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Modeling Strategy to Simulate Recycled Concrete Structures in a Physically Nonlinear Analysis

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Abstract. In order to mitigate the environmental impacts caused by the construction industry, efforts have been made to minimize the production of waste and to promote its reuse in different applications, including construction projects. This reuse of construction and demolition waste is not only a sustainable practice but also economically attractive for the industry, combining waste reduction, decreased extraction of raw materials, and lower expenditure on the acquisition of new inputs. Concrete is a material widely used in construction and, consequently, large quantities of it are found among construction and demolition waste. One option for the circular use of concrete waste is its partial substitution for natural coarse aggregates (gravel) in conventional concrete, resulting in what is known as recycled aggregate concrete. Currently, numerous experimental studies are underway to evaluate the potential and applicability of recycled concrete. However, there remains a gap in the literature regarding computational modeling aimed at better understanding the mechanical behavior of this new material. In response to this demand, the current work presents a strategy to simulate recycled concrete structures, accounting for its heterogeneity by explicitly representing the mainly constituent phases, namely, the cementitious matrix of conventional concrete, the natural coarse aggregate, and the recycled concrete aggregate—which includes the recycled cementitious matrix and recycled gravel. The computational modeling will be performed using the finite element method, employing nonlocal damage models to represent material degradation, thereby preventing spurious localization phenomena typically observed in refined meshes. Through this study, it is intended to foster the development of further research dedicated to recycled concrete, so that, by combining experimental and numerical investigations, its behavior and limitations can be thoroughly understood, promoting its widespread use while ensuring structural safety and high performance.

Keywords: Heterogeneity, Nonlocal Damage Models, Nonlinear Analysis, Recycled Concrete

1 Introduction

The heterogeneity of materials, such as concrete, can be represented in numerical simulations under different approaches. Traditionally, two methodologies, as described by Schlangen and van Mier [1], arise. The first strategy called direct representation, consists of a geometric description of the phases in the discrete model, as performed by several authors [2, 3]. The second approach, known as the indirect method, distributes the physical properties of the materials at the internal points of the model [4–6]. In this work, a hybrid strategy will be adopted [7]. Such a strategy combines aspects of the direct and indirect methods to represent the different phases of concrete.

A significant application of simulation using heterogeneous models is the representation of concrete structures composed of recycled concrete aggregate. A comprehensive simulation of the media encompasses four distinct phases of the material: the cementitious matrix, natural aggregate, the cementitious matrix of recycled concrete, and the natural aggregate found in the recycled material. The latter two phases together compose the recycled concrete aggregate.

Several authors have investigated the properties of concrete produced with recycled concrete aggregate (RCA) as a replacement for natural coarse aggregate (NA)[8–11]. Additionally, some studies focus on the use of recycled concrete as fine aggregate (less than 4 mm) such as the research conducted by Nedeljković et al. [12]. Some authors have also explored methods to enhance the physical properties of concrete containing recycled aggregates [13–15]. However, it is important to note that, despite the extensive scientific literature in the field of materials, studies involving numerical simulations — as presented by Xin Tan [16], Ying et al. [17] — are limited. Furthermore, even these studies tend to be restricted to simplified simulations and do not apply to conventional concrete structures.

In the numerical simulation of concrete heterogeneity, various approaches can be employed in the constitutive relationships of materials. One possibility is modeling based on damage mechanics, in which the concrete fracturing process is phenomenologically represented by the degradation of its elastic properties. Several studies have explored models based on damage mechanics, notably the work of Mazars [18], Lemaitre and Chaboche [19] and de Vree et al. [20] which present different strain criteria, referred to as equivalent strain, to compute the damage evolution.

To avoid issues related to strain localization, it is essential to couple damage models with regularization strategies. One effective approach is the nonlocal method, which can be based on integral formulations [21, 22] or by gradient-based techniques [23, 24].

In light of the above, this work presents a study of numerical simulations of concrete structures using recycled concrete aggregate as a coarse aggregate. The heterogeneous description of the material will be carried out through the hybrid methodology. Additionally, a nonlocal damage model will be adopted for the constitutive description of the material. Finally, numerical simulations will be performed to analyze different percentages of RCA added to the mixture, allowing for an evaluation of the effects on the model results.

2 Theoretical Foundation

2.1 Standard Isotropic Damage Model

A standard formulation of an isotropic damage model is described by the constitutive relationship given by:

$$\sigma_{ij} = (1 - D)E^0_{ijkl}\varepsilon_{kl}\,, \tag{1}$$

where σ_{ij} are the components of the stress tensor, ε_{kl} are the components of the strain tensor, and D is the damage variable, which can assume values between 0 and 1 to indicate whether the material is intact ($D = 0$) or completely degraded ($D = 1$).

The loading function of the damage model relates the evolution of stresses and strains with strength criteria and its historical variables. In general, the strain-based loading function is given by $F(\varepsilon, \kappa(D)) = \widetilde{\varepsilon} - \kappa(D)$. $\widetilde{\varepsilon}$ is the equivalent strain and $\kappa(D)$ is a historical variable of equivalent strain which is κ_0 if $D = 0$ or $\max\left[\widetilde{\varepsilon}, \kappa(D)\right]$ if $D > 0$. κ_0 is the initial limit value of elastic strain. The loading function and the rate of the historical variable ($\dot{\kappa}$) must satisfy the Kuhn-Tucker conditions for loading and unloading, which are $F \leq 0$, $\dot{\kappa} \geq 0$ and $F\dot{\kappa} = 0$.

The model can be developed according to the formulation presented by Carol et al. [25]. Thus, the differentiation of the equation eq. (1) leads to eq. (2), where the upper point indicates a variation in pseudo-time,

$$\dot{\sigma}_{ij} = E^t_{ijkl}\dot{\varepsilon}_{kl}, \text{ where: } E^t_{ijkl} = E_{ijkl} - \frac{\partial D}{\partial \widetilde{\varepsilon}}\frac{\partial \widetilde{\varepsilon}}{\partial \varepsilon_{ij}}E^0_{klpq}\varepsilon_{pq}. \tag{2}$$

To compute the damage evolution, an exponential law, such as the one presented by de Borst and Guitiérrez [23], is adopted, which is given by:

$$D(\tilde{\varepsilon}) = 1 - \frac{\kappa_0}{\tilde{\varepsilon}}\left[1 - \alpha + \alpha e^{-\beta(\tilde{\varepsilon} - \kappa_0)}\right]\,, \tag{3}$$

where $\tilde{\varepsilon}$ is the equivalent strain, κ_0 is the value of the equivalent strain value correspondent to the limit of the elastic domain, α is a parameter controlling the rate of damage accumulation, β is the intensity of the damage evolution.

In this work, the equivalent strain proposed by de Vree et al. [20] was employed, which is defined by:

$$\tilde{\varepsilon} = \frac{k_r - 1}{2k_r\left(1 - \nu\right)}I^{\varepsilon}_1 + \frac{1}{2k_r}\sqrt{\frac{\left(k_r - 1\right)^2}{\left(1 - 2\nu\right)^2}\left(I^{\varepsilon}_1\right)^2 + \frac{12k_r}{\left(1 + \nu\right)^2}J^{\varepsilon}_2}\,, \tag{4}$$

where $k_r = f_c/f_t$, f_c is the compressive strength of the material and f_t is the tensile strength of the material, I^{ε}_1 is the first invariant of the strain tensor, and J^{ε}_2 is the second invariant of the deviatoric strain tensor.

2.2 Nonlocal Model applied to heterogeneous model

Numerically induced strain localization is a recurring problem in damage models based on elastic degradation. This issue can be effectively addressed using a nonlocal strategy. Among many other authors, Jirásek [22] presents the basis of the integral formulation. In this approach, a given variable at a point is replaced by its nonlocal value, which is obtained through a weighted average of values from neighboring points, as presented in the following equation:

$$\bar{f}(\boldsymbol{r}) = \frac{1}{V_r}\int_V w(||\boldsymbol{r}-\boldsymbol{s}||)f(\boldsymbol{s})dV\,,\ \text{where: } V_r = \int_V w(||\boldsymbol{r}-\boldsymbol{s}||)dV\,, \tag{5}$$

where: $w(||\boldsymbol{r}-\boldsymbol{s}||)$ is the weight function that depends on the Euclidean distance $||\boldsymbol{r}-\boldsymbol{s}||$ between the origin point $\boldsymbol{r}$, and the target point $\boldsymbol{s}$; the local value associated with a point $\boldsymbol{s}$ is given by $f(\boldsymbol{s})$, while $\bar{f}(\boldsymbol{r})$ is the nonlocal quantity associated with the point $\boldsymbol{r}$; the volume of the influence domain of the point $\boldsymbol{r}$ is V_r. In this work, the Gaussian weight function was adopted, given by:

$$w(||\boldsymbol{r}-\boldsymbol{s}||) = \exp\left[-\left(\frac{k||\boldsymbol{r}-\boldsymbol{s}||}{\ell}\right)^2\right]\,, \tag{6}$$

where k is a parameter that defines the shape of the curve and ℓ is the nonlocal radius that defines the size of the domain V_r.

Considering the heterogeneity, nonlocal models with phase distinction must be used, as presented by Fish et al. [26], Magri et al. [27] and Lenz and Mahnken [28]. Therefore, in this work, only points with the same phase were considered in the calculation of the nonlocal variable. Therefore, the weight function was redefined to $w = 0$ if $\mathbf{s} \notin V_{r\eta}$. Where $V_{r\eta}$ are within the domain V, with phase η, which is the same phase as the analysis point $\boldsymbol{r}$. Additionally, when performing the non-local domain of aggregates, only points within the aggregate itself are considered for the composition of the nonlocal domain.

2.3 Heterogeneity Representation

To represent heterogeneity, a so-called hybrid methodology is used. This strategy distributes the properties of the materials at the integration points, similar to indirect methods. However, the integration points will spatially represent the aggregates as the direct method. In this way, there is a representation of the aggregate, but no re-meshing process is necessary, like in the direct methods.

In the case of recycled concrete, the strategy for distributing heterogeneity is illustrated in Fig. 1. In the first step, the properties of the cementitious matrix (indicated by the gray area) are assigned to all integration points within the domain. In the next step, the points corresponding to the natural gravel of concrete (shown in orange areas) are established within this matrix. Finally, the recycled aggregate (represented by the pink areas, which represent the recycled matrix, and purple areas, which indicate the recycled gravel) is distributed throughout the recycled concrete cementitious matrix. At the end of this procedure, a discrete model for concrete composed of recycled concrete aggregates is generated.

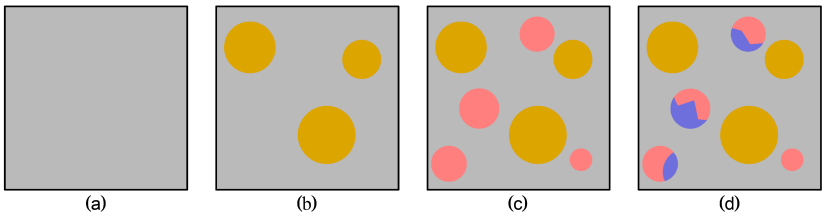


Figure 1. Process for defining the integration points in the hybrid strategy for concrete with RCA. (a) Definition of the points as cementitious matrix. (b) Generation of NA. (c) Generation of the cementitious matrix of the RCA. (d) Generation of the aggregate of the RCA.

A cumulative distribution function for particle size must be adopted in numerical models of concrete heterogeneity. The most traditional in the literature, and used in this work, is proposed by Fuller and Thompson [29], given by:

$$F^{\texttt{Fuller}}(d) = \left(\frac{d}{d_{Max}}\right)^{n^{\texttt{Fuller}}} \tag{7}$$

where $n^{\texttt{Fuller}}$ is the Fuller number that varies from 0.45 and 0.7 according to Wriggers and Moftah [30] and modifies the accumulated value in the particle size distribution curve, d_{Max} is the diameter maximum of the aggregate and d is the diameter of the aggregate considered in the function.

3 Numerical Simulation

A numerical model of an L-shaped panel, as presented by Winkler et al. [31], was developed to analyze the numerical response of the heterogeneous structure resulting from the partial replacement of natural aggregates (NA) with recycled concrete aggregates (RCA). The analysis employed a damage model based on the equivalent strain proposed by de Vree et al. [20] and used an exponential damage law.

The material parameters defined by Winkler et al. [31] were based on the macroscopic properties of concrete. These parameters are: modulus of elasticity $E_0 = 24469$ N/mm^2, Poisson's ratio $\nu = 0.18$, the tensile strength of the material $f_t = 2.7$ MPa, fracture energy $G_f = 0.065$ N/mm.

Details regarding the mesoscopic parameters are limited. As noted by Sun [32], obtaining the characteristics of the mesostructure is challenging, as many studies focus solely on the parameters of concrete. Consequently, the homogeneous material was parameterized using the exponential damage law, and the properties of the phases were adjusted based on these homogeneous values. The elastic moduli of the cement matrix and the aggregate were calculated using eq. (8), as referenced by Eckardt and Könke [33].

$$\frac{1}{E^C} = \frac{\phi^M}{E^M} + \frac{\phi^A}{E^A}, \tag{8}$$

where E^C is the modulus of elasticity of homogeneous concrete, E^M is the modulus of elasticity of the cementitious matrix, E^A is the modulus of elasticity of the aggregate, ϕ^M is the volume fraction of cementitious matrix in concrete, and ϕ^A is the volume fraction of aggregate in concrete.

The values of the exponential damage law for the cementitious matrix are adjusted to align with those of conventional concrete. Additionally, the parameters for the aggregate are modified to reflect a linear elastic response, which promotes the propagation of damage within the mortar. This methodology is widely used by different authors and is discussed in the literature review conducted by Thilakarathna et al. [34]. For the recycled concrete aggregate, the parameters will match those of the natural aggregate. However, the recycled cementitious matrix will be assigned lower values compared to the conventional cementitious matrix. This work employed a percentage of 70% cementitious matrix to 30% aggregates, both in the heterogeneous composition of the structure and the RCA. The damage law values employed for each material are presented in Table 1.

Table 1. Properties of the Damage Law for each material

Material	E	α	β	κ
Concrete	24469	0.999	950	$1.35 \cdot 10^{-4}$
Mortar	23000	0.999	650	$1.12 \cdot 10^{-4}$
Natural Aggregate	28750	0.999	20000	$3.25 \cdot 10^{-4}$
Mortar of Recycle Concrete Aggregate (RCA)	23000	0.999	747.5	$9.60 \cdot 10^{-5}$
Natural Aggregate of Recycle Concrete Aggregate (RCA)	28750	0.999	20000	$3.25 \cdot 10^{-4}$

The geometric data from the numerical test of the L-shaped panel, along with the adopted mesh and the representation of heterogeneity through integration points for the structure composed solely of natural aggregates (NA), are presented in Fig. 2. In Fig. 2c, black integration points represent the cement matrix, red points denote the NA, and gray points indicate the concrete. For computational efficiency, the heterogeneous medium was defined only in the region of interest, where crack propagation is expected (the area of material degradation). In Fig. 3 the heterogeneous region are presented in Fig. 3 by varying the percentages of NA and RCA composition in the structure, wherein Fig. 3a there is a heterogeneous region in for simulation with 100%NA, in Fig. 3b there is a heterogeneous region for simulation with 70%NA and 30%RCA, in Fig. 3c there is a heterogeneous region for simulation with 50%NA and 50%RCA, in Fig. 3d there is a heterogeneous region for simulation with 30%NA and 70%RCA. In all figures, the green integration points represent the cementitious matrix of the RCA, while the blue integration points represent the gravel of the RCA.

The mesh was generated with linear triangle elements. The aggregates ranged in size from 4 mm to 8 mm, according to the cumulative probability density function shown in eq. (7) with $n_{fuller} = 0.5$. Displacement increments of $4 \cdot 10^{-3}$ at the point of force application were considered, using displacement control. The load-displacement curves at the point of force application for each simulation are presented in Fig. 4. In all curves, the nonlocal model with phase distinction, based on the Gaussian weight function, was used. For the cementitious matrices, a nonlocal radius of 28 mm and k_c of 1 were considered. However, the cementitious matrix of the RCA has a nonlocal group within the RCA itself. The nonlocal radius for the natural aggregates was $1 \cdot 10^{-4}$ and k_c of 1. The experimental result obtained by Winkler et al. [31] is also illustrated with the responses of the numerical simulations.

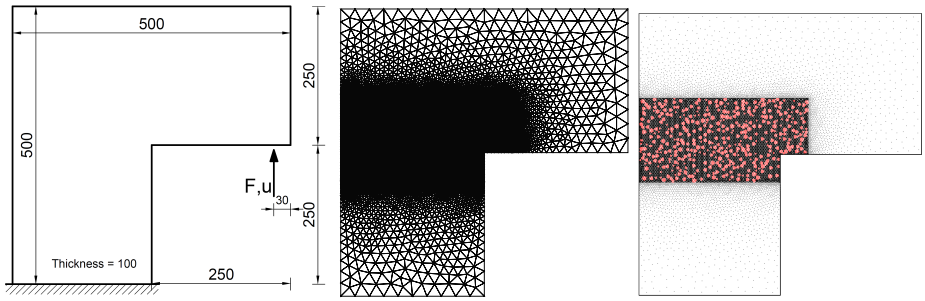


Figure 2. L-shaped panel. (a) Problem settings with measurements in millimeters.(b) T3 mesh ($h = 25$ mm in the unrefined region and $h = 2.5$ mm in the refined region). (c) Representation of heterogeneity in the model with only NA.

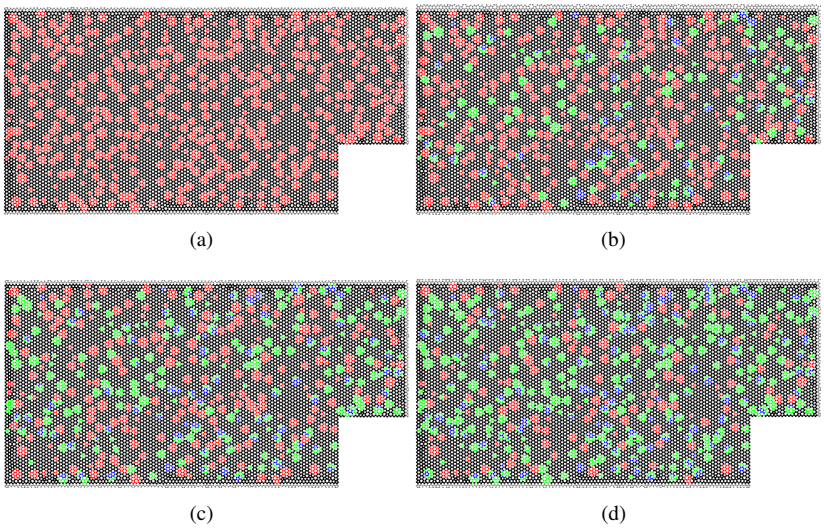


Figure 3. Heterogeneous Region for each simulation composition.

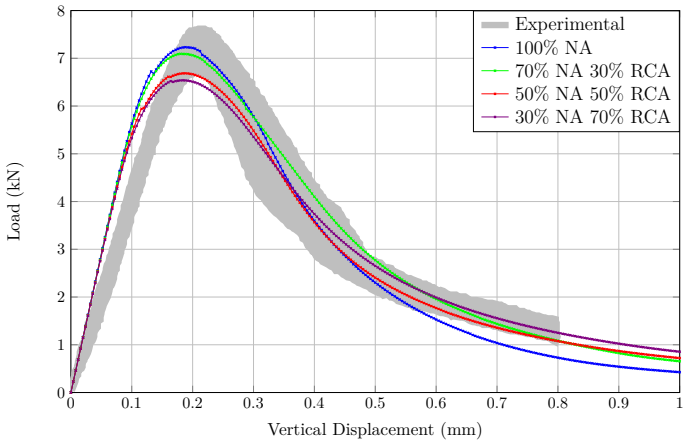


Figure 4. Load-displacement curves for heterogeneous simulations for each simulation composition.

The numerical simulations indicate that as the proportion of RCA replacing NA increases, the peak load decreases while the displacements of the structure increase. This behavior is consistent with the understanding that the strength parameters of RCA are significantly lower than those of natural aggregates. However, it is evident that, despite the reduction in peak load, RCA can be effectively employed in place of NA without a significant variation in peak performance, depending on the percentage of RCA. For instance, in the case of a composition of 30% RCA and 70% NA, the structure presents acceptable performance.

4 Conclusions

In conclusion, this work presents an initial numerical simulation of a concrete structure with partial replacement of natural coarse aggregates (NA) by recycled concrete coarse aggregates (RCA). The study aimed to stimulate academic research in numerical modeling of concrete structures with RCA, encouraging the investigation of experimental tests beyond studies focused only on materials characterization or tension and compression simulations. The work indicates that, when replacing NA with RCA, there is a decrease in peak load, an increase in displacements, and a larger damaged area due to the presence of more brittle aggregates. In future studies, several topics warrant exploration: the use of sampling methods like Monte Carlo simulations; a more comprehensive investigation into the parameters that influence the damage laws of the materials comprising the heterogeneity; and the potential integration of interfaces between materials utilizing the hybrid method.

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