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Assessment of Inherent Variability from SPT Data and Its Application to Shallow Foundations under Axial Loading

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Abstract. This study proposes a practical framework for evaluating the inherent spatial variability of geotechnical parameters based on Standard Penetration Test (SPT) data. The approach includes the implementation and validation of numerical routines for calculating mean, standard deviation, probability distribution fitting, and the scale of fluctuation using vertical SPT profiles. The method is designed to support probabilistic and random field modeling in geotechnical applications. To illustrate the applicability of the developed routines, a case study is presented involving shallow foundations under axial loading. The geotechnical variability inferred from SPT data is incorporated into the bearing capacity assessment using random field theory. This allows for quantifying the influence of spatial variability on design parameters and failure probabilities. The study emphasizes the importance of adequate preprocessing of field data, including trend removal and normalization, and demonstrates how variability metrics—especially the scale of fluctuation—can be effectively integrated into reliability-based design approaches.

Keywords: Spatial Variability, Standard Penetration Test (SPT), Random Field Modeling (up to 5 keywords).

1 Introduction

Only The assessment of inherent spatial variability of geotechnical parameters is a fundamental issue in modern foundation engineering, particularly for reliability-based design and risk quantification. Standard Penetration Test (SPT) data, routinely collected in site investigations, provides a practical basis for characterizing this variability in situ. However, the direct application of SPT results in deterministic design ignores the natural heterogeneity and uncertainty that are critical for understanding serviceability and failure probabilities in shallow foundations [1].

In recent years, significant advances have been made in the statistical analysis of SPT profiles, including the estimation of spatial statistics such as mean, standard deviation, and the vertical scale of fluctuation [2,3]. Incorporating these spatial features into probabilistic models—particularly through the use of random field theory and Monte Carlo simulations—allows for a more realistic quantification of the effects of soil variability on the performance of shallow foundations [4,5]. Despite these advances, the integration of random field theory into practical geotechnical design is still limited, and case studies using real site data are relatively scarce.

This paper addresses this gap by developing and applying a practical framework for simulating and analyzing the spatial variability of SPT data in the context of shallow foundation design. The study highlights the importance of adequate preprocessing of field data, the calibration of spatial correlation structures, and the probabilistic evaluation of foundation settlements and reliability. The framework is illustrated with a case study using marine sediment SPT data, aiming to provide geotechnical engineers with a transparent and reproducible methodology

for incorporating spatial variability into design decisions.

2 Materials and Methods

2.1 Site and SPT Dataset

This study is based on SPT data collected from two sites in Florianópolis, SC, Brazil, as reported by Milan et al. [2]. Both sites, located approximately 5 km apart, belong to the same Pleistocene marine beach deposit, representative of the sandy coastal geology of southern Brazil. The SPT dataset includes a total of 40 vertical profiles (20 per site), each extending to depths of 15 m and 17 m, respectively, reflecting the typical stratigraphy relevant to foundation design. All available measurements within these depths were used for statistical and numerical analyses. The geographic location of both sites and examples of the SPT profiles are shown in Fig. 1.

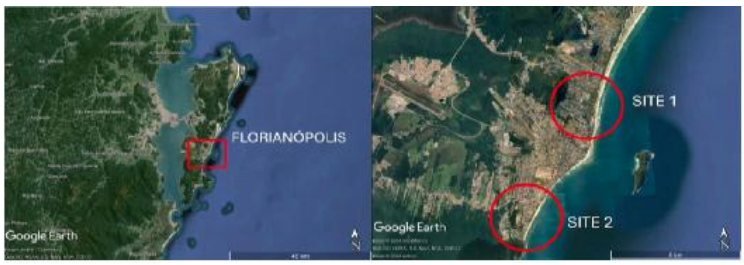


Figure 1. Location of Site 1 and Site 2 in Florianópolis, SC, Brazil (sourced: Milan et al., 2025).

2.2 Statistical treatment of SPT data

The statistical treatment of the SPT dataset begins with the computation of the depth-dependent mean and standard deviation profiles. The mean value at depth z_i, denoted by $\mu(z_i)$, is calculated as the arithmetic average of all available N_{SPT} values at that depth across the N profiles, as shown in Equation (1):

$$\mu(z_i) = \frac{1}{N}\sum_{j=1}^{N} N_{SPT}(z_i, j) \tag{1}$$

Similarly, the standard deviation at each depth, $\sigma(z_i)$, is computed using Equation (2):

$$\sigma(z_i) = \sqrt{\frac{1}{N-1}\sum_{j=1}^{N}[\,N_{SPT}(z_i, j) - \mu(z_i)\,]^2} \tag{2}$$

To remove deterministic trends and obtain stationary residuals for spatial correlation analysis, the mean profile at each depth was subtracted from the original SPT data. Although trend fitting and removal facilitate the generation of stationary random fields, it is necessary to correct the simulated realizations by adding back the measured mean at each depth. This ensures that the expected value of the simulated data matches the empirical mean observed in the field data, as shown in Equation (3):

$$r(z_i, j) = N_{SPT}(z_i, j) - \mu(z_i) \tag{3}$$

This detrending procedure ensures that only the random component is modeled, consistent with recommendations by Phoon & Kulhawy [1] and Fenton & Griffiths [3]. The vertical autocorrelation structure of the residuals, $C(h)$, is estimated by averaging the normalized product of residual pairs separated by lag h along each profile, as shown in Equation (4):

$$C(h) = \frac{1}{N(n_h)}\sum_{j=1}^{N}\sum_{i=1}^{n_h}\frac{r(z_i,j)\,r(z_i+h,j)}{\sigma(z_i)\sigma(z_i+h)} \tag{4}$$

The vertical correlation structure was quantified by fitting several candidate autocorrelation functions (ACFs) to the residual profiles. The best-fitting model, selected based on the coefficient of determination (R^2) model to estimate the scale of fluctuation δ_z [1,3], summarized in Table 1:

Table 1. Autocorrelation Functions

Model	ACF
Exponential	$\rho(\tau) = \exp\left(-\frac{2\,\lvert\tau\rvert}{\delta}\right)$
Exponential quadratic	$\rho(\tau) = \exp\left(-\pi\left(\frac{\tau}{\delta}\right)^2\right)$
Cosine exponential	$\rho(\tau) = \cos\left(\frac{\tau}{\delta}\right)\exp\left(-\frac{\lvert\tau\rvert}{\delta}\right)$
Second Order Markov	$\rho(\tau) = \left(1+\frac{4\,\lvert\tau\rvert}{\delta}\right)\exp\left(-\frac{4\lvert\tau\rvert\rvert}{\delta}\right)$

2.3 Random field generation

Random fields for synthetic SPT profiles were generated using the Cholesky decomposition method. For each simulation, a covariance matrix C was assembled as equation 5:

$$C_{ij} = \sigma(z_i)\,\sigma(z_j)\,\rho(|z_i - z_j|) \tag{5}$$

where $\sigma(z_i)$ and $\sigma(z_j)$ are the standard deviations at depths z_i and z_j, respectively, and $\rho(|z_i - z_j|)$ is the fitted autocorrelation function. The Cholesky decomposition yields a lower-triangular matrix L, and a synthetic profile X is generated as equation 6:

$$X = \mu + Lz \tag{6}$$

where μ is the trend vector and z is a vector of independent standard normal random variables [3,4]. This process yields random SPT profiles that preserve the measured spatial statistics and variability of the original field data, enabling robust probabilistic analysis in geotechnical design.

2.4 Numerical model implementation

The numerical simulations presented in this study were carried out using a custom-developed finite element routine based on the OpenSees framework [6]. OpenSees (the Open System for Earthquake Engineering Simulation) provides an open-source platform for advanced numerical analysis in geotechnical and structural engineering, supporting efficient implementation of spatially variable material properties and custom constitutive models. In this work, the finite element mesh, random field assignment, material properties, boundary conditions, and loading procedures were fully automated using Python scripts interfaced with OpenSeesPy. This allowed for rapid execution of thousands of realizations and systematic variation of applied loads, supporting the probabilistic evaluation of shallow foundation settlements. The overall workflow ensures reproducibility and flexibility, making it suitable for reliability-based geotechnical design applications.

2.5 Simulation workflow

The numerical model consists of a rigid shallow foundation (footing) with a width of 4.0 meters, resting on a heterogeneous soil domain characterized by spatially variable Standard Penetration Test (SPT) fields. The domain is discretized into quadrilateral finite elements, with automatic generation of nodal coordinates and element connectivity for each realization. Soil properties, specifically Young's modulus, are assigned to each element by linearly interpolating from the simulated SPT values to the centroids, ensuring local variability is

captured throughout the mesh.

The rigid foundation is represented as a rectangular concrete block at the ground surface, with the load applied as a uniform line load distributed among the nodes along its upper face. Appropriate boundary conditions are enforced: the lateral boundaries are fixed in the horizontal direction, while the bottom boundary is fixed in both horizontal and vertical directions, simulating field conditions (see Fig. X).

A Monte Carlo simulation scheme is implemented, with 1000 independent realizations, each corresponding to a unique 2D SPT field. For each realization, the finite element model is constructed, boundary and loading conditions are imposed, and settlements are computed at key points beneath the foundation. This approach enables a robust probabilistic evaluation of the influence of spatially variable soil properties on foundation performance.

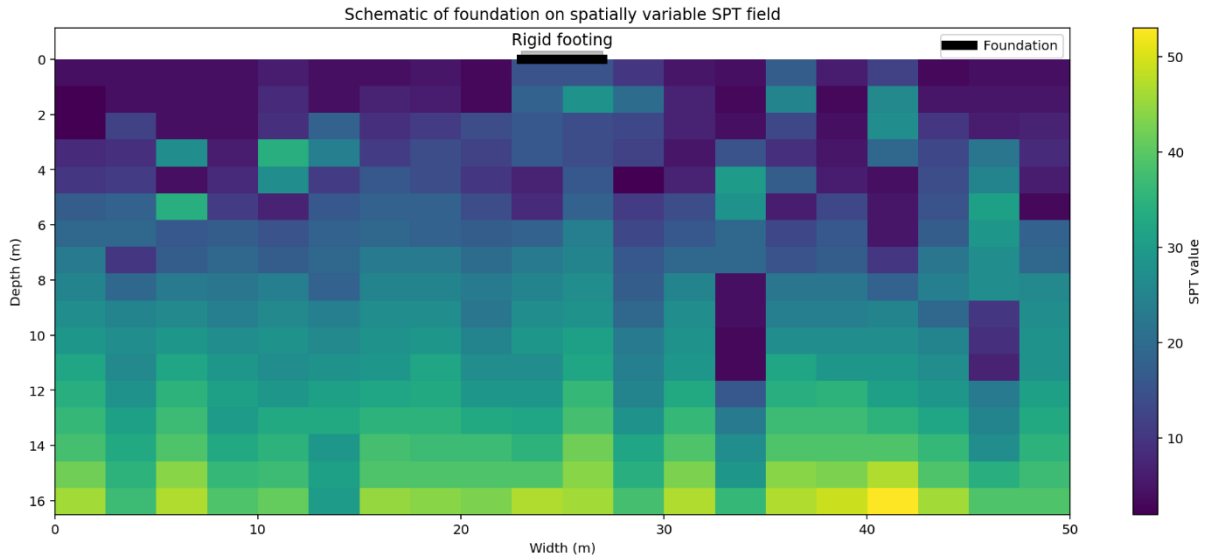


Figure 2. Schematic representation of the finite element mesh, spatially variable SPT field, and rigid shallow foundation.

3 Results and Discussion

3.1 Spatial variability of SPT data

To illustrate the spatial structure of the SPT data, Figure 3a and Figure 3c present the complete set of SPT profiles and the corresponding mean trend for each site. For both locations, the original SPT data matrix was analyzed to calculate the mean profile and standard deviation (σ) at each depth. The results are summarized in Table 2 (Site 1 and Site 2), showing both the statistics for the original measurements and those obtained from the simulated 2D SPT random fields generated via the Cholesky decomposition method. The excellent agreement between the original and simulated statistics demonstrates the validity of the random field simulation procedure.

To further characterize the spatial variability, Figure 3b and Figure 3d display the depthwise residuals (i.e., deviations from the mean) for each profile. The residuals provide a measure of local variability after removing the deterministic depth trend, which is critical for the subsequent spatial correlation analysis.

The empirical vertical autocorrelation function (ACF) of the residuals was then computed for each site, following the procedure of Phoon & Kulhawy [1]. Several theoretical models—including exponential, Gaussian, cosine-exponential, and Markov-2—were fitted to the empirical ACF using least-squares regression, and the best fit was selected based on the R^2 value. The fitted models and the empirical ACFs are plotted for each site in Figure 2a and Figure 2b. The vertical scale of fluctuation, δ, was determined from the best-fitting model for each site and used in the generation of the spatially correlated SPT fields.

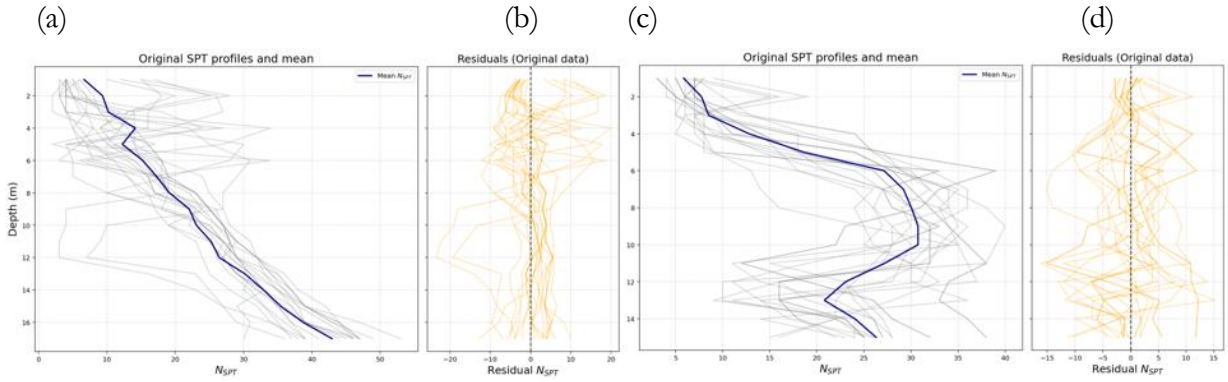


Figure 3. a) SPT profiles and depth wise mean for Site 1. (b) Depth wise residuals (deviations from the mean) for Site 1. (c) SPT profiles and depth wise mean for Site 2. (d) Depth wise residuals for Site 2. The residuals quantify local variability after removing the deterministic depth trend.

The empirical vertical autocorrelation function (ACF) of the residuals was then computed for each site, following the procedure of Phoon & Kulhawy [1]. Several theoretical models—including exponential, Gaussian, cosine-exponential, and Markov-2—were fitted to the empirical ACF using least-squares regression, and the best fit was selected based on the R^2 value. The fitted models and the empirical ACFs are plotted for each site in Figure 4a and Figure 4b. The vertical scale of fluctuation, δ, was determined from the best-fitting model for each site being 3.18 m (Site 1) and 2.76 (Site 2) to be used in the generation of the spatially correlated SPT fields.

This combined approach—trend removal, residual analysis, empirical ACF evaluation, and model fitting—ensures that the simulated fields reproduce not only the global statistics (mean and standard deviation), but also the spatial continuity and variability observed in the measured data.

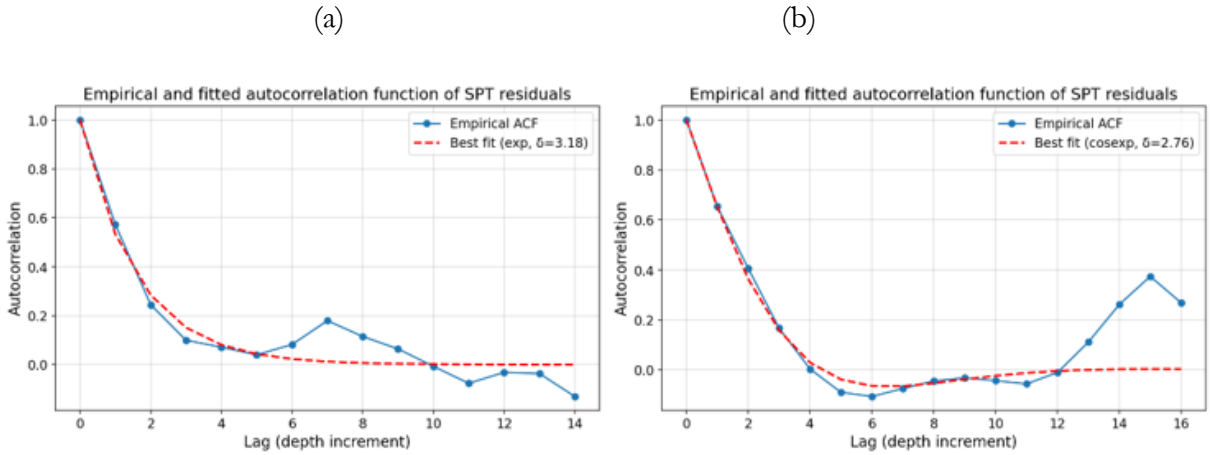


Figure 4. (a) Empirical and best-fit theoretical autocorrelation functions (ACF) of SPT residuals for Site 1. (b) Empirical and best-fit ACF for Site 2.

Table 2. Statistical resume Nspt vs Depth data and simulations

Site 1					Site 2				
Depth (m)	Mean Nspt (Original)	Sigma residual (Original)	Mean Nspt (Simulated)	Sigma residual (Simulated)	Depth (m)	Mean Nspt (Original)	Sigma residual (Original)	Mean Nspt (Simulated)	Sigma residual (Simulated)
1	6.65	4.49	6.88	4.08	1	5.85	1.79	5.88	1.77
2	9.35	8.67	10.11	7.42	2	7.75	4.09	7.88	3.92
3	10.2	+6.01	10.35	5.67	3	8.5	2.33	8.51	2.32
4	14.15	7.71	14.27	7.38	4	12.8	5.52	12.88	5.43
5	12.25	7.81	12.43	7.28	5	18.65	6.67	18.74	6.6
6	15.25	8.28	15.34	7.96	6	27.15	6.67	27.21	6.68
7	17.3	4.55	17.28	4.53	7	29.2	4.96	29.26	4.94
8	19.15	4.43	19.14	4.43	8	30.05	4.77	30.05	4.72
9	22.05	5.19	22.08	5.18	9	30.75	4.55	30.73	4.52
10	23.1	6.13	23.18	6.17	10	30.75	4.76	30.7	4.75
11	25.25	6.88	25.35	6.88	11	27.2	8.83	27.11	8.8
12	26.45	7.7	26.51	7.66	12	23.05	8.32	22.95	8.28
13	30.2	4.71	30.22	4.66	13	20.8	7.96	20.73	7.88
14	32.95	4.02	32.97	3.99	14	24.05	5.53	24.04	5.48
15	35.45	4.19	35.4	4.17	15	26.3	5.74	26.3	5.7
16	38.75	4.73	38.69	4.74					
17	42.95	5.44	42.94	5.43					

3.2 Foundation settlement analysis

A total of 1000 Monte Carlo simulations were performed for each site, applying a range of foundation loads to the probabilistically generated SPT fields. For each simulated realization, the settlement at the center of the footing was computed using the finite element model described in Section 2.5. The probability of failure for each load level was estimated as the fraction of simulations resulting in settlements exceeding the serviceability criterion of 25 mm.

3.3 Reliability assesment

The calculated probabilities of failure (Pf) were converted to reliability indices (β) using the standard normal transformation. Figure 5(a) and 5(b) show the relationship between applied load and the reliability index β for Site 1 and Site 2, respectively. This approach enables direct quantification of the uncertainty in settlement predictions and provides a more sensitive reliability-based interpretation of allowable load.

For Site 1, a reliability index $\beta = 2.33$ ($Pf \approx 1\%$) corresponds to an applied load of approximately 2608 kN; for $\beta = 2.00$ ($Pf \approx 2.3\%$), the load is about 2641 kN.

For Site 2, a reliability index $\beta = 2.33$ is achieved at a load of approximately 2455 kN, while $\beta = 2.00$ occurs near 2497 kN.

This reliability-based framework allows practitioners to select design loads corresponding to target safety levels, explicitly accounting for spatial variability and model uncertainty. The results highlight how reliability decreases rapidly with increasing load and emphasize the importance of probabilistic design in foundation engineering.

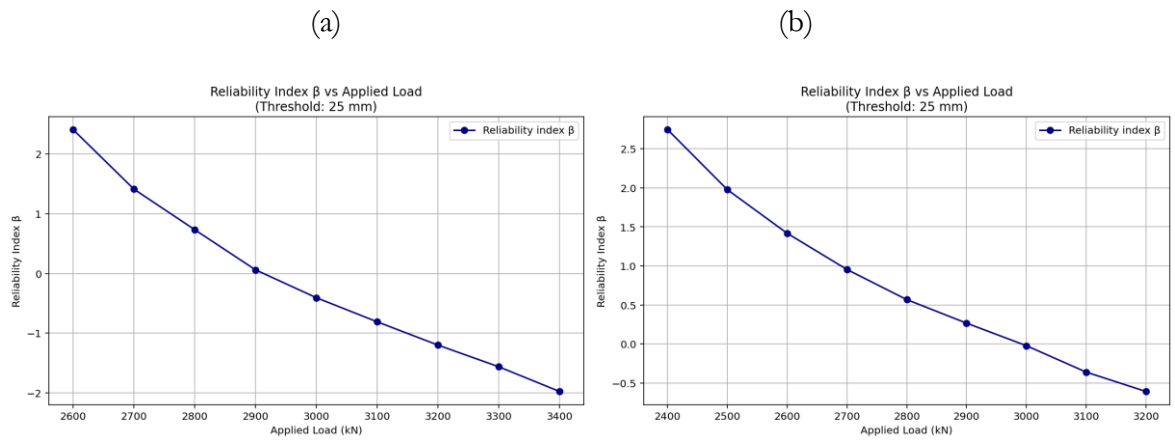


Figure 5. Reliability index β as a function of applied load for (a) Site 1 and (b) Site 2, using a settlement serviceability limit of 25 mm.

4 Conclusions

This study demonstrated a practical and robust framework for assessing the impact of inherent spatial variability in SPT-based parameters on the reliability of shallow foundation performance. The simulation procedure preserved both the mean and the standard deviation (sigma) profiles observed in the original site data, validating the effectiveness of the random field generation. The fitted spatial correlation models enabled the creation of realistic, site-consistent 2D SPT fields, which translated into settlement and reliability estimates that directly reflect the natural variability of the soil. Results highlight that adopting a probabilistic design approach, rather than a conventional deterministic one, provides a more sensitive and realistic quantification of failure probability and performance. The methodology and tools presented here can be directly applied to other sites and support the transition toward reliability-based geotechnical design. Future studies may explore 3D effects or more advanced constitutive models for further refinement.

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