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Structural reliability of casing subjected to annular pressure build-up (APB) in oil and gas wells

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Abstract. The work objective is to estimate the probability of rupture in oil and gas well casing subjected to loads induced by Annular Pressure Build-Up (APB) through First-Order Reliability Method (FORM). During the well lifecycle, temperature changes from different operations can lead to pressure variations in annular spaces. These fluctuations may compromise the structural integrity of casing and tubing strings. Due to manufacturing uncertainties, casing may exhibit variations in geometric and material properties. Along with uncertainties in geomechanical rock properties obtained from laboratory testing, these factors affect simulation accuracy. Estimating failure probability provides a more reliable safety metric than deterministic approaches. The methodology includes: i) identifying deterministic and random variables; ii) Application of FORM to estimate casing failure probabilities (burst); iii) sensitivity analysis to rank the influence of input variables; and iv) organizing and interpreting the results. This work contributes by quantifying the casing failure probability under APB conditions and identifying the most influential uncertain parameters for the studied scenario. These insights support better-informed decisions in casing design, aiming to ensure structural integrity under varying thermal and pressure conditions.

Keywords: Annular pressure buid-up, Structural reliability, Probability of failure

1 Introduction

The discovery of Brazil's pre-salt layer fueled the expansion of oil exploration activities, a trend that will continue in the medium and long term. Drilling wells in ultra-deep regions, such as the pre-salt layer, poses challenges associated with extreme pressure and temperature conditions.

According to Zhang [1], due to the continuous heat transfer from the fluid flowing through the production/injection tubing, the column annulus temperature at each layer of the well varies, and the annulus fluid can then expand or contract due to the heat, resulting in annulus pressure variations. This phenomenon is called Annular Pressure Buildup (APB) (Halal and Mitchell [2]; Sathuvalli et al. [3]; Oudeman and Kerem [4]; Perez [5]; Vasconcelos [6]; Santos [7]).

In onshore wells, the APB can be relieved by draining the fluid. However, in offshore wells, access to the annuli is limited or, in most cases, impossible due to the complexity and operational risks associated with them (Barcelos [8]). Excessive APB can damage the casing, compromising their integrity. Therefore, accurately predicting the APB between casing before testing is crucial to ensuring well integrity of deepwater oil and gas wells (Zhang [1]).

Due to the complexity of the problem and the large number of variables involved, Annular pressure build-up (APB) models are typically implemented through computational methods. Traditionally, these models use a deterministic approach, where each parameter has a fixed value. However, few studies explore how variability in these parameters affects the resulting annular pressures.

In structural analysis, uncertainties related to design variables, including geometric dimensions, applied loads, and material mechanical properties, significantly influence the structural response. Accurately accounting for these uncertainties is essential for reliable performance assessment and safety evaluation (Silva; Lima Junior;

Souza, 2023) [9].

In the context of APB, these uncertainties may be related to the casing construction process, reflected in variations in the values of the modulus of elasticity, Poisson's ratio, and coefficient of thermal expansion, as well as in geometric aspects such as the external diameter and thickness of the pipes.

Additionally, the rock properties formation traversed by the well are also subject to uncertainties, both about the rock physical characteristics and the diameter drilled by the drill bit.

Neglecting these uncertainties, and, consequently, the variability in pressure estimates, can lead to non-conservative safety factors for the casing and potentially compromise overall well integrity.

In this sense, the present work aims to use reliability methods to determine the probability of failure, the reliability index, and the safety factor of the production casing, considering a reference scenario, as well as identifying the most influential variables in the magnitude of the pressure developed.

2 Methodology

The methodology adopted in this work development consists of four macro steps, as illustrated in Figure 1.

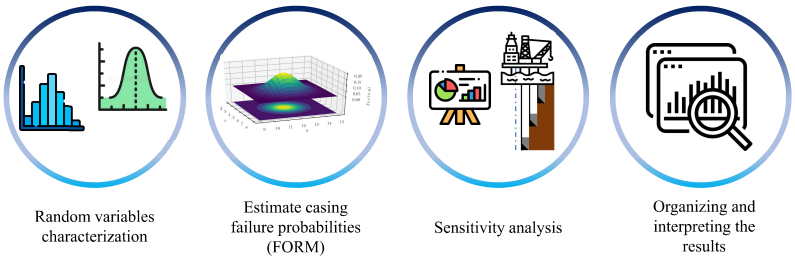


Figure 1. Methodology in this work development

The first step consists of identifying the physical properties of the system that exhibit significant uncertainty in the context of APB simulation. Section 3 presents a discussion of APB mathematical modeling. In this study, the random variables are related to the properties of casing, cement, and surrounding rock. Each variable is assigned a suitable probability distribution, informed by data from the technical literature (Nunes et al. [10]; Gouveia [11]). These distributions are fully characterized using statistical parameters such as the mean and coefficient of variation (COV).

In the second step, the FORM (First-Order Reliability Method) is applied from the reference well using the random variables (r.v.) generated in the previous step. Section 4 presents a discussion of FORM. At this stage, a computational APB simulator (Vasconcelos [6]) is used to calculate the pressures in the annuli of the reference well. The third step involves using FORM to perform a sensitivity analysis quantifying the influence of each random variable on the probability of production casing failure in the reference well. The fourth step involves building graphs and tables to visualize and interpret the results.

3 Annular Pressure Build-up review

The APB is determined by equalizing the fluid volume change (ΔV_{fl}) with the annular volume change (ΔV_{an}) (Oudeman and Kerem [4]), as shown in eq. (1).

$$\Delta V_{fl} = \Delta V_{an} \tag{1}$$

The fluid volume change trapped in the annular is associated with its thermodynamic properties. One of the approaches found in the literature uses the fluid thermal expansion coefficient (α_{fl}) and its compressibility (k_t) to calculate ΔV_{fl}, according to eq. (2). Where ΔT is the temperature variation and ΔP is the pressure variation.

$$\Delta V_{fl} = V_{fl} \left[\alpha_{fl} \Delta T - k_t \Delta P\right], \tag{2}$$

To consider fluid properties as constants generally appropriate when the initial temperature and pressure in the fluids distribution do not vary significantly throughout the annular volume, the difference between the initial and final fluid temperatures is small, and the temperature change is positive (Sathuvalli et al. [12]). Like these conditions are unusual in typical oil well operations, strategies that relate α_{fl} and k_t with the fluid density according

to pressure and temperature conditions are used. In Zamora et al. [13] a methodology for modeling drilling fluids is described. In this sense, Perez [5], Vasconcelos [6] and Santos [7] describe expressions for updating density, coefficient of thermal expansion and compressibility.

Furthermore, ΔV_{an} is determined according to the Lamé equations for thick-walled tubes, as a consequence of the linear elasticity applied to axisymmetric solids. This approach is used to calculate the wall displacements in hollow cylinders and, therefore, the variation in their volume.

eq. (1) can be used to estimate the APB value in a single annular (single string analysis). For analyses with multiple annulars (multi string analysis), the pressure variation in one annular directly interferes with the pressures of the adjacent annular. Therefore, applying eq. (1) to each annular of the well, obtain eq. (3), whose solution provides the APB values in the annulars (Sathuvalli et al. [3]).

$$[\Delta V_{fl}] = [\Lambda][\Delta P] + [\eta][\Delta T] \tag{3}$$

In eq. (3), $[\Delta V_{fl}]$ is the vector that denotes the fluid volume change in the well annular, $[\Lambda]$ is the flexibility matrix, $[\Delta P]$ is the APB vector, $[\eta]$ is the matrix whose terms provide the annular volume change of each annulus due to the casing thermal expansion that limit it and $[\Delta T]$ is the vector of casing columns temperature changes (Sathuvalli et al. [3]).

4 First-Order Reliability Method - FORM

According to Várady Filho et al. [14], the First-Order Reliability Method (FORM) is an semi-analytical approach that transforms the original random variables of the problem into equivalent standard normal variables and linearizes the limit state function to estimate the probability of failure. According to Gouveia [11], it is common to call the limit state equation $G(X) = 0$, where X is a vector with all the random variables of the analysis. Thus, the limit state equation is used to specify failure events and safe events as follows:

$$\begin{cases} G(X) > 0 \Rightarrow & \text{Safe event} \\ G(X) \leq 0 \Rightarrow & \text{Failure event.} \end{cases} \tag{4}$$

The probability of failure corresponds to the ratio between the number of failure events and the total number of events. In structural analysis, limit state functions typically adopt the classical form involving resistance (R) and load (S), where failure occurs when the load exceeds the resistance:

$$G(R, S) = R - S = 0. \tag{5}$$

In this context, according to Várady Filho et al. [14], the reliability problem is formulated as a constrained nonlinear optimization problem, in which the so-called design point is sought, defined in the transformed space of equivalent normal variables. This point contains the values of the random variables most likely to lead to structural failure. This definition gives rise to the concept of the reliability index β, which consists of the shortest distance between the origin of the transformed space and the failure surface, as illustrated in the Figure 2.

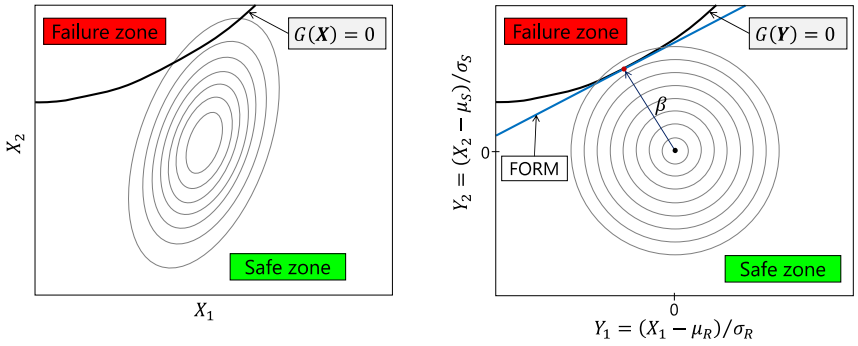


Figure 2. Illustrative diagram of the FORM. Source: Várady Filho et al. [14]

In the application of a FORM-type reliability method, the failure function evaluation and its gradients become necessary, as these quantities define the failure probability of the problem. Considering the implicit nature of

$G(X)$, its derivatives are calculated numerically by finite differences for each call to the APB simulator during the calculation of the pressure differentials over the production casing that will compose the action load (S), as illustrated in Figure 3.

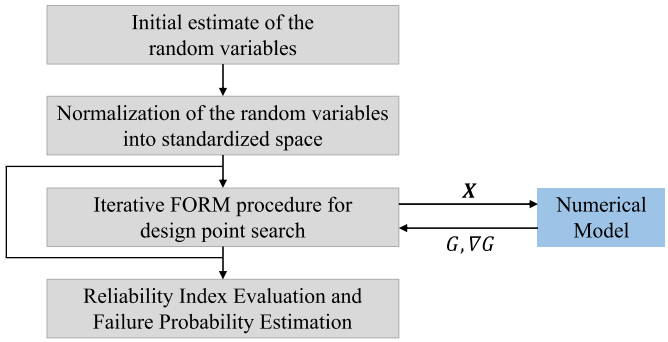


Figure 3. FORM application flowchart. Source: Lima Junior et al. [15]

5 Case Study

To calculate the probability of failure (PoF) of production casing, a reference well is used, illustrated in Figure 4, which presents two annulars in the well with different boundary conditions. For this particular case, axial pressure is not considered.

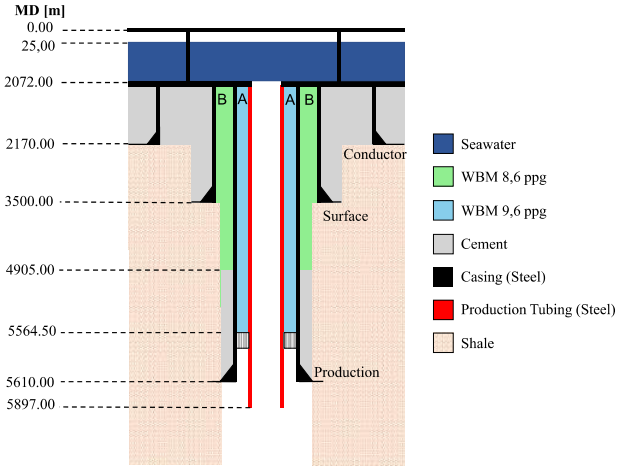


Figure 4. Reference well

Table 1 presents the specifications of the analyzed well. The well has a rotary table height of 25 m and a water depth of 2047 m. A single shale layer with behavior in a linear elastic regime is adopted. The packer depth in the first annulus is 5564.5 m.

Table 1. Application well specification

Name	Type	Top (m)	Bottom (m)	TOC (m)	ID (pol)	OD (pol)	Hole Size (pol)	Linear weight (lbf/pé)	Grade
Conductor	Casing	2072.00	2170.00	2072.00	33.00	36.00	42.00	554.00	B
Surface	Casing	2072.00	3500.00	2072.00	18.00	16.25	20.00	162.00	X-80
Production	Casing	2072.00	5610.00	4905.00	9.66	10.75	14.75	60.70	L80
Production	Tubing	2072.00	5897.12	-	5.79	6.625	-	28.00	SDSS/125KSI

The reference well simulates an oil production operation with a flow rate of 11,000 m³/day (with a production pressure of 9,814.11 psi and a perforation temperature of 99.94 °C). The fracture gradient at the surface casing shoe is 9.0 ppg. In addition, there is 1 bbl injection in annulus A and 1 bbl drainage in annulus B. The simulation

assumes the well to be initially in geothermal equilibrium and determines steady-state temperature profiles based on production conditions. The graphs in Figure 5 present the thermal simulation results.

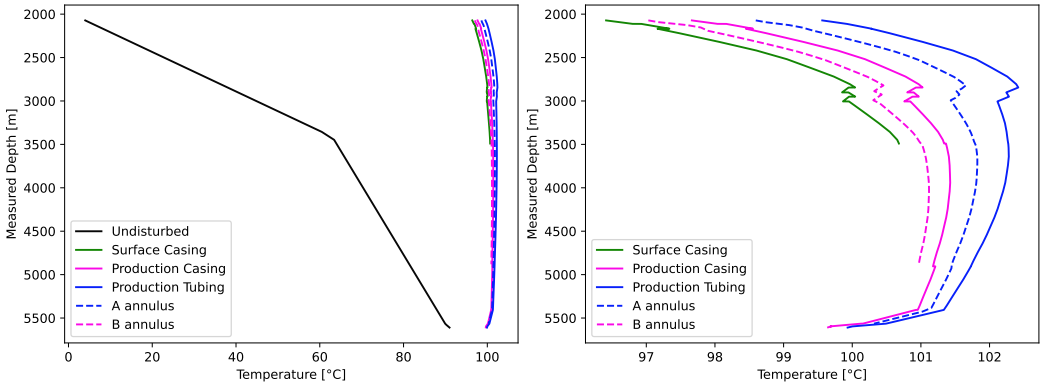


Figure 5. Temperature gradients: (a) Thermal profiles; (b) Thermal profiles of casing and annulars

Table 2 presents the random variables considered in this study. The standard deviation results from the product of the mean value and the coefficient of variation (COV). The probability distribution assigned to each variable follows the definitions provided by Gouveia [11] and Nunes et al. [10]. In this work, the analysis assumes statistical independence among the variables.

Table 2. Statistical parameters of variables

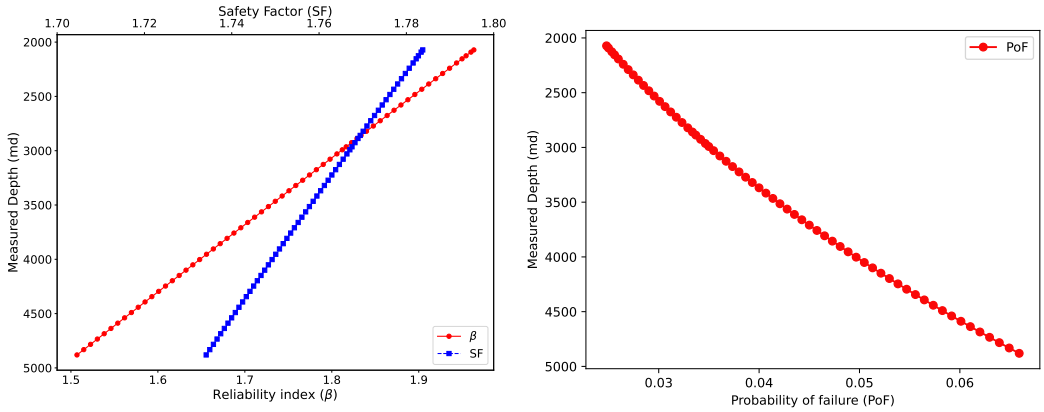
Casing				
Property	Nominal	Mean /Nominal	COV	Distribution
Elastic Modulus [psi]	3.12E+07	1.0	0.05	Normal
Poisson ratio	0.300	1.0	0.05	Normal
Outer Diameter [pol]	10 3/4	1.0059	0.00181	Normal
Ultimate Tensile Strength [psi]	9500	1.0853	0.0333	Normal
Minimum wall thickness [pol]	0.545	0.941	0.0157	Normal
Thickness [pol]	0.545	1.0069	0.0259	Normal
Cement				
Property	Mean	Mean /Nominal	COV	Distribution
Elastic Modulus [psi]	1.50E+06	1.0	0.02	Lognormal
Poisson ratio	0.210	1.0	0.25	Lognormal
Shale				
Property	Mean	Mean /Nominal	COV	Distribution
Elastic Modulus [psi]	2.54E+06	1.0	0.9	Normal
Poisson ratio	0.100	1.0	0.1	Normal

In this case, the limit state equation is given in eq. (6), where the resistance is evaluated using the Klever–Stewart model in accordance with ISO TR 10400 [16], and the load effect is obtained from the APB simulator.

$$G(X) = KS_{design}(f_u, OD, t, t_{min}) - APB(OD, t, E_{casing}, \nu_{casing}, E_{cement}, \nu_{cement}, E_{shale}, \nu_{shale}). \quad (6)$$

6 Results

Figure 6 (a) presents the results of the reliability index (β) and safety factor (SF) along the depth for the production casing. The graph shows that, as depth increases, both β and SF decrease, meaning the most critical conditions occur at deeper levels. Figure 6 (b) presents the probability of failure for the same casing according to depth. The highest failure probabilities occur in the deepest region. This trend is consistent with the reduction in β in graph (a), since a higher β implies a lower PoF.



(a) (b)

Figure 6. Production casing results: (a) Reliability index and safety factor; (b) Probability of failure

One of the steps in the FORM procedure involves calculating the direction cosines, where the subscript i denotes the i-th random variable. They are obtained from the normal vector to the failure surface at the design point. The α_i indicate the relative contribution of each variable to the β and, therefore, can be interpreted as importance factors or sensitivity coefficients. In this sense, Figure 7 presents the results of the sensitivity analysis obtained from the director cosines in FORM. The graph highlights the contribution of each random variable to the production coating reliability index. The variables that do not appear in the graph had percentages below 0.01%.

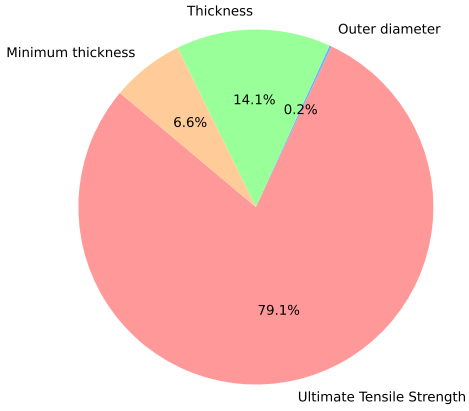


Figure 7. Sensitivity analysis

Ultimate tensile strength accounts for 79.1% of the total influence, making it the parameter with the greatest impact on the probability of failure. This highlights that variations in steel strength exert a dominant effect on the safety margin of the casing. Nominal thickness contributes 14.1% to the total influence, indicating substantial importance, as even small variations in this parameter can significantly affect the structural capacity under internal and external pressures. Minimum thickness represents 6.6% of the total contribution, having a smaller influence than nominal thickness but still relevant to structural performance. Conversely, the outer diameter exhibits negligible influence (0.2%), indicating that, within the variability range considered in this study, this parameter has no significant effect on the structural reliability of the casing.

7 Conclusions

This work presented a methodology to estimate the probability of rupture in oil and gas well casing subjected to loads induced by Annular Pressure Build-Up (APB) through First-Order Reliability Method - FORM. By applying the methodology to a case study, it was possible to calculate the reliability index, the safety factor, and the probability of failure of the production casing. For the case analyzed, the results show that the most critical regions for the structural integrity of the production casing occur at the greatest depths, where β and SF reach their minimum values, while PoF attains its maximum. Additionally, a sensitivity analysis was performed, which showed

that, within the range of variability adopted for the random variables, the casing reliability is strongly controlled by the material's mechanical properties (primarily tensile strength) and, secondarily, by the wall thickness.

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