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Nonlinear Structural Dynamics of Folding Wingtip: A Flexible Multibody Approach

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Abstract. In commercial aviation, even incremental improvements in aircraft performance can yield substantial environmental benefits. Increasing wing span is a proven strategy to enhance aerodynamic efficiency and reduce emissions, but practical limitations arise as many airports cannot accommodate larger-wing aircraft. Folding wingtips have emerged as an innovative solution to reconcile efficiency gains with operational flexibility. However, analyzing such adaptive structures requires addressing geometric nonlinearities induced by large displacements, which traditional linear models fail to capture accurately. This study proposes a computationally efficient framework for static and dynamic analysis of highly flexible structures, including folding wingtips. The method combines a linear structural model based on the Rayleigh-Ritz method with flexible multibody dynamics, where geometric nonlinearities are modeled through rigid connections between multiple flexible bodies. This hybrid approach avoids the high computational costs of fully nonlinear structural models while retaining fidelity in capturing nonlinear dynamic behavior. Results demonstrate that the framework accurately predicts the structural dynamics of highly flexible structures with folding parts, validating its capability to handle complex, morphing geometries. The proposed methodology not only supports the design of next-generation adaptive wings but also offers versatility for broader applications in aerospace engineering, where flexibility and large deformations play a critical role.

Keywords: Flexible multibody dynamics, Rayleigh-Ritz method, Geometric nonlinearities, Folding wingtips, Highly flexible structures

1 Introduction

The continuous pursuit of greater efficiency in commercial aviation has driven the development of aircraft with higher aspect ratio wings. This design approach aims to reduce induced drag, which directly translates into lower fuel consumption and, consequently, a decrease in pollutant emissions. However, the increased wingspan imposes significant operational challenges, as many existing airport infrastructures are not compatible with large-span aircraft, restricting their operations. As a solution to this limitation, Folding Wing-Tips (FWTs) have emerged as a promising technology. They allow for the retraction of the wingtips on the ground, ensuring compatibility with gates and taxiways, and offer potential for in-flight applications, such as gust load alleviation systems and wing configuration optimization for different flight phases [1].

The main challenge in analyzing such aircraft lies in the accurate modeling of geometric nonlinearities arising from large displacements and rotations, characteristic of highly flexible structures. Traditional linear models, such as those analyzed by Guimarães Neto et al. [2], fail to accurately capture these effects as wing flexibility increases. The articulation of the FWT introduces additional nonlinearities into the system's dynamics, which are crucial for predicting aeroelastic behavior. Although fully nonlinear Finite Element Models (FEM) or nonlinear beam models [3–5] can represent this complexity, their high computational cost often makes them impractical for design and optimization phases.

The academic literature has explored various methodologies to address this challenge. Studies such as those by Castrichini et al. [6] have used multibody dynamics with lumped-mass models. More recent approaches, like that of Álvaro Cea Esteban [7], have demonstrated the effectiveness of combining linear finite element models with multibody dynamics to capture nonlinear effects. Other researchers, such as Kruger [8] and Zhao and Ren [9], have focused on simulating flexible behavior by coupling rigid beam segments with equivalent joints and springs. More recently, research has concentrated on experimental validation and the development of control strategies, both passive, based on joint stiffness [10], and active, which use actuators for load alleviation and maneuvering

[11, 12].

To overcome the dichotomy between fidelity and computational cost, this work proposes a linear structural model coupled with a multibody called Flexible Multibody Rayleigh-Ritz (FMBRR), initially presented in da Luz et al. [13]. This methodology combines a linear structural model, based on the Rayleigh-Ritz method to describe the local deformation of each body, with flexible multibody dynamics. The global geometric nonlinearities, resulting from large displacements, are modeled through constraint equations that define the connections between the multiple flexible bodies that make up the complete structure. This hybrid approach offers a robust and lower-cost computational alternative compared to fully nonlinear methods, while maintaining fidelity in capturing nonlinear dynamic behavior.

The objective of this paper is, therefore, to present and validate the FMBRR framework for the structural dynamics analysis of highly flexible wings equipped with folding tips. The methodology is detailed, focusing on the constraint equations for revolute joints and the evaluation of polynomial shape function bases for the Rayleigh-Ritz method. The results are validated against reference models, demonstrating the accuracy and efficiency of the method. The structure of the paper is as follows: Section 2 details the FMBRR methodology, covering the fundamentals of flexible multibody dynamics and the structural representation via the Rayleigh-Ritz method. Section 3 presents the model's validation with a case of a cantilever beam with a folding part. Finally, Section 4 offers the conclusions and discusses future work.

2 Methodology: The FMBRR Framework

Multibody dynamics modeling is based on Shabana [14, 15], which uses the floating frame approach. In the floating frame of reference formulations, two sets of coordinates are used to describe the configuration of flexible bodies, one set describes the location and orientation of a selected body coordinate system, while the second set describes the deformation of the body with respect to its coordinate system [15].

The motion of a flexible body is represented by the translation and rotation of the body frame in relation to the inertial frame. The body frame, denoted as j and defined by x_j, y_j and z_j, is rigidly attached to the body with its origin not necessarily coincident with the center of mass of the body. The inertial frame, denoted as i and defined by x_i, y_i and z_i, is fixed to an initial position. To describe the vectors from the inertial frame to the body frame of the body j, the rotation matrix $\overline{A}_{j/i}$ is used, considering the conventional 3-2-1 rotation sequence [14].

Then, by calculating the kinetic energy of a flexible body with both flexible and rigid-body degrees of freedom one finds the equations of motion of a flexible body given by the matrix equation [14, 16],

$$\overline{M}_j \dot{\mathbf{x}}_j = \mathbf{Q}_j, \tag{1}$$

where $\overline{M}_j$ is the generalized mass matrix, $\dot{\mathbf{x}}_j = [\dot{\mathbf{V}}_{O,j}^T,\ \dot{\boldsymbol{\omega}}_j^T, \ddot{\mathbf{q}}_j^T]^T$ is the vector of translational, rotational, and flexible accelerations, and Q_j is the vector of generalized forces, including quadratic velocity terms, external forces, and internal structural forces [13].

To describe the dynamics of a multibody system, the two flexible bodies j and k, are considered. Then, as presented in Shabana [14], the concatenation of the dynamics of the two bodies takes place

$$\underbrace{\begin{bmatrix} \overline{M}_j & \overline{0} \\ \overline{0} & \overline{M}_k \end{bmatrix}}_{\overline{M}_t} \cdot \underbrace{\begin{bmatrix} \ddot{\mathbf{x}}_j \\ \ddot{\mathbf{x}}_k \end{bmatrix}}_{\ddot{\mathbf{x}}_t} = \underbrace{\begin{bmatrix} \mathbf{Q}_{i,j} + \mathbf{Q}_{e,j} \\ \mathbf{Q}_{i,k} + \mathbf{Q}_{e,k} \end{bmatrix}}_{\mathbf{Q}_t}, \tag{2}$$

and the union of the systems occurs through a system augmented by Lagrange multipliers $\boldsymbol{\lambda}$ that contains the equation of restriction kinematics, as

$$\begin{bmatrix} \overline{M}_t & \overline{C}_x^T \\ \overline{C}_x & \overline{0} \end{bmatrix} \cdot \begin{bmatrix} \ddot{\mathbf{x}}_t \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{Q}_t \\ \mathbf{Q}_x \end{bmatrix}, \tag{3}$$

where $\overline{C}_x$ denotes the constraint matrix and $\mathbf{Q}_x$ represents the vector of generalized constraint forces, derived from the constraint equations. In the context of a holonomic system, these are specified by [14]:

$$\mathbf{C}(\mathbf{x}_t, t) = \mathbf{0}, \tag{4}$$

where $\mathbf{C}(\mathbf{x}_t, t)$ constitutes the vector of linearly independent constraint functions.

Simulating multibody systems involves solving complex Differential-Algebraic Equations (DAEs), and a common method to simplify them can lead to a drift problem where the solution becomes physically inaccurate. To correct this, this study employs the Post-Stabilization (PStab) method [17, 18]. Unlike traditional approaches that can be difficult to tune, PStab avoids defining additional parameters by applying corrective adjustments directly to the velocities after each integration step. This process guides the numerical solution back toward its physical constraints, ensuring the results remain physically relevant.

2.1 Rayleigh-Ritz method for Local Flexibility

The local flexibility of each structural component is modeled using the Rayleigh-Ritz method. This approach represents the displacement of any point on a body as a weighted sum of predefined shape functions. These functions are then used to formulate the beam's total strain energy from bending and torsion, which directly yields the system's stiffness matrix ($\overline{K}$). A corresponding damping matrix ($\overline{D}$) is also defined, established as a proportional relationship with the mass and stiffness matrices.

The shape functions themselves are derived from an orthogonal basis of polynomial functions tailored to meet specific boundary conditions. For bending, Two Boundary Condition Polynomial Shape Functions (TPSF) are used to satisfy the clamped-beam conditions of zero displacement resulting in:

$$\mathbf{N}_{z,TPSF} = \left[\frac{y^2}{L^2} \quad -\frac{6\left(\frac{5\,L\,y^2}{6} - y^3\right)}{L^3} \quad \frac{28\left(\frac{15\,L^2\,y^2}{28} - \frac{3\,L\,y^3}{2} + y^4\right)}{L^4} \quad -\frac{120\left(\frac{7\,L^3\,y^2}{24} - \frac{7\,L^2\,y^3}{5} + \frac{21\,L\,y^4}{10} - y^5\right)}{L^5} \right], \tag{5}$$

for the first four shape functions, as presented in da Luz et al. [13]. For torsion, a similar approach was developed here, and Essential Boundary Condition Polynomial Shape Functions (EPSF) are utilized to satisfy the clamped condition for twist, resulting in

$$\mathbf{N}_{\theta,EPSF} = \left[\frac{y}{L} \quad -\frac{4\left(\frac{3\,L\,y}{4} - y^2\right)}{L^2} \quad \frac{15\left(\frac{2\,L^2\,y}{5} - \frac{4\,L\,y^2}{3} + y^3\right)}{L^3} \quad -\frac{56\left(\frac{5\,L^3\,y}{28} - \frac{15\,L^2\,y^2}{14} + \frac{15\,L\,y^3}{8} - y^4\right)}{L^4} \right], \tag{6}$$

for the first four shape functions.

2.2 Connections modeling

To model the connections between bodies in a flexible multibody system, the constraints are handled by treating translational and rotational motion separately.

Translational constraints ensure that two flexible bodies remain connected at a specific point. This is achieved by formulating a kinematic equation that relates the position vectors of both bodies relative to a global inertial frame and the connection point. This expression can then be manipulated to derive the constraint matrix ($\overline{C}_x$) and the vector of generalized constraint forces ($\mathbf{Q}_x$) needed for the system's equations of motion [13, 14].

Rotational constraints are analyzed by defining a local reference frame at the connection point [14]. For a revolute joint, which allows rotation about a single axis, the model uses a set of unit vectors to define the joint frame: one vector for the axis of rotation and two vectors for the constrained axes.

The restriction is enforced by creating equations that ensure there is no relative rotation about the constrained axes. These equations are then differentiated twice with respect to time to obtain the final kinematic constraints for the joint, which can be incorporated into the overall dynamic system.

3 Numerical Results

3.1 Highly flexible cantilever beam

The FMBRR model was verified for bending in the work of da Luz et al. [13] using a cantilever beam with a vertical force at the free end. Then, to verify the model for torsion, the same beam presented in da Luz et al. [13] was used with a torsional moment M_y at the free end, disregarding damping and gravity, as illustrated in Fig. 1. The properties that define the beam used in both problems are found in Table 1.

In the FMBRR model, the beam will be discretized into interconnected segments, called bodies, allowing the capture of nonlinear geometric effects through constraint equations.

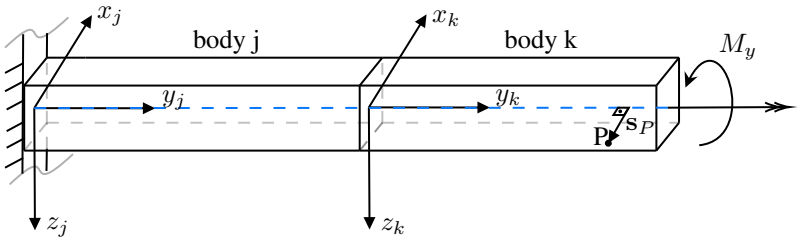


Figure 1. Clamped beam defined by two bodies j and k with a M_y moment at the free end.

Table 1. **Beam Properties.**

Properties	Symbol	Value
Length	L	1.0 m
Torsional Stiffness	GJ	50.0 Nm^2
Flat Bending Stiffness	EI_{xx}	50.0 Nm^2
In-plane Bending Stiffness	EI_{zz}	1000.0 Nm^2
Mass per length	m_L	0.20 $\frac{\text{kg}}{\text{m}}$
Flat Bending Inertia per length	I_{xx}	$1 \cdot 10^{-6}$ kg$\frac{\text{m}^2}{\text{m}}$
Rotational Inertia per length	I_{yy}	$1 \cdot 10^{-4}$ kg$\frac{\text{m}^2}{\text{m}}$
In-plane Bending Inertia per length	I_{zz}	$1 \cdot 10^{-4}$ kg$\frac{\text{m}^2}{\text{m}}$

The results presented will be compared with the results from the commercial software MATLAB® Simscape Multibody and the strain-based geometrically nonlinear structural dynamics code called Aeroflex, developed by Ribeiro et al. [19] and available at Ribeiro [20]. In Simscape, 4 rigidly connected bodies were used to describe the beam, and in Aeroflex, 10 elements were used to describe the beam. Both methods used the *ode15s* function provided by MATLAB® to solve the system. Additionally, in the static results, only Aeroflex was used as a reference for comparison. The global displacements of point P, defined by the unit vector $\mathbf{s}_P = [1,\ 0,\ 0]^T$ at the end of the last body, i.e., at the end of the beam, were used as comparison values.

Static results

To capture the model's behavior, the number of shape functions was varied from 2 to 5, and the number of bodies from 2 to 10. However, since the method is limited to small local displacement conditions on each body, the force for the static calculation varies for each number of bodies defining the total beam to maintain the validity of the small local displacement condition. Thus, the following was defined:$M_z = [5, 10, 15, 20, 25, 30, 35, 40, 45]$, where the columns correspond to each case, with the first term for a number of bodies equal to 2 and the last term for 10 bodies. The force increases with the number of bodies to enhance the global deflection while preserving the validity of the small local displacement condition. Cases involving only one body were not considered because it is a model without considerations of geometric nonlinearities, meaning the results are linear from the perspective of structural dynamics.

Fig. 2 presents a comparative analysis of different function bases against reference standards for beam tip deflections in the Z and X axes. The evaluation employed 10 bodies to describe the beam and 5 shape functions. The EPSF base exhibited results closely aligning with the reference standards. Moreover, it is noted that FMBRR approximates the curve in the Z-axis better than the curve in the X-axis, because the displacement in the X-axis is an exclusive effect of the multibody model, meaning it only arises when we have more than one body describing the beam. Additionally, Fig. 3 shows the local rotation for each body, demonstrating that, across both function bases, the local rotation consistently remained near 5 degrees, aligning with the acceptable criteria for the assumption of small local displacements.

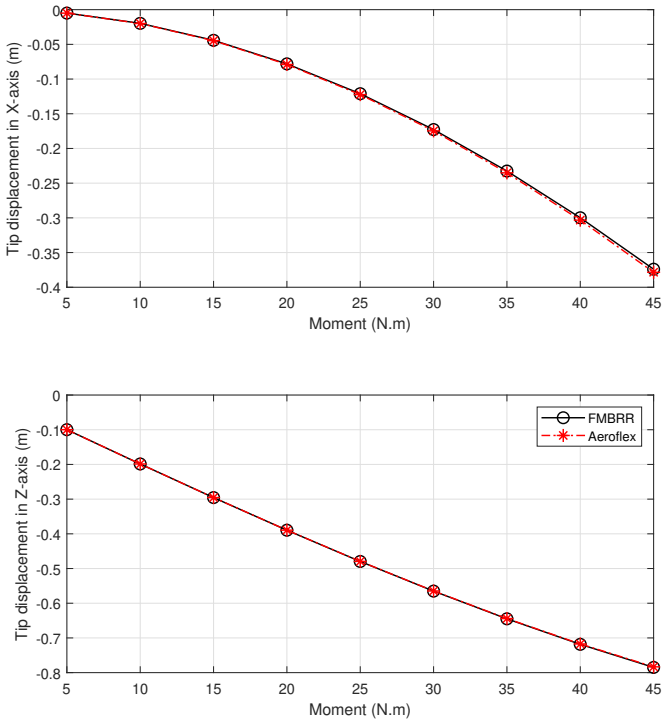


Figure 2. Comparison of global beam tip deflections in the Z-axis and X-axis as a function of the applied moment.

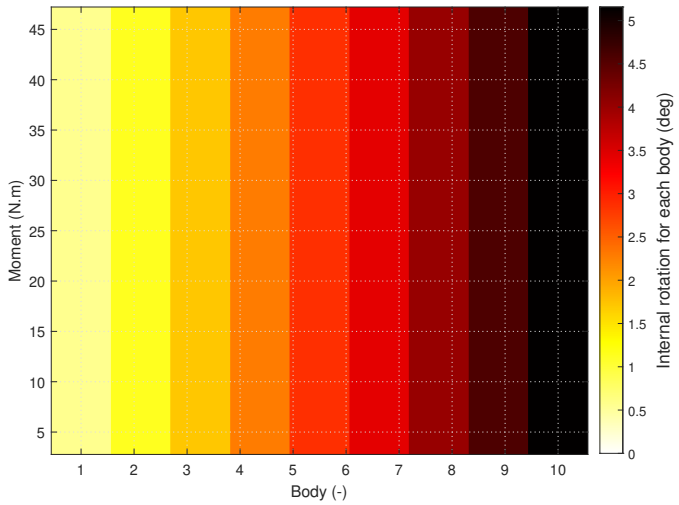


Figure 3. Local rotation for each body as function of the applied moment

Dynamic results

To analyze the dynamic behavior, it was decided to use 4 cases:

- Case 1: $M_y = 5\sin(10t)$;
- Case 2: $M_y = 5\sin(20t)$;
- Case 3: $M_y = 15\sin(10t)$;
- Case 4: $M_y = 15\sin(20t)$.

In the first two cases, we have an excitation with a maximum amplitude of 5 Nm which, based on the static result, provides a maximum global deflection in the Z-axis close to 10% and for the last two cases, a maximum amplitude of 15 Nm which provides a maximum global deflection in the Z-axis close to 30%.

Analyzing the static results, it can be concluded that 4 bodies and 5 shape functions are sufficient to describe the most extreme cases.

In Table 2, the NRMSD values as percentages for both reference models and both displacement axes are presented. Upon analysis, it becomes apparent that the observed differences were more pronounced in the X-axis displacement, as previously noted in the static result discussion. Furthermore, it is clear that FMBRR closely approximates the reference results.

Table 2. NRMSD values in percentage for the global deflection of the beam tip in the Z and X axes.

Case	Aeroflex		Simscape	
	Z-axis	Y-axis	Z-axis	Y-axis
Case 1	4.10	0.36	4.09	0.34
Case 2	4.51	0.71	4.46	0.67
Case 3	4.03	0.36	4.02	0.35
Case 4	4.45	0.71	4.40	0.69

Fig. 4 and Fig. 5 show the global displacements at the beam tip for FMBRR, Aeroflex, and Simscape. Upon examination of the results, it is evident that FMBRR consistently approximates the reference results throughout the entire time interval under study. Furthermore, in Table 3, it can be observed that the FMBRR, using Model 3, has the lowest computational cost compared to the references.

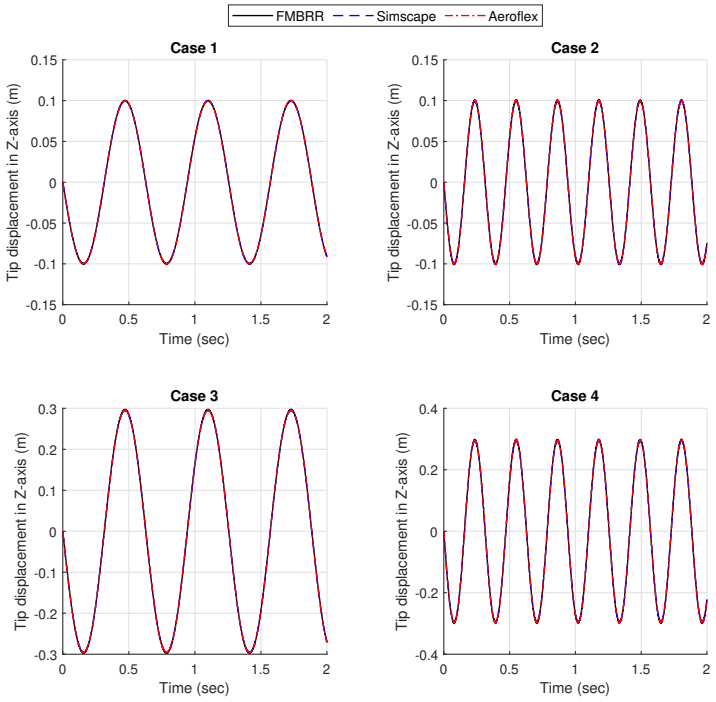


Figure 4. Global beam tip deflections in the Z-axis as a function of time.

3.2 Highly flexible cantilever beam with folding part

To validate the implementation and modeling of the rotational constraints, the same beam from Subsection 3.1 was employed. The configuration consists of a rigid body, defining a folding part, which constitutes 20% of the beam's total length. The remaining 80% is a flexible body, to which the folding part is attached via a revolute joint. To induce dynamic motion, a time-varying force, F_z, and moment, M_y, were applied, in addition to a spring at the

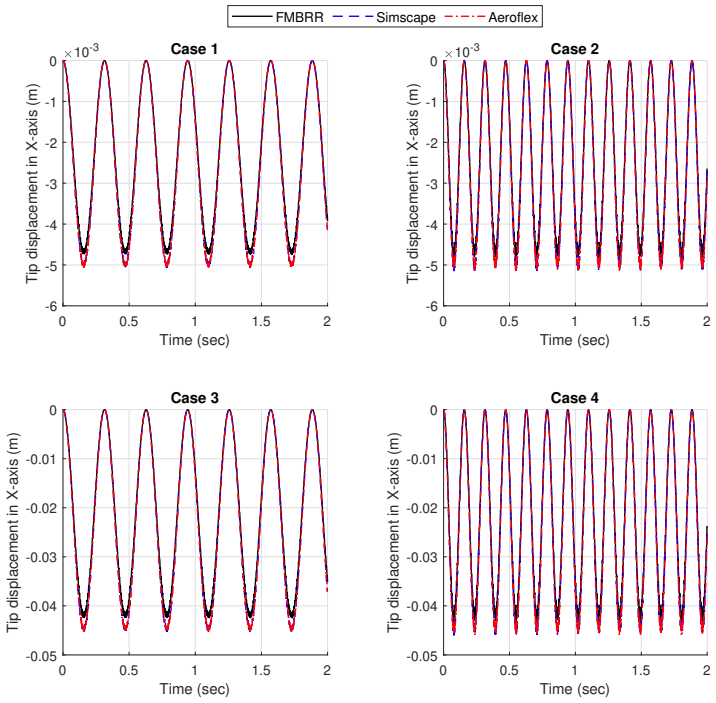


Figure 5. Global beam tip deflections in the X-axis as a function of time.

Table 3. Comparison of computational cost for Case 4: $M_y = 15\sin(20t)$.

Model	Simulation time (s)	Computational time (s)
Aeroflex	1.0	770.00
Simscape	1.0	751.08
FMBRR	1.0	183.95

connection. Furthermore, the effect of gravity was included, while damping was neglected. Figure 6 provides an illustration of the problem configuration.

As a reference model, Simscape was utilized again. The reference model was configured with four rigidly connected flexible bodies and a single rigid body attached to the extremity of the final flexible body through a revolute joint. Figure 7 displays the block diagram used to define the problem in Simscape.

The global displacements of point P, defined by the unit vector $\mathbf{s}_P = [1,\ 0,\ 0]^T$ at the end of the rigid body, were used as metrics for comparison between the reference and the FMBRR models. The applied force was defined as $F_z = 70\sin(10t)$ and the moment as $M_y = 7\sin(10t)$, with a spring constant of $k = 0.05\ \frac{\text{Nm}}{\text{deg}}$.

Two simulations were executed: the first, designated FMBRR-ode15s, utilized the *ode15s* solver without the PStab method. The second, FMBRR-PStab, incorporated the PStab method. The results are presented in Figures 8 and 9.

Analysis of the results in Figure 8 reveals the presence of the numerical drift phenomenon in the FMBRR-ode15s simulation. This drift causes a divergence in the x and y-axis displacements when compared to the Simscape results. Consequently, the application of a stabilization process is demonstrated to be essential. This was implemented in the FMBRR-PStab simulation, which yielded results in close agreement with the reference. Furthermore, examination of Figure 9 makes it evident that the model accurately captured the dynamics of the revolute joint connection. The reaction moment at the joint, obtained via Lagrange multipliers, also shows excellent correlation with the values obtained from Simscape.

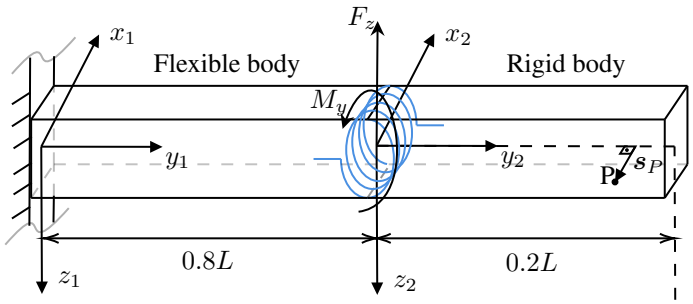


Figure 6. Clamped beam with folding part.

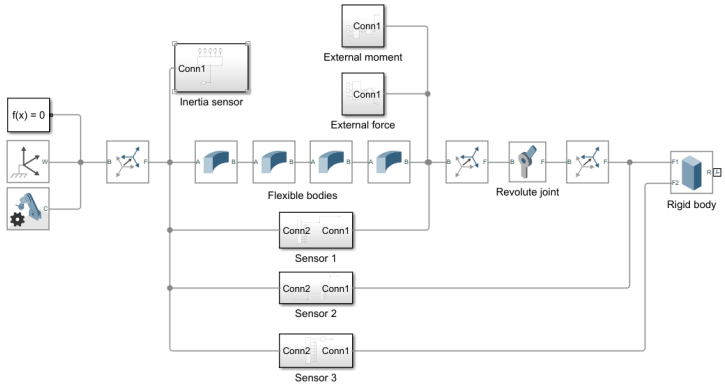


Figure 7. Simscape diagram for the clamped beam with a folding part.

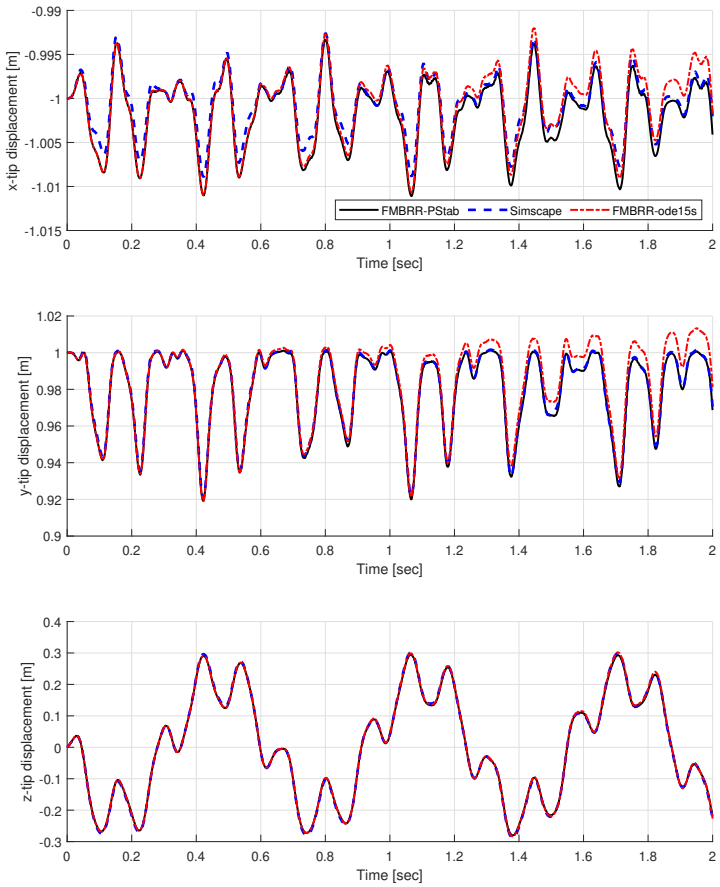


Figure 8. Comparison of the global displacements of point P at the end of the rigid body.

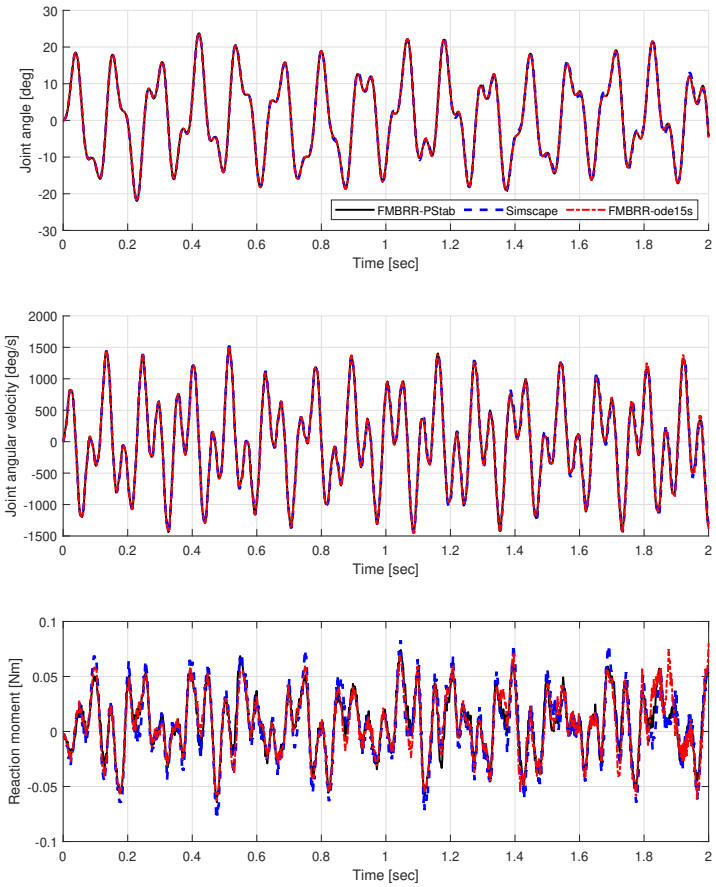


Figure 9. Comparison of angular displacements and reaction moment at the revolute joint.

4 Conclusion

This paper presented the development and validation of the FMBRR framework, a novel approach for analyzing the nonlinear structural dynamics of highly flexible structures with articulated parts, such as folding wingtips. The methodology successfully combines the strengths of flexible multibody dynamics to capture global geometric nonlinearities with the Rayleigh-Ritz method to model local structural deformations linearly. This hybrid formulation provides a valuable alternative that circumvents the complexity and, most notably, the high computational expense associated with fully nonlinear structural models.

The validation process demonstrated the framework's accuracy, robustness, and computational efficiency. Through rigorous comparisons with the established codes Aeroflex and MATLAB® Simscape, it was shown that the FMBRR model accurately predicts the static and dynamic behavior of highly flexible beams. Critically, the results revealed that the FMBRR framework is significantly more computationally efficient, achieving a fourfold reduction in simulation time compared to the benchmark models for the analyzed cases, without a loss of fidelity. Furthermore, the analysis of a cantilever beam with a revolute joint successfully validated the formulation for articulated structures. This case highlighted the model's capability to handle complex connections and underscored the necessity of numerical stabilization techniques, such as the PStab method, to ensure a physically consistent solution.

In summary, the FMBRR framework stands as a reliable and computationally powerful alternative for the preliminary design and dynamic analysis of morphing aircraft. Its versatility allows for the straightforward modeling of complex, interconnected systems, and its superior computational performance makes it an exceptionally valuable tool for aerospace engineering challenges where flexibility and large deformations are critical, particularly in design and optimization phases that require numerous simulations.

Future work will focus on expanding the framework's capabilities by integrating it with aerodynamic models

to conduct comprehensive aeroelastic and flight dynamics simulations. This will enable the investigation of the influence of FWTs on aircraft stability and control, as well as the development and testing of passive and active control laws for gust load alleviation and maneuver enhancement.

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