a}/a$.

3 Flow Through Packed Discs

We simulate flow over a bed of circular inclusions using the lattice Boltzmann method [9], employing a multi-relaxation-time (MRT) scheme and a ghost-fluid approach to resolve fluid–solid interfaces [10]. A constant external force $G = 10^{-6}$ drives the flow, and the viscosity is chosen to maintain laminar conditions.

The erosion and deposition dynamics are implemented using a phenomenological model based on the local wall shear stress, with equal transport rates $\kappa_{\mathrm{er}} = \kappa_{\mathrm{dep}} = 10$. The computational domain spans 256×384 nodes, with a central packed region surrounded by empty inlet and outlet zones. Particles have radii drawn from a Gaussian distribution (mean 20, 5% standard deviation), and are compressed to fill a square domain. Subsequently, their sizes are rescaled to 75% to achieve the desired porosity.

4 Wall Shear Stress Estimation at the Pore Scale

To estimate wall shear stress in porous structures, we begin by analyzing a benchmark case: viscous flow between two flat, parallel plates spaced by a distance h. The velocity profile for steady-state Poiseuille flow in this geometry is given by

$$v(y) = \frac{G}{2\rho\nu} y(h - y), \tag{7}$$

where G is the applied body force per unit volume, ρ the fluid density, and ν the kinematic viscosity.

The wall shear stress is obtained by evaluating the velocity gradient at the boundary:

$$\tau_w = \rho\nu \left.\frac{\partial v}{\partial y}\right|_{y=0} = 6\rho\nu\frac{\langle v \rangle}{h} = 6\rho\frac{\langle v \rangle^2}{\mathrm{Re}}, \tag{8}$$

where $\langle v \rangle$ is the average velocity and Re the Reynolds number defined using the average pore size.

In disordered porous media composed of packed inclusions, we approximate the local flow using this capillary-based expression. To determine a representative pore length scale, we apply the watershed segmentation technique to identify regions between neighboring particles, using their separation as a proxy for pore diameter, following

the method described in [11]. Figure 1(g) illustrates these watershed partitions for three configurations: initially, after erosion, and after deposition. The shear stress measured from pore-scale simulations shows good agreement with the analytical prediction, even across samples with varying erosion thresholds.

We explored three regimes by varying the erosion threshold τ_{er}. For low thresholds ($\sim 8 \times 10^{-7}$), erosion dominates; at high thresholds ($\sim 8 \times 10^{-5}$), deposition prevails; and at intermediate values ($\sim 8 \times 10^{-6}$), both mechanisms compete, leading to localized channel formation. In all cases, the relationship between rescaled shear stress and velocity matches the theoretical trend. Deviations at late times—especially after deposition—are attributed to pore occlusion and disconnected regions, which affect pore size estimation.

5 Effect of Initial Porosity Disorder on Flow Localization

To investigate how structural heterogeneity affects erosion-driven channel formation, we introduce disorder directly in the initial porosity field. Specifically, the porosity at each spatial location is sampled from a truncated exponential distribution:

$$\phi_i = \phi_{\max} \exp\left[\beta_\phi (x_i - 1)\right], \tag{9}$$

where x_i is drawn from a uniform distribution on the interval $[0, 1)$, and β_ϕ quantifies the strength of the disorder. This formulation introduces spatial variability while maintaining porosity within the physically reasonable bounds of $[0.1, 0.9]$ to ensure connectivity and avoid overcrowding.

The numerical simulations are performed using a lattice Boltzmann method (LBM) to resolve the fluid flow, coupled with an Euler integration scheme to evolve the porosity field in time. The computational domain has periodic lateral boundaries and fixed inlet and outlet velocities, promoting a uniform driving force across the sample. We explore different domain sizes and aspect ratios to verify the robustness of our results.

To quantify the emergence of preferential flow paths, we define an order parameter O based on the spatial distribution of kinetic energy:

$$O = 1 - \frac{\langle e \rangle^2}{\langle e^2 \rangle}, \quad \text{with} \quad e = \|\mathbf{u}\|^2. \tag{10}$$

This metric ranges from 0 for completely uniform flows to 1 in the limit of fully localized flow.

Figure 1 illustrates the development of flow localization across different disorder strengths β_ϕ. Even modest disorder levels trigger channelization, as evident from both the velocity and porosity fields. The time evolution of the order parameter (panel a) confirms that the system settles into a steady state after approximately 5×10^5 time steps, independent of the initial fluctuation amplitude. Panel (b) shows that the degree of localization increases smoothly with β_ϕ, with minimal dependence on the aspect ratio or system width—justifying our use of an elongated simulation domain. The lower panels (c–f) compare the initial and final states of porosity and velocity. High-flow regions strongly correlate with regions of initially higher porosity, indicating a positive feedback mechanism: larger pores induce higher shear, which accelerates erosion and reinforces flow through those regions.

These findings show that initial porosity heterogeneity, even in the absence of variability in material strength, is sufficient to drive self-organization of flow into persistent channels.

6 Conclusion

We presented a continuum framework for simulating erosion- and deposition-driven evolution in porous media, incorporating pore-scale physics into a Darcy-scale model. By validating the wall shear stress estimates through pore-scale simulations of flow through packed discs, we demonstrated that the capillary approximation remains accurate under various erosion and deposition regimes. This establishes confidence in using the model to explore large-scale pattern formation phenomena.

Focusing on heterogeneity in the initial porosity field, we found that even weak disorder is sufficient to trigger persistent flow localization. Our simulations reveal that the emergence of preferential channels is not abrupt but rather continuous with increasing disorder strength. Importantly, these channels align with regions of initially higher porosity, indicating that early microstructural variations are amplified by a positive feedback loop: higher local permeability increases flow, which in turn promotes more erosion and further enhances conductivity in those regions.

This work highlights the central role of structural disorder in determining flow morphology, even in the absence of variation in material properties such as erosion resistance. It also supports the use of reduced-order models to study long-term dynamics in systems where fully resolved pore-scale simulations are infeasible.

In future work, the model can be extended to incorporate additional physical effects such as particle transport and clogging, as well as generalized to three-dimensional geometries and more realistic porous media. These

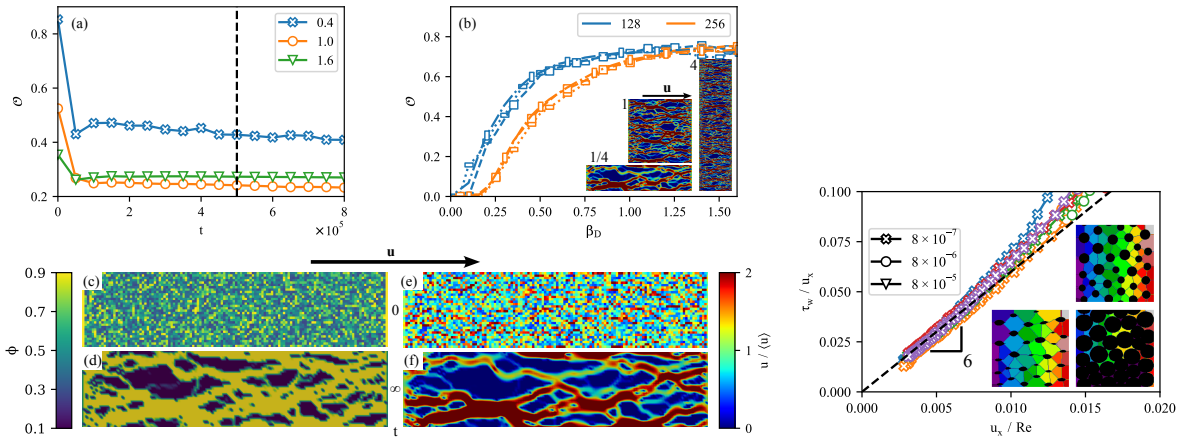


Figure 1. (a) Plot of the time evolution of the order parameter for different β_ϕ (markers and color). For the main results, the simulations ended after 5×10^5 iterations, marked with the vertical dashed line. (b) Plot of the order parameter as a function of the disorder for different aspect ratios (markers and lines) and system width (colors). The insets represent the steady-state velocity for the different aspect ratios ($\beta_D = 1$). Changing the aspect ratio of the system does not impact the measured order parameter. Thus, we chose the aspect ratio of $1/4$ that allow us to simulate larger widths. (c)-(d) Porosity and (e)-(f) velocity field for the initial time (top) and steady-state (bottom) for $\beta_D = 1$. The velocity field strongly depends on the porosity field. (g) Pore-scale wall shear stress as a function of the average fluid velocity. The symbols correspond to different erosion/deposition thresholds and the colors to different samples. The dashed line is given by the analytical prediction, Eq. (8). The inset contains examples of a porous medium with the particles (black) and the water basins (colors) determined using the watershed method for the initial system (top), after erosion (bottom left), and after deposition (bottom right). The lines separating the basins correspond to the pore size between each pair of particles used to determine the Reynolds number (Re).

steps are essential to bridge the gap between idealized simulations and practical applications in filtration, energy extraction, and geomorphological processes.

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