

Modeling of Non-isothermal Single-phase Flow in Porous Media with Well-reservoir Coupling

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Abstract. This study explores the numerical modeling of single-phase flow in porous reservoirs equipped with production/injection wells, focusing on the application of well-reservoir coupling techniques. The approach considers hydrodynamic coupling between the well and reservoir systems. The nonlinear equations governing flow are solved using the Control Volume Finite Difference method with fully implicit time discretization. To determine the pressure and temperature distributions, an operator-splitting method is employed to decouple the governing equations. The resulting parabolic subproblems are solved using Picard iteration for linearization, and the algebraic systems are subsequently resolved with the Conjugate Gradient method. The coupling framework includes the calculation of productivity indices while accounting for transient near-well behavior. Several simulations are performed, including mesh refinement tests and sensitivity analyses to assess the influence of key physical and operational parameters. The results demonstrate that hydrodynamic coupling between wellbore and reservoir is fundamental for complete representation of non-isothermal flow conditions.

Keywords: Heat Transfer, Non-isothermal Flow, Numerical Simulation, Porous Media, Thermal Well-reservoir Coupling

1 Introduction

A key responsibility of petroleum engineers is enhancing process efficiency for the economical extraction of oil and gas, a task that fundamentally relies on a comprehensive understanding of production systems. Since the 1960s, reservoir numerical simulation has evolved into an indispensable tool for predicting the complex behavior of hydrocarbon flow within subsurface formations under various production scenarios, enabling the calculation of crucial parameters such as pressure and temperature distributions throughout the reservoir and within production wells [1]. This study specifically focuses on the intricate challenge of accurately estimating pressure and, critically, temperature within these wells by employing advanced well-reservoir coupling methodologies, with a particular emphasis on the complexities introduced by non-isothermal flow conditions.

A typical oil or gas production system comprises a series of interconnected components, extending from the geological reservoir to surface processing facilities [2]. While traditional numerical simulators have primarily concentrated on modeling flow within the porous reservoir using fundamental principles like Darcy's law [3], the critical interface at the wellbore requires specialized treatment through well-reservoir coupling models [4]. These coupling techniques are essential for bridging the distinct flow characteristics between the wellbore and surrounding porous medium. However, accurately describing the complex physical phenomena occurring within the well, including multiphase and multicomponent flow, and, most relevantly for this work, non-isothermal flow regimes, continues to pose a significant challenge [5].

This study directly addresses the numerical modeling of non-isothermal, single-phase flow in porous reservoirs specifically equipped with static heating wells. The central focus is the application and refinement of coupling techniques that incorporate hydrodynamic interactions governing fluid flow. Achieving accurate estimations of both wellbore and reservoir pressures is of paramount importance for optimizing production strategies and gaining a deeper understanding of overall system performance in scenarios where intentional heating near the wellbore can significantly alter the physical properties of the reservoir fluids and consequently impact their flow behavior [6]. Traditional assumptions of isothermal flow, frequently adopted to simplify the mathematical derivation of governing equations, may no longer be valid or sufficiently accurate in operational scenarios characterized by significant pressure drawdown, naturally tight or compact geological formations, or the intentional introduction of thermal

energy through injection or heating wells [7]. Therefore, a comprehensive understanding of these systems necessitates the incorporation of the energy balance equation alongside the fundamental flow equations [8]. The primary aim is to investigate and enhance coupling techniques tailored for non-isothermal flow conditions in reservoirs with static heating wells, thereby improving the reliability and applicability of reservoir simulation studies.

2 Non-isothermal flow in reservoirs

A physical-mathematical model for non-isothermal single-phase flow in a petroleum reservoir involves solving coupled partial differential equations for pressure and temperature, subject to appropriate boundary and initial conditions.

The reservoir is modeled as a porous medium with porosity (ϕ) and permeability tensor (\mathbf{k} , assumed diagonal). Key assumptions include a Newtonian fluid, no chemical reactions, heterogeneous and isotropic permeability, small and constant rock compressibility, constant thermal conductivities, low flow velocities (justifying Darcy's law), single-phase and non-isothermal flow in two dimensions, negligible gravity and radiation, and idealized well conditions. A volume-averaged temperature T is used, defined based on fluid (T_o), oil, and rock (T_r) temperatures and their thermal capacities [9].

2.1 Governing Equation for Flow

Based on mass conservation and Darcy's law, and incorporating the formation volume factor $B = \rho_{sc}/\rho$, the flow equation is [10]:

$$\frac{\partial}{\partial t} \left(\frac{\phi}{B} \right) - \nabla \cdot \left(\frac{\mathbf{k}}{B\mu} \nabla p \right) - \frac{q_{sc}}{V_b} = 0. \quad (1)$$

where ρ is the density, and q_{sc} is a source term at standard conditions.

The transient term's expansion, considering pressure and temperature dependence of ϕ and B [11], leads to the final pressure equation:

$$\Gamma_p \frac{\partial p}{\partial t} - V_b \nabla \cdot \left(\frac{\mathbf{k}}{B\mu} \nabla p \right) = \Gamma_T \frac{\partial T}{\partial t} + q_{sc}, \quad (2)$$

with Γ_p and Γ_T involving fluid compressibility (c_o), oil thermal expansion (c_{oT}), rock compressibility (c_ϕ), and rock thermal expansion ($c_{\phi T}$) [10]

$$\Gamma_p = V_b \left(\frac{\phi c_o}{B^0} + \frac{\phi^0 c_\phi}{B} \right), \quad \Gamma_T = V_b \left(\frac{\phi c_{oT}}{B^0} + \frac{\phi^0 c_{\phi T}}{B} \right), \quad (3)$$

where V_b is the bulk volume and the superscript "0" denotes reference values.

Viscosity (μ) is temperature-dependent [12],

$$\mu = a \exp \left(\frac{b}{T_o - T_{visc}} \right) \quad \text{for } T_o > T_{visc} \quad (4)$$

where a and b are specific coefficients pertinent to the oil considered, and T_{visc} is a reference temperature.

2.2 Governing Equation for Energy Transport

Employing a one-equation energy balance model without local thermal equilibrium [9], and accounting for heat input from wells (q_H) [12], the energy equation is:

$$\frac{\partial}{\partial t} [(\rho c_p)T] + \nabla \cdot (\rho_o h_o \mathbf{v}_o) - \nabla \cdot (\boldsymbol{\kappa} \nabla T) - \frac{q_H}{V_b} - \frac{\rho_o h_o q_{sc}}{V_b} = 0, \quad (5)$$

where $(\rho c_p)T = \phi(\rho c_p)_o T_o + (1 - \phi)(\rho c_p)_r T_r$, $\rho c_p = \phi(\rho c_p)_o + (1 - \phi)(\rho c_p)_r$, \mathbf{v}_o is the Darcy velocity of the fluid, $\boldsymbol{\kappa} = \phi \boldsymbol{\kappa}_o + (1 - \phi) \boldsymbol{\kappa}_r$ is the equivalent thermal conductivity tensor [9], contributions from tortuosity and hydrodynamic dispersion are neglected, and enthalpy $h_o \approx c_{p_o} T$ [12].

2.3 Initial and Boundary Conditions

Initial pressure (p_{ini}) and temperature (T_{ini}) are specified. No-flow conditions are applied at reservoir boundaries for both pressure and temperature:

$$\left(\frac{\partial p}{\partial x}\right)_{x=0,L_x} = \left(\frac{\partial p}{\partial y}\right)_{y=0,L_y} = 0, \quad \left(\frac{\partial T}{\partial x}\right)_{x=0,L_x} = \left(\frac{\partial T}{\partial y}\right)_{y=0,L_y} = 0, \quad (6)$$

where L_x and L_y are the respective lengths of the reservoir in the x and y directions.

2.4 Well-reservoir Coupling

The accurate modeling of well-reservoir coupling is a central aspect of this study, as it governs the exchange of mass between the wellbore and the surrounding reservoir.

Concerning the hydrodynamic coupling, the flow of fluids between the reservoir and the wellbore is typically characterized by the productivity index (J_w), which relates the production rate to the pressure drawdown in the well [10]:

$$q_{sc} = -J_w (p - p_{wf}), \quad (7)$$

where p_{wf} is the wellbore pressure. The productivity index J_w is not a constant but depends on reservoir and fluid properties, and well geometry. For transient flow conditions, particularly relevant in the near-wellbore region, J_w becomes time-dependent, reflecting the evolving pressure distribution around the well.

3 Discretization of the governing equations

The governing flow equation is discretized using the Control Volume Finite Difference (CVFD) method on a block-centered grid [10]. For a two-dimensional domain in Cartesian coordinates, this approach, applied to Equation (2) with a fully implicit time discretization (using backward Euler), yields the following discretized equation for cell (i, j) at time level $n + 1$:

$$\begin{aligned} & \mathbb{T}_x \Big|_{i+1/2,j}^{n+1} (p_{i+1,j}^{n+1} - p_{i,j}^{n+1}) - \mathbb{T}_x \Big|_{i-1/2,j}^{n+1} (p_{i,j}^{n+1} - p_{i-1,j}^{n+1}) + \mathbb{T}_y \Big|_{i,j+1/2}^{n+1} (p_{i,j+1}^{n+1} - p_{i,j}^{n+1}) \\ & - \mathbb{T}_y \Big|_{i,j-1/2}^{n+1} (p_{i,j}^{n+1} - p_{i,j-1}^{n+1}) = \frac{(\Gamma_p)_{i,j}^{n+1}}{\Delta t} (p_{i,j}^{n+1} - p_{i,j}^n) + \frac{(\Gamma_T)_{i,j}^{n+1}}{\Delta t} (T_{i,j}^{n+1} - T_{i,j}^n) + (q_{sc})_{i,j}^{n+1}, \end{aligned} \quad (8)$$

where \mathbb{T}_x and \mathbb{T}_y represent the transmissibilities in the x and y directions, respectively, calculated using harmonic averages for permeability and arithmetic averages for fluid properties at the cell interfaces [10].

Similarly, the energy transport equation, Equation (5), is discretized using the CVFD method and a fully implicit scheme, resulting in:

$$\begin{aligned} & \mathbb{K}_x \Big|_{i+1/2,j}^{n+1} (T_{i+1,j} - T_{i,j})^{n+1} - \mathbb{K}_x \Big|_{i-1/2,j}^{n+1} (T_{i,j} - T_{i-1,j})^{n+1} + \mathbb{K}_y \Big|_{i,j+1/2}^{n+1} (T_{i,j+1} - T_{i,j})^{n+1} \\ & - \mathbb{K}_y \Big|_{i,j-1/2}^{n+1} (T_{i,j} - T_{i,j-1})^{n+1} = V_b \left[\frac{(\rho c_p)}{\Delta t} \right]_{i,j}^{n+1} (T_{i,j}^{n+1} - T_{i,j}^n) + (\rho_o h_o q_{sc})_{i,j}^{n+1} + \Phi_{i,j}^{n+1} + (q_H)_{i,j}^{n+1}, \end{aligned} \quad (9)$$

where \mathbb{K}_x and \mathbb{K}_y are the thermal conductivities at the cell interfaces, and $\Phi_{i,j}^{n+1}$ represents the convective heat flux terms derived from Darcy's velocity.

3.1 Determination of well productivity index

The accurate representation of well-reservoir coupling is crucial for capturing the dynamic interaction between the wellbore and the reservoir during production processes.

The relationship between the production rate at standard conditions, $(q_{sc})_{i,j}^{n+1}$, and the pressure drawdown is discretized as:

$$(q_{sc})_{i,j}^{n+1} = -(J_w)_{i,j}^{n+1} [p_{i,j}^{n+1} - (p_{wf})_{i,j}^{n+1}]. \quad (10)$$

Under transient flow conditions prevalent near the wellbore, the productivity index $(J_w)_{i,j}^{n+1}$ is not constant but evolves with time. It is calculated as [10]:

$$(J_w)_{i,j}^{n+1} = \left\{ 4\pi k L_z \left[E_i \left(\frac{r_w^2}{4\eta_t t} \right) - E_i \left(\frac{r_{eq}^2}{4\eta_t t} \right) \right]^{-1} \right\}_{i,j}^{n+1}, \quad (11)$$

where r_w is the wellbore radius, r_{eq} is the time-dependent equivalent wellbore radius, L_z is the reservoir thickness, $\eta_t = k/(\phi c_t \mu)$, $k = \sqrt{k_x k_y}$, $c_t = c_o + c_\phi$, and E_i is the exponential integral function.

In the determination of the equivalent radius r_{eq} , present in Equation (11), the Newton-Raphson algorithm is used [13]:

$$(r_{eq})_{i+1} = (r_{eq})_i - \frac{f(r_{eq}_i)}{f'(r_{eq}_i)}, \quad (12)$$

where for pressure we have

$$f(r_{eq}_i) = \frac{q_{sc}}{4\pi k L_z} E_i \left(\frac{r_{eq}_i^2}{4\eta_t t} \right) - \Delta p, \quad f'(r_{eq}_i) = \frac{q_{sc}}{2\pi k L_z r_{eq}} \exp \left(\frac{r_{eq}_i^2}{4\eta_t t} \right), \quad (13)$$

where $\Delta p = p_i - p_{i,j}$ corresponds to the difference between the initial well pressure and the pressure of the cell containing the well.

Thus, in the case of vertical production wells, the calculation of the transient equivalent radius is performed for hydrodynamic coupling by means of:

$$r_{eq_{i+1}} = r_{eq_i} \left[1 - \frac{q_{sc} E_i(r_{eq_i}^2/(4\eta_t t)) - 4\pi k L_z \Delta p}{2q_{sc} \exp[r_{eq_i}^2/(4\eta_t t)]} \right]. \quad (14)$$

The use of a transient equivalent radius in hydrodynamic coupling model allows for a more accurate representation of the near-wellbore behavior, particularly during the early stages of production when transient effects are dominant. We considered the idea of dimensionless time and considering the normal area A , which represents the drainage area of the well in a developed reservoir [13],

$$t_{DA} = \frac{kt}{\phi \mu c_t A}. \quad (15)$$

for a well located at the center of the reservoir and that its drainage area has a square shape. In this case, for values of t_{DA} below 0.09, the flow behavior is analogous to that of an infinite system. On the other hand, starting from $t_{DA} = 0.1$, the pseudo-steady-state regime is reached [13]. Thus, it is assumed that the equivalent radius no longer is time dependent. The product $\phi \mu$ was computed based on the initial pressure and temperature, as an approximation.

3.2 Operator Splitting and Linearization

The coupled non-linear algebraic systems arising from the discretization of the flow (Equation (8)) and energy (Equation (9)) equations are solved using an operator splitting technique [12]. This involves decoupling the system into two sub-problems, one for pressure and one for temperature, with information exchanged between them. Each sub-problem is then linearized using Picard iteration [10]. The resulting linear systems are solved iteratively using the Conjugate Gradient method. The overall solution process involves inner iterations for pressure and temperature, followed by outer Picard iterations to ensure convergence of the coupled system. Convergence is assessed based on the absolute difference between solutions in successive iterations, compared against predefined tolerances for pressure and temperature.

4 Numerical results

The numerical simulations were performed to investigate the non-isothermal, single-phase flow in porous reservoirs equipped with static heating wells, with a specific focus on the hydrodynamic coupling between the reservoir and production wells. The governing nonlinear equations for flow and heat transfer were discretized using

the Control Volume Finite Difference method with a fully implicit time scheme. The resulting algebraic systems were efficiently solved using an operator splitting method combined with Picard linearization and the Conjugate Gradient method. Our approach, which incorporates hydrodynamic coupling, allowed for a detailed analysis of pressure and temperature distributions. The following results section presents the outcomes of several simulations, including a mesh refinement study to ensure numerical convergence and accuracy, along with sensitivity analyses to evaluate the impact of key physical and operational parameters on system performance and simulation fidelity. Table 1 provides a comprehensive list of all parameters and properties utilized throughout the simulations. It is worth noting that Δt_i is the initial time step, which is successively multiplied by $F_{\Delta t}$ until the final time step, $\Delta t_{f_{max}}$, is reached, as the successive simulation time steps are executed. The simulation finishes when the final time, t_{max} , is reached.

Table 1. Rock, fluid, and geometry general parameters

Parameter	Value	Unit	Parameter	Value	Unit
a	0.6×10^{-3}	Pa·s	$p_{ini} = p^0$	69,000	kPa
b	166.67	K	q_{sc}	-200	std·m ³ /day
B^0	1.1	m ³ /std·m ³	q_H	75	kW
c_o	7.25×10^{-7}	kPa ⁻¹	r_w	0.1	m
c_p	2,100	J/kg·K	t_{max}	180	day
$c_{p\phi}$	1,200	J/kg·K	tol_p	1×10^{-6}	kPa
c_{T_o}	9.2×10^{-4}	K ⁻¹	tol_T	1×10^{-6}	K
c_ϕ	5×10^{-7}	kPa ⁻¹	$T_i = T^0$	330	K
$c_{\phi T}$	1.8×10^{-3}	K ⁻¹	T_{visc}	277.78	K
$F_{\Delta t}$	1.1	–	κ_o	0.45	W/m·K
$k_x = k_y = k$	0.025	μm^2	κ_r	3.5	W/m·K
$L_x = L_y$	320	m	$\phi_i = \phi^0$	0.25	–
$L_z = L_{wf}$	50	m	ρ_o	950	kg/m ³
$\Delta t_{f_{max}}$	10	day	ρ_r	2,500	kg/m ³
Δ_h	20	m	Δt_i	0.01	day

During reservoir heating, four static heater wells are used, each with an individual power of 75 kW, positioned equidistantly at 20 m (Δ_h) from the producer well. With respect to grid refinement, we employed four grid sets: G1 (17 × 17), G2 (33 × 33), G3 (65 × 65), and G4 (129 × 129).

4.1 Grid refinement

In this subsection, we examine the effects of grid refinement on the numerical simulation of non-isothermal flow within the reservoir, comparing two well–reservoir coupling models under both steady-state [10] and transient regimes (considered in this work) with reservoir heating (Figure 1). The analysis is restricted to heated scenarios and aims to evaluate how spatial discretization influences the accuracy of the coupled solutions. Particular attention is given to potential variations in the predicted thermal and flow fields arising from the chosen coupling strategy and the level of grid resolution.

As can be observed from the results, the curves converge toward each other as the grids are progressively refined. Therefore, we consider the method to be convergent, and grid G4 was selected for obtaining all subsequent results. The productivity index as a time function was adequately implemented to mitigate the numerical artifact associated with the assumption of a steady-state considered for the conventional productivity index [4].

4.2 Sensitivity analysis

The following analysis examines the results obtained for both coupling models, considering scenarios with and without reservoir heating (Figure 2), with a focus on the variation of the wellbore pressure. The study also investigates how changes in absolute permeability and fluid viscosity (a in Equation (4)) affect the pressure response over time (see Figure 3). By systematically varying these parameters, we aim to elucidate their role in controlling the coupled thermo-hydraulic behavior of the reservoir. This analysis provides insights into the com-

bined effects of rock and fluid properties under both thermal and non-thermal transient conditions, contributing to a more comprehensive understanding of well–reservoir interactions.

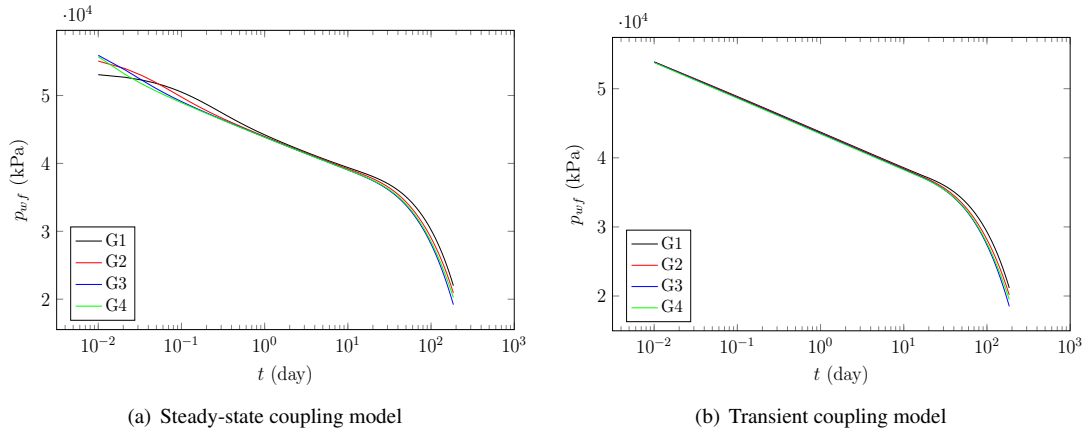


Figure 1. Time evolution of producer well pressure (p_{wf}) with heating

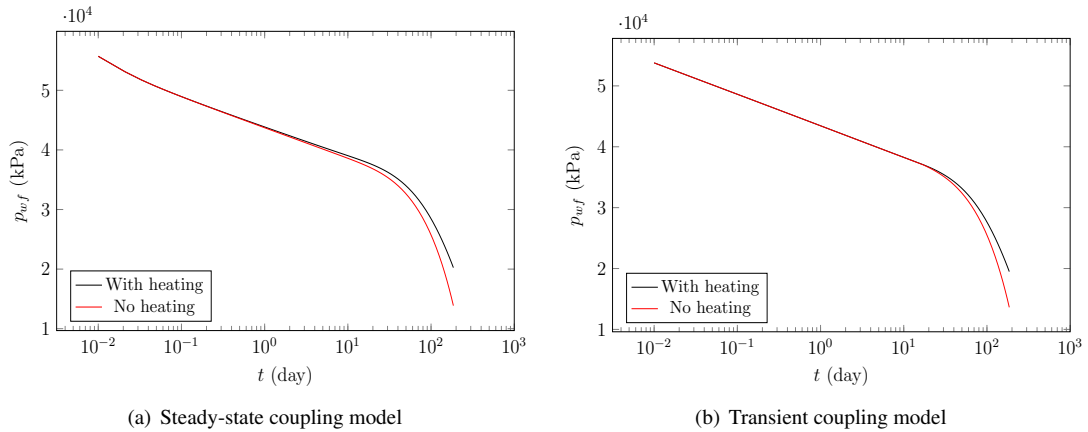


Figure 2. Time evolution of producer well pressure (p_{wf}) with and without heating

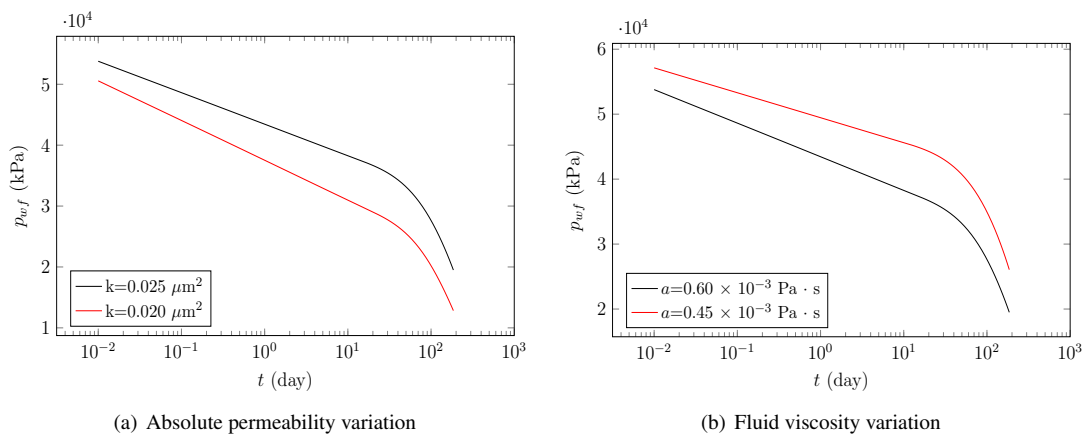


Figure 3. Time evolution of producer well pressure (p_{wf}) for the transient coupling model with heating

From Figure 2, we observe that the main differences when employing the coupling model assuming steady-state conditions occur at early times. After a sufficiently long period, both models yield pressure values that are practically identical. We also highlight the effect of heating on the pressure variation at the producer well in both coupling models, which helps maintain higher pressure values compared to scenarios without reservoir heating. Consequently, this contributes to sustaining production over a more extended period.

Finally, Figure 3 shows that the expected behavior is obtained when varying the values of absolute permeability and viscosity, through the variation of the parameter a in Equation (4). The higher the permeability, the lower the resistance to fluid flow within the reservoir, resulting in a smaller pressure drop at the producer well. Regarding viscosity, the less viscous the fluid (i.e., the lower the value of a), the smaller the pressure drop will be, since the fluid can flow more easily through the reservoir.

5 Conclusions

In this study, a numerical framework was successfully developed and applied to model non-isothermal, single-phase flow in porous reservoirs equipped with static heating wells. By advancing beyond classical hydrodynamic formulations, our approach systematically integrated hydrodynamic interactions between the wellbore and the reservoir, leading to a more complete representation of the system in non-isothermal conditions. The use of a fully implicit time scheme, combined with an operator splitting method and the Conjugate Gradient solver, proved effective in efficiently determining pressure and temperature distributions. Through mesh refinement tests and sensitivity analyses, we validated the model's accuracy and stability across various physical and operational parameters. The results conclusively demonstrate that a comprehensive hydrodynamic coupling framework in non-isothermal conditions is essential for enhancing simulation fidelity and providing a more accurate assessment of system performance, thereby underscoring the critical role of these advanced techniques in modern reservoir simulation.

Acknowledgements. This study was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior – Brasil (CAPES) – Finance Code 001.

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