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Body-fitted mesh approach for structural topology optimization considering stress constraints

Lucas O Siqueira[1], Emílio C. N. Silva[1], Renato Picelli[2]

[1]*Department of Mechatronics and Mechanical Systems Engineering, University of São Paulo*
Av. Prof. Mello Moraes, São Paulo, 05508-030, São Paulo, Brazil
lucas.osiqueira@usp.br, ecnsilva@usp.br
[2]*Department of Naval Architecture and Ocean Engineering, University of São Paulo*
Av. Prof. Mello Moraes, São Paulo, 05508-030, São Paulo, Brazil
rpicelli@usp.br

Abstract.
Effective control of stress distribution in structural optimization is critical to avoid structural failure and ensure efficient material utilization. Based on that, a stress-based topology optimization framework is constructed to include stress constraints in the structural design through binary design variables. The solid structure is modeled using the Navier-Cauchy equation, considering linear elasticity. The governing equations are solved using the finite element method. The optimization problem is compliance minimization subjected to volume and stress constraints. The optimization is performed using the Topology Optimization of Binary Structures with Geometry Trimming (TOBS-GT) method, which employs Sequential Integer Linear Programming (SILP). The TOBS-GT method decouples the optimization and Finite Element Analysis (FEA) meshes. This decoupling offers several advantages, such as the flexibility to employ different sets of tools for geometry treatment, mesh building, FEA analysis, updating design variables, and post-processing results. Two classical benchmarking examples are explored: the L beam and the T pier. The proposed framework achieves the volume and stress constraints in both examples.

Keywords: Topology optimization, p-norm stress, TOBS-GT, Explicit smooth interface, Integer linear programming

1 Introduction

Topology optimization (TO) is a powerful and systematic approach for conceptual structural design, enabling the discovery of innovative solutions in engineering by defining only the design domain and boundary conditions [1]. Stress is a primary criterion for structural design in critical fields like aerospace, aeronautics, and mechanical engineering [2]. However, incorporating stress into topology optimization presents three significant and well-documented challenges [3, 4]:

- Singularity Problem: Elements with very low densities can still exhibit high stress, making their removal difficult. Discrete methods like TOBS naturally avoid this due to their binary nature.
- Local Nature of Stress: Stress must be controlled at every material point, leading to a large number of constraints. In this case, aggregation techniques such as the p-norm function provide a global measure.
- Highly Nonlinear Behavior of Stress: Stress levels are highly sensitive to subtle topological changes, particularly at stress concentrations like reentrant corners, causing numerical instabilities. This is often managed by filtering techniques and sensitivity normalization strategies.

Furthermore, traditional fixed-grid, density-based TO methods often produce jagged boundaries. These irregular boundaries can lead to artificial stress concentrations and reduced accuracy in stress calculations, necessitating post-processing [3]. In contrast, Level Set Methods (LSM) inherently provide explicitly defined and smooth structural boundaries [4]. More advanced approaches, such as the Topology Optimization of Binary Structures (TOBS) method combined with Geometry Trimming (GT), achieve smooth boundaries by identifying the solid-void contour and re-meshing the domain for accurate Finite Element Analysis (FEA) [1]. Similarly, the integration of Isogeometric Analysis (IGA) with untrimming techniques allows for a unified and precise representation of both geometric and mechanical models, ensuring accurate stress computations directly on the evolving boundaries [5]. This class of methods provides well-defined boundaries and can intrinsically handle nonlinear behaviors, reducing

reliance on extensive penalty methods [1].

Based on that, this work proposes to apply the TOBS-GT method to solve the stress-constrained problem in structural topology optimization. This methodology integrates explicit boundaries representation with sequential integer linear programming to handle the volume and stress constraints.

2 Topology Optimization Formulation

2.1 Optimization Problem and Material Models

In this work, we propose to solve the optimization problem of compliance minimization subject to a volume and a stress constraint. The p-norm function formulation defines the global measure of the stress as the aggregated stress. The optimization problem can be expressed as:

$$\begin{aligned} \underset{\boldsymbol{\alpha}}{\text{Minimize}} \quad & C(\boldsymbol{\alpha}) = \frac{1}{2}\int_{\Omega_s} \boldsymbol{\varepsilon} : \mathcal{C} : \boldsymbol{\varepsilon} \quad d\Omega_s, \\ \text{Subject to} \quad & V(\boldsymbol{\alpha}) = \frac{1}{V_0}\sum_{j=1}^{N} \alpha_j V_j \leq \overline{V}, \\ & \sigma_{PN} = \left(\int_{\Omega_s} \sigma_{vm}^{p_n} d\Omega_s\right)^{\frac{1}{p_n}} \leq \overline{\sigma}_{PN}, \\ & x_j \in [0,1], \quad j \in [1, N_d], \end{aligned} \tag{1}$$

where, $C(\boldsymbol{\alpha})$ is the compliance, $V(\boldsymbol{\alpha})$ is the volume fraction, $\overline{V}$ is the constrained volume fraction, V_0 is the volume of the full solid domain, V_j is the volume of the element j, σ_{PN} is the p-norm stress, $\overline{\sigma}_{PN}$ is the constrained p-norm stress, σ_{vm} is the von Mises stress, and p_n is the penalization factor for the p-norm function.

In order to evaluate the derivatives of the structural compliance and the stress by semi-automatic differentiation using the adjoint method, the physical model should be interpolated with the design variables. For this, the classical SIMP material model is adopted, which is expressed as:

$$E(\alpha_j) = \alpha_j^p E_0, \tag{2}$$

$$\hat{\sigma}_j(\alpha_j) = \alpha_j \sigma_j, \tag{3}$$

where E is the interpolated material property with respect to the design variable α_j, E_0 is Young's modulus of the solid element, p is the penalty exponent factor, and $\hat{\sigma}_j$ is the interpolated stress.

Considering the von Mises stress criteria, the stress for every element in the domain is calculated as:

$$\sigma_{vm} = \sqrt{\hat{\sigma}_j(\alpha_j)\mathbf{V}\hat{\sigma}_j(\alpha_j)}, \tag{4}$$

where,

$$\mathbf{V} = \begin{bmatrix} 1 & -\frac{1}{2} & 0 \\ -\frac{1}{2} & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}. \tag{5}$$

2.2 TOBS Method

The TOBS method, proposed by Sivapuram and Picelli [6], employs binary design variables $\{0,1\}$. This methodology linearizes the objective and constraint functions associated with integer linear programming [7]. The

method uses Taylor's series expansion and truncates it in the linear part to express the approximate structural mean compliance objective and volume constraint functions. Therefore, the approximate integer linear subproblem to be solved is given by:

$$\begin{aligned} \underset{\Delta\boldsymbol{\alpha}^k}{\text{Minimize}} \quad & \left.\frac{\partial f}{\partial \alpha}\right|_{\boldsymbol{\alpha}^k} \Delta\boldsymbol{\alpha}^k, \\ \text{Subject to} \quad & \left.\frac{\partial g_i}{\partial \boldsymbol{\alpha}}\right|_{\boldsymbol{\alpha}^k} \Delta\boldsymbol{\alpha}^k \leq \bar{g}_i - g_i^k \quad i \in [1, N_g], \\ & ||\Delta\boldsymbol{\alpha}^k||_1 \leq \beta N_d, \\ & \Delta\alpha_j \in \{-\alpha_j, 1-\alpha_j\} \quad j \in [1, N_d], \end{aligned} \tag{6}$$

where $f(\alpha)$ is the objective function, bounded by the constraints $g_i(\alpha) \leq \bar{g}_i$. $i \in [1, N_g]$, where N_g and N_d are respectively the numbers of inequality constraints and elements in the vector of design variables. β is the truncation error parameter and $||\Delta\alpha^k||_1$ is the truncation error. The term $g_i(\alpha^k)$ is the value of the constraint g_i in the k_{th} optimization iteration. The ILP solver is used to find the optimal change $\Delta\alpha$ for the integer design variables α. After each iteration, the design variables are updated as $\alpha_{k+1} = \alpha_k + \Delta\alpha_k$.

3 Numerical Implementation

3.1 Smoothing Technique

The TOBS-GT, proposed by Picelli et al. [8], is employed in this work. The optimization is carried out in MATLAB® via the TOBS implementation available at Picelli et al. [9]. The static structural analysis is solved using COMSOL Multiphysics® and the sensitivities are obtained by semi-automatic differentiation. In addition, a smoothing technique based on a level-set distribution from the filtered design variables is used to represent the topology as proposed in Siqueira et al. [10]. Based on Fig. 1, the smoothing contour procedure is to apply a nodal filter to the distribution of design variables $\{0, 1\}$, which becomes a surface similar to a level-set function. The smooth contour can be extracted by setting a value in the plane for the design variables ($\alpha_j = 0.5$). In this work, the MATLAB function `contourf` was used in order to extract the contour points of the smooth contour. After that, the contour points are used to generate CAD files in a .dxf extension to be exported to the FEA software.

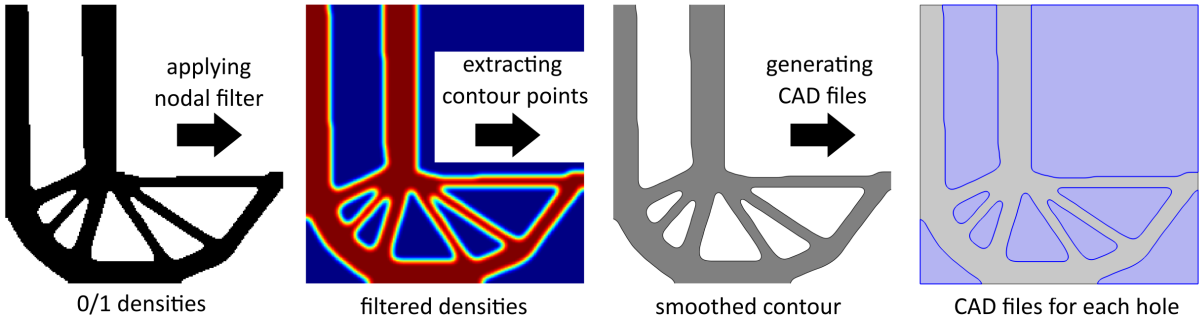


Figure 1. Illustration of the smoothing technique based on the level-set procedure.

3.2 Strategy for the Stress Constraint

The classical strategy proposed by Le et al. [3] is employed, which consists of a normalized global stress measure to approximate the maximum stress. Since the p-norm stress measure σ_{PN} lacks physical meaning, the constraint is enforced on the maximum stress using the p-norm stress. The normalized global stress measure uses information from the previous optimization iteration to scale, i.e. normalize, the global p-norm measure as $c\sigma_{PN}$ in order to provide a better approximation of the maximum stress. The normalized global p-norm constraint at each iteration k is defined as:

$$\sigma_{max} \approx c\sigma_{PN} \leq \overline{\sigma}, \tag{7}$$

where the normalization parameter c is calculated at each optimization iteration $k \geq 1$ as follows:

$$c^k = \eta^k \frac{\sigma_{max}^{k-1}}{\sigma_{PN}^{k-1}} + (1-\eta^k)c^{k-1}, \tag{8}$$

The parameter η controls the variation between c^k and c^{k-1}. In this work, we defined $\eta = 0.5$ and $c = 1$.

4 Numerical Examples

This section presents the results obtained for 2D structural stress-based problems. Two examples were explored: the L-beam and the T-pier. When not specified, the optimization problem is compliance minimization subjected to a volume and a stress constraint, Eq. 1. A filter radius of 5 grid sizes is adopted for densities and sensitivity filtering. Material models are interpolated considering the penalization factor $p = 3$. The constraint relaxation parameter is $\varepsilon = 0.01$, and the truncation parameter is $\beta = 0.02$. The convergence is defined by averaging the changes in the compliance function over 6 consecutive iterations for a tolerance of $\tau = 0.001$.

4.1 L-beam

The design domain of the L-beam is illustrated in Fig. 2. The properties of the structure are assumed as qthe aluminum: Young's modulus of $E = 70$ GPa, Poisson's ratio of $\nu = 0.33$, and density of $\rho = 2700$ kg/m^3. An external distributed load of $F = 25$ N is considered. The optimization grid consists of 200×200 quadrilateral elements. Figure 3 presents the final topology for compliance, stress, and stress-constrained design[1].

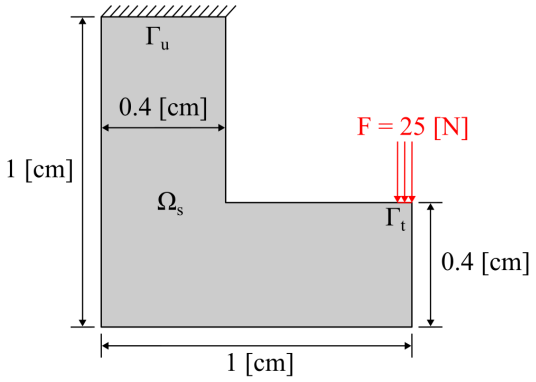


Figure 2. Design domain with dimensions and boundary conditions for the L-beam.

Figure 3 illustrates the trade-off between compliance and the maximum stress. The compliance design case achieves the lowest compliance; however, it results in higher maximum stress acting on the structure. In contrast, applying a stress constraint enables achieving significantly better compliance values compared to the stress design cases, while still satisfying the specified maximum stress limit. Finally, Figure 4 displays the contours of the solutions for both the stress-constrained and compliance design cases. From these contours, it can be observed that only minor local adjustments to the boundaries of the structure are required to satisfy the stress constraint.

[1]Compliance design refers to compliance minimization subjected to a volume constraint. Stress design refers to stress minimization subjected to a volume constraint. Finally, the stress- constrained design is the optimization problem presented in Eq. 1

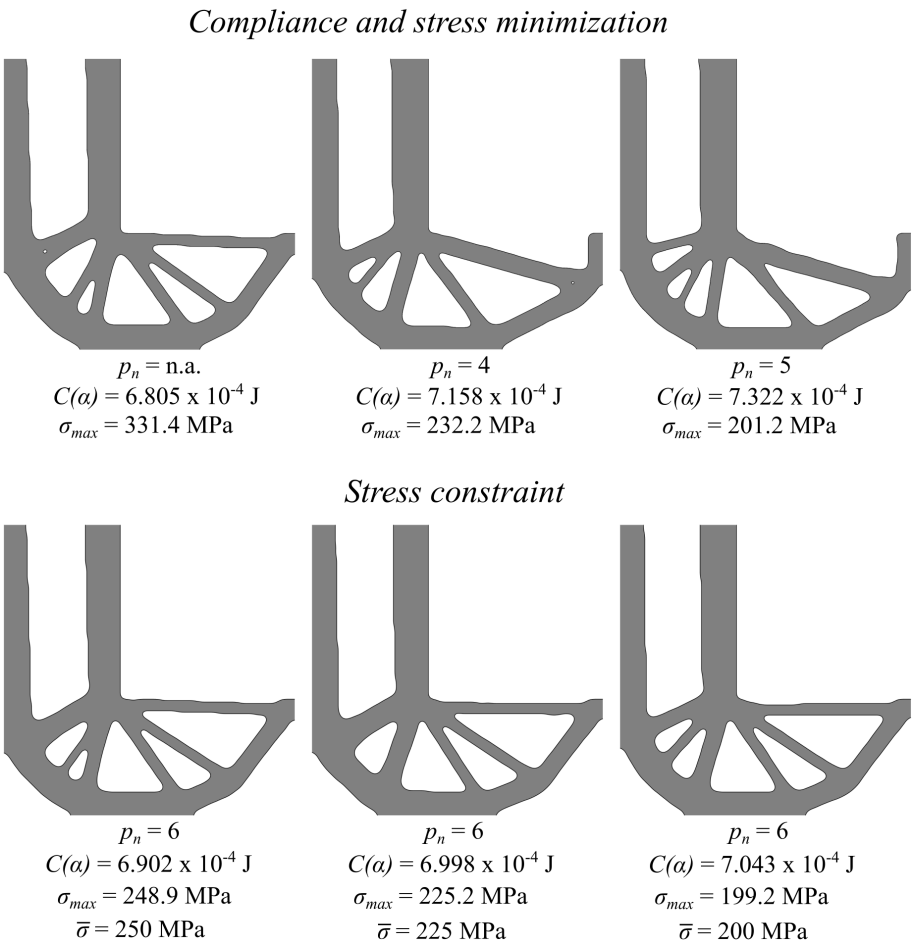


Figure 3. Final topology with final compliance and maximum stress values for compliance design, stress design, and stress-constrained design cases for the L-beam problem.

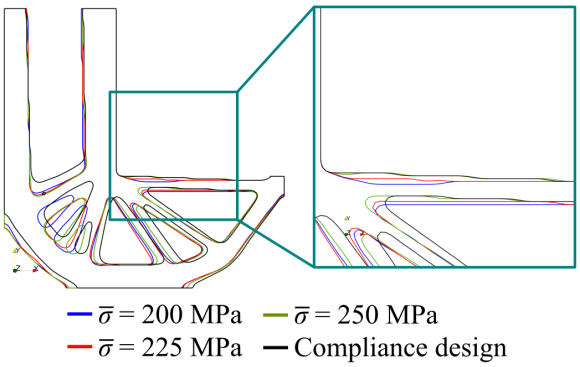


Figure 4. Contour of the final topology for the compliance design and stress-constrained cases for the L-beam problem.

4.2 T-pier

The design domain of the T-pier is illustrated in Fig. 5. The properties of the structure are assumed as the aluminum: Young's modulus of E = 70 GPa, Poisson's ratio of ν = 0.33, and density of ρ = 2700 kg/m^3. An external distributed load of F = 25 N is applied in the pier deck. The optimization grid consists of 240×240 quadrilateral elements. Figure 6 presents the final topology for compliance, stress, and stress-constrained design.

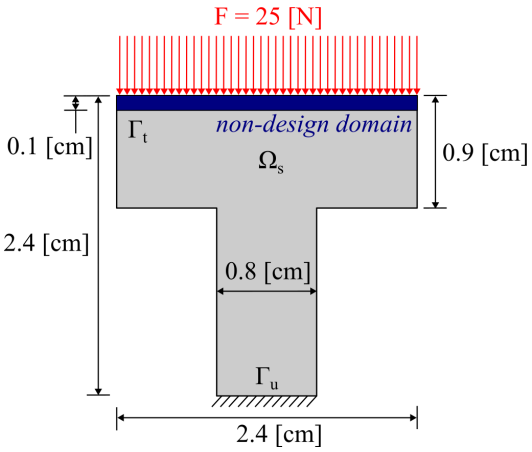


Figure 5. Design domain with dimensions and boundary conditions for the T-pier.

Similar to the previous case, Figure 6 illustrates the trade-off between compliance and maximum stress. The stress-constrained design achieves a balance between compliance and the target stress value. Figure 7 shows the contours of the final topologies, where it can be observed, as expected, that the optimizer tends to eliminate the corner regions in cases where stress is considered. This behavior is not observed in the compliance design case, which results in increased maximum stress in this region of the structure.

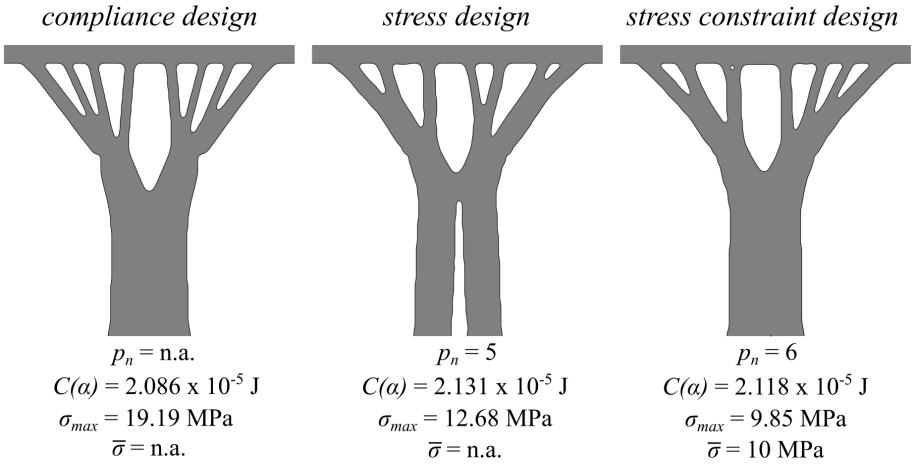


Figure 6. Final topology with final compliance and maximum stress values for compliance design, stress design, and stress-constrained design cases for the T-pier problem.

5 Conclusions

The stress constrained topology optimization problem was solved using the TOBS-GT method. Two 2D static structural examples were explored: the L-beam and the T-pier. The optimization problem was compliance minimization subjected to a volume and a stress constraint. The TOBS-GT method was employed in order to obtain body-fitted meshes and solve the multiple constraints problem by using sequential integer linear programming. The results demonstrate that the stress constraints can be achieved even using a body-fitted mesh approach.

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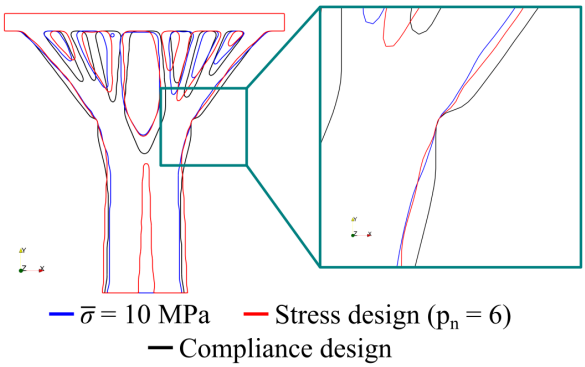


Figure 7. Contour of the final topology for the compliance design, stress design, and stress-constrained cases for the T-pier problem.

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